A Dynamic Condensation for Tall Buildings with Active Tuned Mass Damper

능동 동조질량감쇠의 고층빌딩 해석을 위한 동적압축법

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국문요약

자유도 수가 많은 고층 구조물의 해석하기 위해 모든 층에 sensors를 설치하는 것은 비 실용적이다. 따라서 이러한 문제를 해결하기 위해 "reduced-order control" 방법이 소개되었다. 본 논문은 동적압축법(dynamic condensation method)이 제안되었다. 이 압축법은 반복법으로 "Guyan condensation"의 initial approximation을 적용하였다. 본 논문에서 제시된 동적압축법(dynamic condensation)은 원하는 값을 얻을 때까지 지속적으로 updated가 되며, 결과는 기존의 "Guyan condensation"보다 정확한 결과를 나타내었다. 또한 "eigenvalue shifting technique"을 적용하여 iteration으로 계산되는 시간을 크게 단축하였다. "Reduced-order system"을 도입하기 위한 두가지 schemes이 토의되었다. 제시된 동적압축법 효과의 증명을 위해 능동 동조질량감쇠 고층빌딩의 수치 해석이 토의되었고, 단지 두 번의 반복(iterations)을 통한 결과는 매우 정확한 것으로 나타났다.

주요어 : 능동제어, 동적압축법, 동조질량감쇠

ABSTRACT

It is impractical to install sensors on every floor of a tall building to measure the full state vector because of the large number of degrees of freedom. This makes it necessary to introduce reduced order control. A kind of system reduction scheme (dynamic condensation method) is proposed in this paper. This method is iterative and Guyan condensation is looked upon as an initial approximation of the iteration. Since the reduced order system is updated repeatedly until a desired one is obtained, the accuracy of the reduced order system resulting from the proposed method is much higher than that obtained from the Guyan condensation method. An eigenvalue shifting technique is applied to accelerate the convergence of iteration. Two schemes to establish the reduced order system by using the proposed method are also presented and discussed in this paper. The results for a tall building with active tuned mass damper show that the proposed method is efficient for the reduced order modelling and the accuracy is very close to exact only after two iterations.

Key words : active control, dynamic condensation, tuned mass damper

1. Introduction

The response of tall buildings under strong wind turbulence or earthquake has a severe influence on the structural safety and comfort of occupants. Hence a variety of control algorithms have been developed to control the displacement and acceleration responses. Among them, the active vibration control has received considerable attention by a lot of researchers. Different active control devices have been investigated and constructed in the U.S., Japan and elsewhere during the last two decades (Soong et al.[1], Abiru et al.[2]). The principle of the active control is to provide external corrective forces in strategic points in the structure to constrain the response within predetermined performance limits(Soong[3],).

Active bracing systems and active variable stiffness systems are systems built of conventional structural components of structures enhanced with external forces that modified either the effective damping, or the natural frequency of the system to produce more efficient vibration suppression. However, the use of such systems requires an intervention or modification of the structural systems which is usually prohibitive in the existing large structures (Cao et al.[4]). Mass dampers, which include tuned mass dampers (TMDs) and active tuned mass dampers (ATMDs), are additional large weights added to the structure or isolated from the main structural system. They can absorb energy transferred from the main structure through their large movement. Their action is similar to increasing the damping in the main structure. The advantage of this kind of system is that it does not require major intrusion in the structural system. Therefore, it is suitable for retrofit cases. Although the performance of TMDs has been dem-

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onstrated to be quite effective, passive dampers may not achieve satisfactory performance for some structures with space and physical limitations, and hence ATMDs are necessary (Ankireddi et al.[5]).

For the reduced-order control, a reduced-order system should be derived from the full-order structural system at first by using system reduction scheme. Many kinds of system reduction methods, such as optimal projection (Wilson[6]; and Hyland et al.[7]), critical mode reduction (Yang et al.[8]), Guyan condensation (Guyan[9]) and so on, have been proposed during the past three decades.

In the Guyan condensation method, the total degrees of freedom of the full model are divided into two parts which are called the master and slave degrees of freedom. The former will be retained in the reduced model and the latter will be condensed out. Then a condensation matrix is used to transfer the full model into the reduced model defined in the subspace which is defined by the master degrees of freedom. The advantage of the condensation method is that the corresponding reduced-order system is defined in the subspace of the physical space of the full-order system; hence, the coordinate has specific physical meaning. Due to this, the technique has been broadly used for the active vibration control of journal bearing (Sun et al.[10]), spacecraft solar array (Sowmianarayanan et al.[11]), flexible payloads (Gaudenzi et al.[12]).

Unfortunately, the dynamic effects are ignored in the Guyan condensation matrix. It is only exact for static problems. For dynamic problems, the accuracy is usually much lower and fully depends on the selection of the master and slave degrees of freedom. If the chosen master degrees of freedom are improper, the accuracy will be very low. Hence, several selection schemes were proposed (Henshell et al.[13], Matta[14]). According to the rules of the condensation technique (Qu[15]), the degrees of freedom on which the actuators and sensors are mounted and on which the displacements (velocities and accelerations) are dependent should be selected as the master degrees of freedom. This makes these selection schemes fully or partially invalid for active control systems.

To improve the accuracy of the Guyan condensation, a dynamic condensation method is first proposed to establish the reduced-order system in this paper. This method is derived directly from the eigenvalue equation of the full-order system. It is an iterative method and the static condensation (Guyan[9]) is looked upon as an initial approximation of the reduced-order system. Hence, the accuracy of reduction is much higher than the static. Then the method is used for the vibration control of a tall building with an active tuned mass damper. Finally, numerical examples, a tall building, a tall building with TMD and a tall building with ATMD, are included to demonstrate the efficiency of the proposed method.

2. System Reduction Scheme

The general form of dynamic equations for a structure or system can be written as

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(t) \tag{1}$$

where, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices of size $n \times n$, respectively. $\mathbf{M}$ is assumed to be positive definite. $\mathbf{F}(t)$ is an external force vector. $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$ are the displacement, velocity and acceleration vector of the system. For a structure, especially for a large and/or complex structure, the damping matrix is usually assumed to be proportional to stiffness and/or mass matrices, that is

$$\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M} \tag{2}$$

where α and β are constants.

The general eigenvalue problem of this structure can be expressed as

$$\mathbf{K\Phi} = \mathbf{M\Phi\Omega} \tag{3}$$

where $\mathbf{\Phi}$ and $\mathbf{\Omega}$ are, respectively, the eigenvector and eigenvalue matrices of size $n \times n$. The eigenvalues in the matrix are arranged in ascending order. When an eigenvalues-shifting technique is applied to Eq. (3), one has

$$\mathbf{D\Phi} = \mathbf{M\Phi\Lambda} \tag{4}$$

where

$$\mathbf{D} = \mathbf{K} - q\mathbf{M} \quad \mathbf{\Lambda} = \mathbf{\Omega} - q\mathbf{I} \tag{5}$$

and q is an eigenvalue-shifting value and has two functions: (1) accelerating the convergence of iteration and (2) making the reduced-order system close to the dynamic characteristics of the full order in any given frequency range $[\omega_{\min}, \omega_{\max}]$ or around a given frequency ω. For case 1, q should satisfy

$$0 \le q < \lambda_1 \tag{6}$$

where λ_1 is the lowest eigenvalue of the full-order system. For the second case,

$$q = \frac{\omega_{\min}^2 + \omega_{\max}^2}{2} \quad \text{or} \quad q = \omega^2 \tag{7}$$

If only m eigenvalues and eigenvectors are considered, Eq. (4) can be rewritten as

$$\mathbf{D\Phi}_{\mathrm{m}} = \mathbf{M\Phi}_{\mathrm{m}}\mathbf{\Lambda}_{\mathrm{mm}} \tag{8}$$

Matrices $\mathbf{\Phi}_{\mathrm{m}} \in R^{n \times m}$ include the m eigenvectors and eigenvalues, respectively.

Suppose that the total degrees of freedom (n) of the full system is divided into master degrees of freedom (m), which will be retained in the reduced model, and slave degrees of freedom (s), which will be condensed. The numbers of the two groups are m and s, respectively. According to the practical rules of the condensation technique, the degrees of freedom on which the actuators and sensors are mounted and on which the displacements (velocities and accelerations) are dependent should be selected as the master degrees of freedom. Based on this division, Eq. (8) can be partitioned as

$$\begin{bmatrix} \mathbf{D}_{\mathrm{mm}} & \mathbf{D}_{\mathrm{ms}} \\ \mathbf{D}_{\mathrm{sm}} & \mathbf{D}_{\mathrm{ss}} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{\mathrm{mm}} & \mathbf{M}_{\mathrm{ms}} \\ \mathbf{M}_{\mathrm{sm}} & \mathbf{M}_{\mathrm{ss}} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{bmatrix} \mathbf{\Lambda}_{\mathrm{mm}} \tag{9}$$

After rearranging the second equation of Eq. (9), one has

$$\mathbf{\Phi}_{\mathrm{sm}} = \mathbf{D}_{\mathrm{ss}}^{-1}[\mathbf{M}_{\mathrm{sm}}\mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Lambda}_{\mathrm{mm}} + \mathbf{M}_{\mathrm{ss}}\mathbf{\Phi}_{\mathrm{sm}}\mathbf{\Lambda}_{\mathrm{mm}} - \mathbf{D}_{\mathrm{sm}}\mathbf{\Phi}_{\mathrm{mm}}] \tag{10}$$

Define system reduction matrix $\mathbf{R} \in \mathrm{R}^{n \times m}$, which relates the deformations or eigenvectors associated with the master and slave degrees of freedom, as

$$\mathbf{\Phi}_{\mathrm{sm}} = \mathbf{R\Phi}_{\mathrm{mm}} \tag{11}$$

Substituting Eq. (11) into both sides of Eq. (10) and then postmultiplying both sides by the inversion of matrix $\mathbf{\Phi}_{\mathrm{mm}}$, we get

$$\mathbf{R} = \mathbf{D}_{\mathrm{ss}}^{-1}[(\mathbf{M}_{\mathrm{sm}} + \mathbf{M}_{\mathrm{ss}}\mathbf{R})\mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Lambda}_{\mathrm{mm}}\mathbf{\Phi}_{\mathrm{mm}}^{-1} - \mathbf{D}_{\mathrm{sm}}] \tag{12}$$

If we let $q = 0$ and all the eigenvalues in matrix $\mathbf{\Omega}_{\mathrm{mm}}$ are zero which means ignoring the dynamic effects, Eq. (12) becomes

$$\mathbf{R}_{\mathrm{G}} = -\mathbf{K}_{\mathrm{ss}}^{-1}\mathbf{K}_{\mathrm{sm}} \tag{13}$$

This is the so-called Guyan condensation matrix. Obviously, it is exact for static problems. For dynamic problems, the accuracy will reduce with the increase of the eigenvalues.

For the reduced system, the eigenvalue problem can be expressed as

$$\mathbf{K}_{\mathrm{R}}\mathbf{\Phi}_{\mathrm{mm}} = \mathbf{M}_{\mathrm{R}}\mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Omega}_{\mathrm{mm}} \tag{14}$$

where $\mathbf{M}_{\mathrm{R}}$ and $\mathbf{K}_{\mathrm{R}} \in \mathrm{R}^{m \times m}$ are the mass and stiffness matrices of the reduced model, respectively. Similarly, when an eigenvalue shift is introduced into Eq. (14), one has

$$\mathbf{D}_{\mathrm{R}}\mathbf{\Phi}_{\mathrm{mm}} = \mathbf{M}_{\mathrm{R}}\mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Lambda}_{\mathrm{mm}} \tag{15}$$

The following equation can be obtained from Eq. (15)

$$\mathbf{M}_{\mathrm{R}}^{-1}\mathbf{D}_{\mathrm{R}} = \mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Lambda}_{\mathrm{mm}}\mathbf{\Phi}_{\mathrm{mm}}^{-1} \tag{16}$$

Introducing Eq. (16) into the right-hand side of Eq. (12) one has

$$\mathbf{R} = \mathbf{D}_{\mathrm{ss}}^{-1}[(\mathbf{M}_{\mathrm{sm}} + \mathbf{M}_{\mathrm{ss}}\mathbf{R})\mathbf{M}_{\mathrm{R}}^{-1}\mathbf{D}_{\mathrm{R}} - \mathbf{D}_{\mathrm{sm}}] \tag{17}$$

Eq. (15) is the governing equation of the system reduction matrix. It can be rewritten in an iterative form as

$$\begin{aligned} \mathbf{R}^{(i+1)} &= \mathbf{D}_{\mathrm{ss}}^{-1}[(\mathbf{M}_{\mathrm{sm}} + \mathbf{M}_{\mathrm{ss}}\mathbf{R}^{(i)})(\mathbf{M}_{\mathrm{R}}^{(i)})^{-1}\mathbf{D}_{\mathrm{R}}^{(i)} - \mathbf{D}_{\mathrm{sm}}] \\ \mathbf{R}^{(0)} &= -\mathbf{D}_{\mathrm{ss}}^{-1}\mathbf{D}_{\mathrm{sm}} \end{aligned} \tag{18}$$

where $i = 0, 1, \ldots$.

Since the proportional damping assumption is used in Eq. (1), the dynamic equation can be uncoupled in real modal space and the dynamic responses can be obtained by using modal superposition, that is,

$$\mathbf{X}(t) = \mathbf{\Phi}_{\mathrm{m}}\mathbf{q}(t) \tag{19}$$

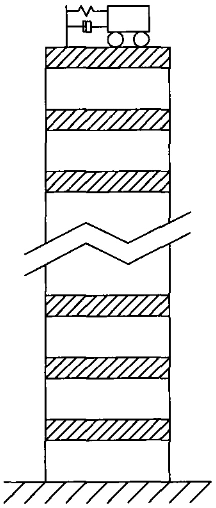

Figure 1 Tall building with ATMD.

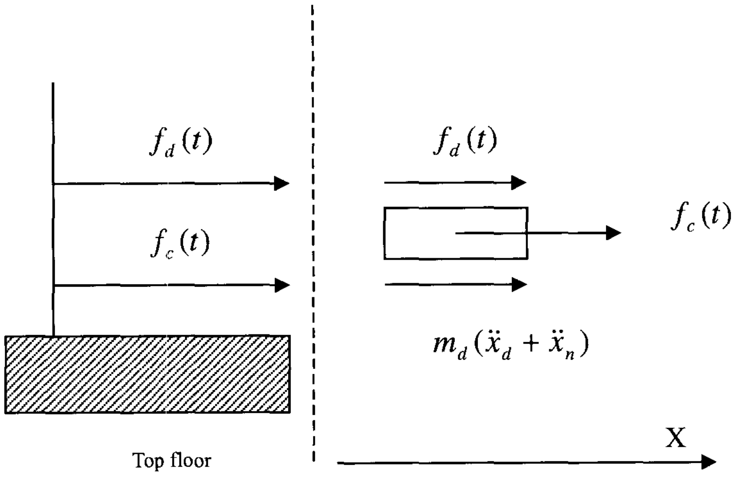

Figure 2 Forces acting on the active control system.

$\mathbf{q}(t)$ is the displacement of the structure in modal space. If the same division of the degrees of freedom is used, Eq. (19) becomes

$$\begin{Bmatrix} \mathbf{X}_{\mathrm{m}}(t) \\ \mathbf{X}_{\mathrm{s}}(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{bmatrix} \mathbf{q}(t) \tag{20}$$

which is equivalent to

$$\mathbf{X}_{\mathrm{m}}(t) = \mathbf{\Phi}_{\mathrm{mm}}\mathbf{q}(t),\ \ \mathbf{X}_{\mathrm{s}}(t) = \mathbf{\Phi}_{\mathrm{sm}}\mathbf{q}(t) \tag{21}$$

Introducing Eq. (11) into Eq. (20) and considering Eq. (21), one has

$$\begin{Bmatrix} \mathbf{X}_{m}(t) \\ \mathbf{X}_{s}(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{mm} \\ \mathbf{\Phi}_{sm} \end{bmatrix} \mathbf{q}(t) = \begin{bmatrix} \mathbf{I} \\ \mathbf{R} \end{bmatrix} \mathbf{\Phi}_{mm}\mathbf{q}(t) = \begin{bmatrix} \mathbf{I} \\ \mathbf{R} \end{bmatrix} \mathbf{X}_{m}(t) \equiv \mathbf{T}\mathbf{X}_{m}(t) \tag{22}$$

where the coordinate transformation matrix $\mathbf{T}$ is defined as

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} \\ \mathbf{R} \end{bmatrix} \tag{23}$$

The transformation matrix is independent of time t, hence we have

$$\dot{\mathbf{X}}(t) = \mathbf{T}\dot{\mathbf{X}}_{m}(t),\ \ \ddot{\mathbf{X}}(t) = \mathbf{T}\ddot{\mathbf{X}}_{m}(t) \tag{24}$$

Substituting Eqs. (22) and (24) into Eq. (1) and premultiplying both sides by the transpose of matrix **T**, one has

$$\mathbf{M}_{\mathrm{R}}\ddot{\mathbf{X}}_{m} + \mathbf{C}_{\mathrm{R}}\dot{\mathbf{X}}_{m} + \mathbf{K}_{\mathrm{R}}\mathbf{X}_{m} = \mathbf{F}_{\mathrm{R}} \tag{25}$$

where the mass, damping, and stiffness matrices and the force vector of the reduced model are defined as

$$\mathbf{M}_{\mathrm{R}} = \mathbf{T}^{\mathrm{T}}\mathbf{M}\mathbf{T},\ \mathbf{C}_{\mathrm{R}} = \mathbf{T}^{\mathrm{T}}\mathbf{C}\mathbf{T},\ \mathbf{K}_{\mathrm{R}} = \mathbf{T}^{\mathrm{T}}\mathbf{K}\mathbf{T},\ \mathbf{F}_{\mathrm{R}} = \mathbf{T}^{\mathrm{T}}\mathbf{F} \tag{26}$$

Since the number of master degrees of freedom is much smaller than the total, it will be very efficient when we use the reduced model to design the active vibration control system.

3. Full Order Control

A tall building with ATMD, as shown in Fig. 1, is considered here. The forces acting on the active control system are shown in Fig. 2. x_n and $x_n + x_d$ are the absolute displacements of the top storey and ATMD. $f_c(t)$ is an active control force acting on the ATMD. It is an internal force for the system, hence the top storey will be subjected a reactive force.

Supposing that the tall building has n degrees of freedom, the dynamic equations of this system are

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(t) + \mathbf{D}f_d(t) - \mathbf{D}f_c(t) \tag{27}$$

$$m_{\mathrm{d}}[\ddot{x}_{\mathrm{d}}(t) + \ddot{x}_n(t)] + f_{\mathrm{d}}(t) = f_{\mathrm{c}}(t) \tag{28}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the building, respectively. $\mathbf{F}(t)$ is an excited force vector. $f_{\mathrm{d}}(t)$ and the force distribution vector $\mathbf{D}$ are defined as

$$f_{\mathrm{d}}(t) = c_{\mathrm{d}}\dot{x}_{\mathrm{d}}(t) + k_{\mathrm{d}}x_{\mathrm{d}}(t) \tag{29}$$

$$\mathbf{D}^{\mathrm{T}} = \{\underbrace{0,0,\ldots,0,1}_{n-1}\} \tag{30}$$

By using matrix form, Eqs. (27)-(30) can be expressed as

$$\overline{\mathbf{M}}\ddot{\overline{\mathbf{X}}}(t) + \overline{\mathbf{C}}\dot{\overline{\mathbf{X}}}(t) + \overline{\mathbf{K}}\overline{\mathbf{X}}(t) = \overline{\mathbf{F}}(t) + \overline{\mathbf{D}}f_c(t) \tag{31}$$

where

$$\overline{\mathbf{M}} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} & 0 \\ m_{21} & m_{22} & \cdots & m_{2n} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} & 0 \\ 0 & 0 & \cdots & m_d & m_d \end{bmatrix}, \quad \overline{\mathbf{C}} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} & 0 \\ c_{21} & c_{22} & \cdots & c_{2n} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} & -c_d \\ 0 & 0 & \cdots & 0 & m_d \end{bmatrix} \tag{32a}$$

$$\overline{\mathbf{K}} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} & 0 \\ k_{21} & k_{22} & \cdots & k_{2n} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} & -k_d \\ 0 & 0 & \cdots & 0 & k_d \end{bmatrix}, \quad \overline{\mathrm{X}}(\mathrm{t}) = \begin{Bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \\ x_d(t) \end{Bmatrix}, \quad \overline{\mathbf{D}} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ -1 \\ 1 \end{Bmatrix}, \quad \overline{\mathbf{F}} = \begin{Bmatrix} \mathrm{F}(\mathrm{t}) \\ 0 \end{Bmatrix} \tag{32b}$$

Eq. (31) can be rewritten in state space as

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}f_c(t) + \mathbf{V}(t) \tag{33}$$

where system matrix $\mathbf{A} \in R^{2(n+1)\times 2(n+1)}$, control vector $\mathbf{B} \in R^{2(n+1)}$, load vector $\mathbf{V}(t) \in R^{2(n+1)}$, and state vector $\mathbf{Z}(t) \in R^{2(n+1)}$ are, respectively, defined as

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\overline{\mathbf{M}}^{-1}\overline{\mathbf{K}} & -\overline{\mathbf{M}}^{-1}\overline{\mathbf{C}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{M}}^{-1}\overline{\mathbf{D}} \end{bmatrix}, \quad \mathbf{V}(t) = \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{M}}^{-1}\overline{\mathbf{F}}(t) \end{bmatrix},$$

$$\mathbf{Z}(t) = \begin{Bmatrix} \overline{\mathbf{X}}(t) \\ \dot{\overline{\mathbf{X}}}(t) \end{Bmatrix} \tag{34}$$

There are many control algorithms for the active vibration control of tall buildings. The LQR (linear quadratic regulator) is only considered here for simplicity. The regular problem is defined as (Meirovitch[16]) the problem of designing a control input so as to drive the structures from some initial state to the zero state. The linear regulator problem is the one in which the control is a linear function of the state. According to the definition of the LQR, one has

$$\mathbf{V}(t) = 0 \tag{35}$$

The object of LQR is to determine an optimal control minimizing the quadratic performance measure

$$J = \frac{1}{2}\mathbf{Z}^{\mathrm{T}}(t_{\mathrm{f}})\mathbf{S}\mathbf{Z}(t_{\mathrm{f}}) + \frac{1}{2}\int_0^{t_f}\left[\mathbf{Z}^{t}(t)\mathbf{Q}(t)\mathbf{Z}(t) + f_c U(t) f_c(t)\right]\mathrm{d}t \tag{36}$$

where the weighting matrices $\mathbf{S}$ and $\mathbf{Q}$ of state variable are real symmetric positive semidefinite. U is usually a real symmetric positive-definite matrix. However, it is a positive variable for the control model considered here. t_{f} is the end of integration time. A minimization of the performance index J in Eq. (36) subject to the constraint of Eq. (33) results in the well-known LQR controller (Meirovitch, 1990),

$$f_c(t) = -\mathbf{\Gamma}\mathbf{Z}(t) = -U^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) \tag{37}$$

in which $\mathbf{P}$ is the Riccati matrix satisfying the algebraic Riccati equation

$$\mathbf{PA} + \mathbf{A}^{\mathrm{T}}\mathbf{P} - \mathbf{PB}U^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{38}$$

It can be proven that Eq. (38) has a unique positive definite solution as long as (A, Q) is observable. $\mathbf{\Gamma}$ is the state feedback matrix. The dynamic equation of the corresponding close-loop control system is

$$\dot{\mathbf{Z}}(t) = \left[\mathbf{A} - \mathbf{B}U^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\right]\mathbf{Z}(t) \tag{39}$$

4. Reduced Order Control

There are two ways to obtain the reduced-order system by using the proposed system reduction scheme. One is to reduce the global dynamic model of building and ATMD. The other is to reduce the dynamic model of building at first and then to construct the reduced-order system by using ATMD and the reduced model of building. There are advantages and disadvantages for both ways. The first way is a little more simple and direct than the latter and the reduced-order system obtained from it can retain all the dynamic characteristics of the full-order system in a given frequency range. However, when the parameters of the ATMD change, the reduced-order system should be reformulated from the beginning. This makes it inconvenient to optimize the parameters of the ATMD and it is usually used in a specific system. For the second way, only several variables should be modified when the parameters of ATMD change. In order to be concise, only the second way is considered in the following.

Suppose that the reduced-order mass, damping and stiffness matrices of the tall building are, respectively, $\mathbf{M}_R$, $\mathbf{C}_R$ and $\mathbf{K}_R$. The dynamic equations of the reduced system are

$$\overline{\mathbf{M}}_R \ddot{\overline{\mathbf{X}}}_m(t) + \overline{\mathbf{C}}_R \dot{\overline{\mathbf{X}}}_m(t) + \overline{\mathbf{K}}_R \overline{\mathbf{X}}_m(t) = \overline{\mathbf{F}}_R(t) + \overline{\mathbf{D}}_R f_c(t) \quad (40)$$

where matrices or vectors $\overline{\mathbf{M}}_R$, $\overline{\mathbf{C}}_R$, $\overline{\mathbf{K}}_R$, $\overline{\mathbf{X}}_m(t)$, $\overline{\mathbf{F}}_R$, and $\overline{\mathbf{D}}_R$ are obtained from Eqs. (31) and (29) by replacing m, c, k, n and $\mathbf{F}$ by m^R, c^R, k^R, m and $\mathbf{F}_R$, respectively. Eq. (39) can be rewritten in state space as

$$\dot{\mathbf{Z}}_m(t) = \mathbf{A}_R \mathbf{Z}_m(t) + \mathbf{B}_R f_c(t) + \mathbf{V}_R(t) \quad (41)$$

The system matrix $\mathbf{A}_R \in R^{2(m+1)\times 2(m+1)}$, control vector, force vector $\mathbf{B} \in R^{2(m+1)}$, force vector $\mathbf{V}(t) \in R^{2(m+1)}$ and state vector $\mathbf{Z}_m(t) \in R^{2(m+1)}$are defined as

$$\mathbf{A}_R = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\overline{\mathbf{M}}_R^{-1}\overline{\mathbf{K}}_R & -\overline{\mathbf{M}}_R^{-1}\overline{\mathbf{C}}_R \end{bmatrix}, \quad \mathbf{B}_R = \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{M}}_R^{-1}\overline{\mathbf{D}}_R \end{bmatrix} \quad (42a)$$

$$\mathbf{V}_R(t) = \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{M}}_R^{-1}\overline{\mathbf{F}}_R(t) \end{bmatrix}, \quad \mathbf{Z}_m(t) = \begin{Bmatrix} \overline{\mathbf{X}}_m(t) \\ \dot{\overline{\mathbf{X}}}_m(t) \end{Bmatrix} \quad (42b)$$

Similarly, we have the following equations by using the LQR control technique

$$f_c(t) = -\mathbf{\Gamma}_R \mathbf{Z}_m(t) = -U_R^{-1}\mathbf{B}_R^T \mathbf{P}_R \mathbf{Z}_m(t) \quad (43)$$

$$\mathbf{P}_R\mathbf{A}_R + \mathbf{A}_R^T\mathbf{P}_R - \mathbf{P}_R\mathbf{B}_R U_R^{-1}\mathbf{B}_R^T\mathbf{P}_R + \mathbf{Q}_R = \mathbf{0} \quad (44)$$

$$\dot{\mathbf{Z}}_m(t) = \left[\mathbf{A}_R - \mathbf{B}_R U_R^{-1}\mathbf{B}_R^T\mathbf{P}_R\right]\mathbf{Z}_m(t) \quad (45)$$

5. Numerical Example

The proposed method has been tested on a 40-storey building with ATMD shown in Fig. 1. Each storey unit of the building is identically constructed with a storey height of 4m, mass m_i = 1290 tonnes, stiffness k_i = 106 kN/m, and damping c_i = 14260 kNs/m for $i = 1, 2, \cdots, 40$. The building is symmetric in both lateral directions and the mass centre coincides with the elastic centre, so that there is no coupled lateral-torsional motion. Only one direction motion will be considered. The mass of the damper is 258 tonnes, which is 20 percent of a floor mass. The stiffness and damping coefficient of the damper are 300.9 kN/m and 83.592 kNs/m, respectively.

Suppose that the 5th, 10th, 15th, 20th, 25th, 30th, 35th and 40th floors are selected as the master degrees of freedom when the proposed system reduction scheme is applied to the tall building.

Table 1 Comparison of frequencies (rad/s) of the reduced and full order models of a tall building

Mode	FOS		ROS (i=0)		ROS (i=1)		ROS (i=2)	
	Real	Imag.	Real	Imag.	Real	Imag.	Real	Imag.
1	−0.0083	1.0798	−0.0083	1.0814	−0.0083	1.0798	−0.0083	1.0798
2	−0.0747	3.2369	−0.0767	3.2796	−0.0747	3.2369	−0.0747	3.2369
3	−0.2072	5.3869	−0.2226	5.5831	−0.2072	5.3871	−0.2072	5.3869
4	−0.4049	7.5250	−0.4639	8.0525	−0.4052	7.5273	−0.4049	7.5251
5	−0.6667	9.6465	−0.8230	10.7124	−0.6691	9.6642	−0.6668	9.6479
6	−0.9909	11.7469	−1.3079	13.4804	−1.0067	11.8400	−0.9934	11.7618
7	−1.3756	13.8217	−1.8594	16.0413	−1.4433	14.1543	−1.3960	13.9227
8	−1.8186	15.8666	−2.3006	17.8151	−1.9327	16.3503	−1.8910	16.1755

Table 2 Comparison of frequencies (rad/s) of the reduced and full order models of a tall building with TMD.

Mode	FOS		ROS (i=0)		ROS (i=1)		ROS (i=2)	
	Real	Imag.	Real	Imag.	Real	Imag.	Real	Imag.
1	−0.0294	1.0719	−0.0297	1.0737	−0.0294	1.0719	−0.0294	1.0719
2	−0.1418	1.0771	−0.1416	1.0769	−0.1419	1.0771	−0.1419	1.0771
3	−0.0768	3.2386	−0.0788	3.2814	−0.0767	3.2387	−0.0767	3.2387
4	−0.2090	5.3878	−0.2246	5.5841	−0.2089	5.3880	−0.2089	5.3878
5	−0.4066	7.5256	−0.4660	8.0532	−0.4068	7.5279	−0.4066	7.5257
6	−0.6683	9.6469	−0.8255	10.7129	−0.6708	9.6646	−0.6685	9.6484
7	−0.9925	11.7472	−1.3106	13.4808	−1.0086	11.8403	−0.9951	11.7621
8	−1.3772	13.8219	−1.8620	16.0416	−1.4455	14.1546	−1.3979	13.9230

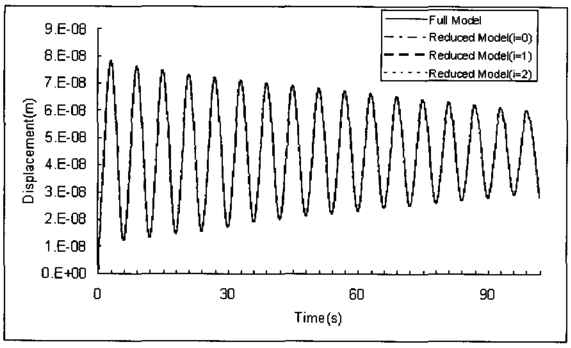

Figure 3 Responses of building with TMD under step load.

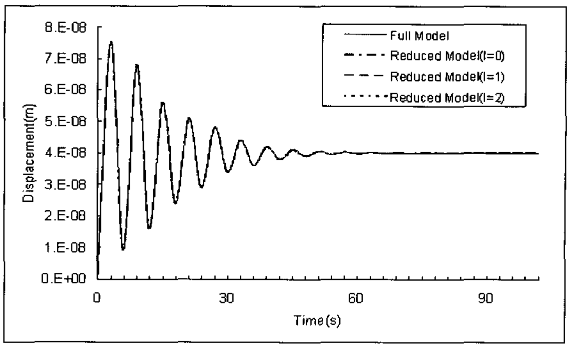

Figure 4 Responses of building with ATMD under step load

Weighting matrices $\mathbf{Q}$ and $\mathbf{Q}_R$ are diagonal. All the diagonal elements corresponding to the master degrees of freedom are 1 and the rest are zero. $U = U_R = 10^{-11}$. The former eight complex frequencies of the reduced-order system of (1) the tall building only, (2) the building with TMD and (3) the building with ATMD are listed in Tables 1, 2 and 3, respectively. The former eight complex frequencies of the corresponding full-order system are also listed in these tables. They are considered as the exact for comparison purpose.

The following three conclusions can be drawn from the results in these tables. (1) The accuracy of the reduced-order system obtained from the initial approximation, static condensation, is very low. Only the former two or three frequencies are close to the exact. (2) The differences between the complex frequencies of the reduced-order system and the former eight frequencies of the full-order system reduce consistently with the increase of the iteration. (3) The frequencies of the reduced-order system are all very close to the full-order system after two or three iterations. Hence, they can accurately replace the dynamic characteristics of the full-order system in low-frequency range.

Assume that there is anunit step load acting on the top floor of the building. The responses of the top floor of the building with TMD and ATMD obtained from the reduced-order and full order systems are shown in Figures 3 and 4, respectively. Obviously, there is a little difference between the initial approximation and the exact, while the responses of the first and second approximations are very close to the exact and it is very difficult to distinguish them from these figures.

6. Conclusions

A system reduction scheme, dynamic condensation method, has been proposed in this paper. Since it is an iterative method and the static condensation method is considered as its initial approximation, the accuracy of the reduced-order system obtained from the present method is much higher than the static. An eigenvalue-shifting technique was also applied to accelerate the convergence and make the reduced-order system close to the dynamic characteristics of the full-order system in any given frequency range. Two schemes for constructing the reduced-order system by using the proposed method were presented. The results of (1) a tall building, (2) a tall building with TMD and (3) a tall building with ATMD showed that

Table 3 Comparison of frequencies (rad=s) of the reduced and full order models of a tall building with ATMD

Mode	FOS		ROS (i=0)		ROS (i=1)		ROS (i=2)	
	Real	Imag.	Real	Imag.	Real	Imag.	Real	Imag.
1	−0.0789	1.0563	−0.0806	1.0578	−0.0788	1.0563	−0.0788	1.0563
2	−0.1249	1.0926	−0.1235	1.0927	−0.1251	1.0927	−0.1251	1.0927
3	−0.0782	3.2386	−0.0804	3.2814	−0.0782	3.2386	−0.0782	3.2386
4	−0.2094	5.3878	−0.2251	5.5841	−0.2094	5.3880	−0.2094	5.3878
5	−0.4068	7.5256	−0.4663	8.0532	−0.4070	7.5279	−0.4068	7.5257
6	−0.6684	9.6469	−0.8257	10.7129	−0.6709	9.6646	−0.6686	9.6483
7	−0.9925	11.7472	−1.3108	13.4807	−1.0087	11.8403	−0.9952	11.7621
8	−1.3772	13.8220	−1.8622	16.0415	−1.4456	14.1545	−1.3980	13.9229

the proposed method was efficient for the reduced -order control and the accuracy was very close to exact only after two iterations.

Notation

$A = (2(n+1)\times 2(n+1))$ system matrix in state space
$B = (2(n+1)\times 1)$ control vector in state space
$c_d =$ damping of active tuned mass damper
$C = (n\times n)$ damping matrix of the full model of building
$\overline{C} = ((n+1)\times(n+1))$ damping matrix of the full model of building with ATMD
$D = (n\times n)$ dynamic stiffness matrix of the full model of building; $(n\times 1)$ force distribution vector of building
$\overline{D} = ((n+1)\times 1)$ force distribution vector of building with ATMD
$f_c =$ active control force
$F = (n\times 1)$ external force vector of building
$\overline{F} = ((n+1)\times 1)$ external force vector of building with ATMD
$I = (n\times n)$ unit matrix
$k_d =$ stiffness of active tuned mass damper
$K = (n\times n)$ stiffness matrix of the full model (of building)
$\overline{K} = ((n+1)\times(n+1))$ stiffness matrix of the full model of building with ATMD
$m_d =$ mass of active tuned mass damper
$M = (n\times n)$ mass matrix of the full model (of building)
$\overline{M} = ((n+1)\times(n+1))$ mass matrix of the full model of building with ATMD
$P = (2(n+1)\times 2(n+1))$ Riccati matrix
$q =$ eigenvalue shifting value
$Q = (2(n+1)\times 2(n+1))$ weighting matrix of state variable
$R = (s\times m)$ dynamic condensation matrix
$S = (2(n+1)\times 2(n+1))$ weighting matrix of state variable
$U =$ weighting of control force
$V = (2(n+1)\times 1)$ load vector in state space
$X = (n\times 1)$ displacement vector of building
$\overline{X} = ((n+1)\times 1)$ displacement vector of building with ATMD
$Z = (2(n+1)\times 1)$ state vector
$\Phi = (n\times n)$ eigenvector matrix
$\Lambda = (n\times n)$ eigenvalue matrix with eigenvalue shifting
$\lambda_1 =$ the lowest eigenvalue of the full model
$\omega =$ frequency (rad/s)
$\Omega = (n\times n)$ eigenvalue matrix

Subscripts

$m =$ parameters associated with the master degrees of freedom
$R =$ parameters associated with the reduced model
$s =$ parameters associated with the slave degrees of freedom

Superscripts

-1 = inverse of matrix
0 = initial approximation
i = ith approximation
T = matrix transpose

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