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An Evolution of Nonlinear Dynamic Response of an Unreinforced Masonry Structure

비보강 조적조의 비선형 동적 거동의 전개

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국문 요약 >> 비보강 조적조는 비균일 재료로 이루어진 합성재료에 가까우므로 그 거동이 하중종류와 구조물의 손상 정도에 따라서 매우 달라지게 된다. 본 연구에서는 여러 차례의 모의 지진을 받는 구조물의 손상거동이 어떻게 전개되어 가는지 살펴보는 간단한 방법을 제시한다. 특히, 시간영역 자료를 여러 구간으로 나누어서 주파수가 어떻게 변화해 가는지 살펴보는 부분 FFT방법을 이용하였다. 또한, 주파수와 강도와의 관계식을 이용하여 단자유도계의 이선형모델도 유도하였다. 본 연구에서 제시하는 이러한 방법들을 이용하여 비보강 조적조의 비탄성 성질들을 합리적으로 구할 수 있었다.

주요어 비보강 조적조, 부분 FFT, 등가 단자유도 시스템, 이선형모델

ABSTRACT >> Unlike homogeneous material structure, the behavior of masonry structure is not perfectly elastic even in the range of small deformations because it is a non-homogeneous and anisotropic composite structural material, consisting of masonry units, mortar, and grout. This paper proposes a simplified way of investigating the evolution of the deformation and damage of the structure subjected to a series of successive ground motions with varying shaking. Especially, the most simple but useful algorithm of Fast Fourier Transformation (FFT) has been adopted to investigate the evolution of the deformation and damage of the structure tested on the shaking table. Moreover, the development of a bi-linear curve for an equivalent SDOF system which is obtained by exploiting the frequency and stiffness relationship was discussed. Finally, some important findings related to inelastic properties of the URM are summarized.

Key words unreinforced masonry structure, windowed FFT, equivalent SDOF system, bi-linear curve

1. Introduction

During the last decades intensive experimental research on unreinforced masonry (URM) structures subjected to seismic loadings has been carried out to understand seismic behavior of URM structures.[(1)] In addition, different testing procedures including static and dynamic, cyclic and monotonic procedures were used to simulate the effects of seismic loads. The experimental test results reveal that the structural performance depends on loading types and the extent of residual damage. Especially, the behavior of masonry structure is not perfectly elastic even in the range of small deformations because it is a *non-homogeneous and anisotropic* composite structural material, consisting of masonry units, mortar, and grout. Such features of URM structures make it difficult to describe nonlinear dynamic responses based on the experimental tests in a parametric formulation with respect to important parameters. In order to understand the realistic extent of probable damage of URM structures against various types of earthquake loadings and site conditions, fragility analysis can be used as comprehensive analytical tools. This fragility analysis considers changes in stiffness and material properties, the variability of seismic source and in-situ soil conditions by performing

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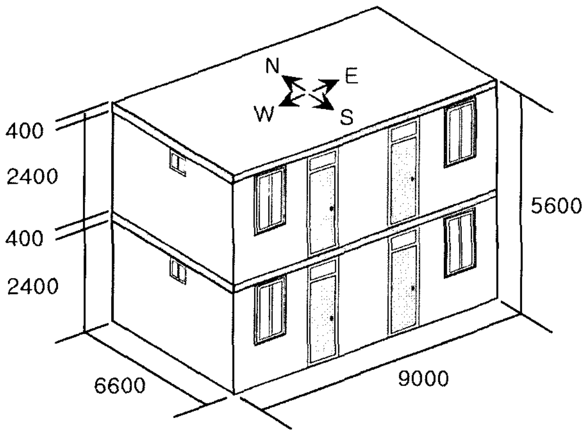
numerous nonlinear time history analyses. However, the nonlinear time history analyses are numerically intensive and it appears as a time and cost-consuming process. This problem has motivated the development of a simplified dynamic model which represents nonlinear properties of URM structures for the fragility analysis to reduce the computational cost associated with nonlinear dynamic analyses.

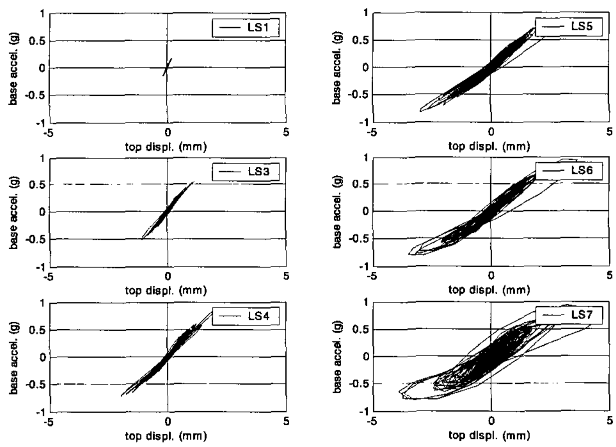
Considering the above issues related to the experimental research on URM structure, this study has mainly focused on the followings: (a) a simplified way of investigating the evolution of the deformation and damage of the structure tested on shaking table; and (2) an equivalent single degree of freedom (SDOF) model using a bi-linear curve. Especially, the experimental results of the shaking table test that were previously presented in (Kim and Kim 2004) were used. An overall behavior of the URM structure was mainly focused in the previous study while gradual changes occurred as the structure experienced mechanical degradation in this paper. For comprehensive investigation of seismic behavior of URM structures, the most simple but useful algorithm of Fast Fourier Transformation (FFT) has been adopted. Then the vibration analysis was further used in the development of an equivalent SDOF system of the URM structure which enables a quantitative assessment of the seismic lateral loading capacity of the test structure in a rational way. When vibration is viewed as a ratio of forces to stiffness, the development of a bi-linear curve can be described by exploiting the frequency and stiffness relationships. Finally, some important findings of inelastic seismic behavior of the URM structures that were obtained by using the proposed windowed FFT analysis and the simplified bi-linear curves are summarized.

2. An Overall Structural Behavior of URM

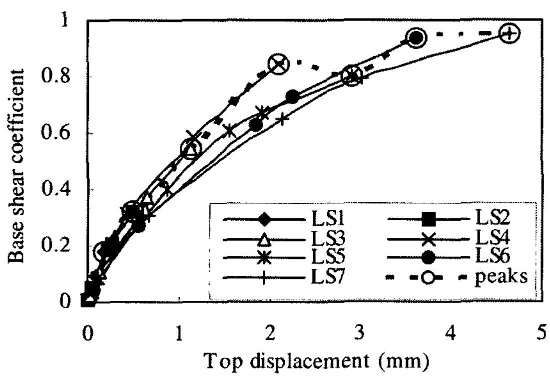
To investigate seismic behavior and damage patterns of URM structure typically constructed in Korea, a series of seven test runs were conducted for the 1/3-scale model



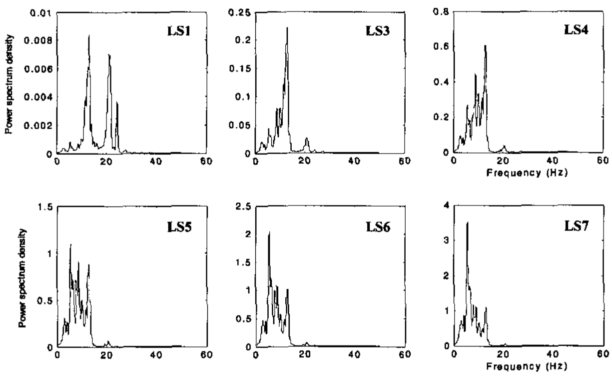
〈Figure 1〉 URM structure (unit:mm) (Kim and Kim 2004)



〈Figure 2〉 Base acceleration vs. top displacement



〈Figure 3〉 Base shear coefficient vs. top displacement

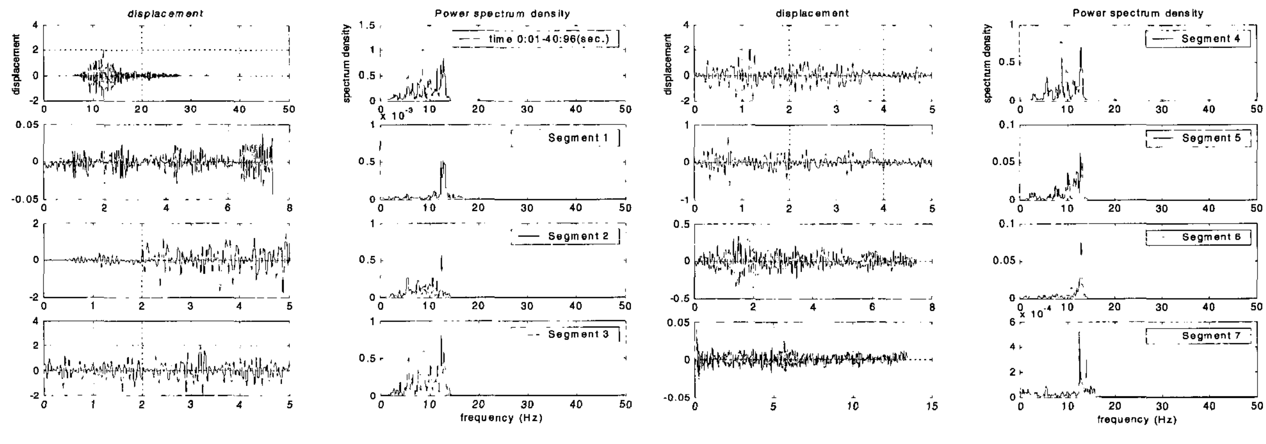


〈Figure 4〉 FFT analysis for the whole excitation of a two-story unreinforced masonry (URM) structure (Figure 1). An overall structural behavior can be summarized in terms of base shear coefficient against top displacement as shown in Figure 2. Also, the natural frequencies of the time history data obtained during the whole excitation of each test run were computed using the FFT analysis. Figure 3 shows dominant natural frequencies for entire time periods which were reduced as structural damage increased. Based on the overall behavior of the URM structure was investigated in the previous study, the detailed investigation into inelastic responses has been addressed in this study.

3. Investigation of Frequency Changes

To investigate changes of frequencies of the test structure during shaking (Kim and Kim 2004), the complete time-history of each load step was analyzed by moving the time window along the whole duration of the test. Prior to the windowed FFT analysis, all the signal data were band-pass filtered according to the following procedures: (1) review of raw data directly obtained from recording instruments; (2) band-pass filterization; (3) baseline correction; and (4) verification of the refined data. All the signal data were band-pass filtered in the frequency ranges of (0.0, 0.5, 14, 19.5). And a size of window was selected as large as to provide an acceptable frequency resolution in the FFT because the ability of the FFT analysis to represent the frequency spectrum of the input signal depends very heavily on the number of the input signal data points being processed. After some trials, the time window duration as shown in Table 1 was determined to realistically capture the evolution of the vibration frequency of the test structure. Some of the signal data were overlapped. Figure 5 shows the windowed FFT analysis results for the displacement time histories in LS4 when the URM structure experienced a certain amount of mechanical damage. It is seen that the frequency for Seg 4 was significantly dropped and the dominant frequency for the whole time-history record is the same as that of Seg 4 where the peak displacement and acceleration of the load step occurred.

Table 2 summarizes the results of frequency analysis made for all the load steps using the displacement and acceleration time-history records. The peak values of displacement and acceleration for each time segment are also provided in the table. The values of the frequency analysis for the whole time-story records provide dominant values of the test structure for each load step while those of time segmental analyses reveal the evolution of frequency change for each load step. Comparison of the two results of frequency analyses for the displacement



⟨Figure 5⟩ Windowed FFT analysis for the displacement time history in LS4.

⟨Table 1⟩ Window time duration.

	Seg 1	Seg 2	Seg 3	Seg 4	Seg 5	Seg 6	Seg 7
Time segment (sec)	0-7.5	7-12	9-14	11-16	15-20	20-27.5	27.5-40.96
Time duration (sec)	(7.5)	(5)	(5)	(5)	(5)	(7.5)	(13.46)

〈Table 2〉 Results of FFT analysis along time windows.

		Whole	Seg 1	Seg 2	Seg 3	Seg 4	Seg 5	Seg 6	Seg 7
LS1 0.05g	D	13.0	5.46	12.4	12.3	13.0	12.9	12.9	12.98
	A	13.0	12.9	12.4	12.3	13.0	12.9	12.9	12.98
	D_m	0.189	0.014	0.1	0.189	0.189	0.038	0.03	0.012
	A_m	0.177	0.008	0.1	0.177	0.177	0.047	0.034	0.012
LS2 0.1g	D	13.0	5.56	12.4	12.3	13.0	12.9	12.9	12.9
	A	13.0	13.6	12.5	12.3	13.0	12.9	12.9	12.9
	D_m	0.502	0.01	0.271	0.502	0.502	0.101	0.049	0.016
	A_m	0.323	0.005	0.204	0.323	0.323	0.09	0.047	0.015
LS3 0.15g	D	13.0	12.9	12.4	12.3	13.0	13.2	12.9	12.9
	A	13.0	12.9	12.4	12.3	13.0	13.2	12.9	12.9
	D_m	1.137	0.022	0.73	1.137	1.137	0.313	0.165	0.025
	A_m	0.546	0.022	0.374	0.546	0.546	0.191	0.113	0.027
LS4 0.2g	D	8.78	12.4	12.4	12.3	8.78	12.9	12.9	12.5
	A	8.78	12.9	12.5	12.3	8.78	12.9	12.95	12.8
	D_m	2.094	0.038	1.412	2.094	2.094	0.663	0.335	0.041
	A_m	0.84	0.038	0.626	0.84	0.84	0.347	0.218	0.041
LS5 0.25g	D	8.78	12.9	11.13	6.784	8.78	12.9	12.9	12.9
	A	8.78	12.9	11.13	6.78	8.78	12.9	12.9	12.9
	D_m	2.897	0.115	1.915	2.897	2.897	0.988	0.534	0.073
	A_m	0.802	0.083	0.67	0.802	0.802	0.365	0.246	0.046
LS6 0.3g	D	5.64	12.9	11.13	6.78	5.7	10.5	12.9	12.5
	A	5.64	12.9	11.13	6.78	5.7	10.5	12.9	12.8
	D_m	3.627	0.062	2.4	3.627	3.627	1.089	0.641	0.09
	A_m	0.938	0.045	0.759	0.938	0.938	0.4	0.272	0.067
LS7 0.35g	D	5.64	12.909	11.127	6.784	5.32	5.539	12.9	12.8
	A	5.64	12.909	11.127	6.784	5.32	5.539	12.9	12.8
	D_m	4.634	0.084	3.02	4.634	4.634	1.244	0.705	0.119
	A_m	0.952	0.064	0.796	0.952	0.952	0.456	0.317	0.076

*D: displacement time history, D_m: the maximum displacement during the test run
*A: acceleration time history, A_m: the maximum displacement during the test run

and acceleration time-histories shows that the test structure oscillated essentially with the same one frequency. Due to this fact, the development of a bi-linear curve for an equivalent SDOF system was motivated for the URM structure. The reduction of the fundamental vibration frequency for the entire time duration was first made during the run of LS4 which was about 23%. Further reduction of the fundamental frequency did not occur during the run of LS5, which indicated that in this case the stiffness degradation was not so dramatic. However, the vibration frequencies of the windowed FFT analysis reveal that there was an event of structural damage occurred during LS5. It has been proved that the windowed FFT analysis provides more beneficial tools to investigate inelastic dynamic responses of the URM structure.

4. Correlation of Frequency with Stiffness

Considering that frequency changes are very closely related with stiffness changes, the results obtained from the frequency analysis of the response of URM can be used to relate the variation of the vibration frequency of the structure with the variation of its mechanical property. The linearization of the response is achieved by defining an average stiffness, K, which depends on the frequency value. When a structure is considered as a single degree of freedom system with mass M and lateral stiffness K, its vibration frequency f is written as follows:

$$f=\frac{1}{2\pi}\sqrt{\frac{K}{M}} \qquad (1)$$

Using the equation (1), if the vibration frequency during a certain time interval is known, the corresponding average stiffness is given by the following equation:

$$K=M(2\pi f)^2 \qquad (2)$$

For the initial frequency f_0, the initial elastic stiffness

⟨Table 3⟩ Frequency changes and corresponding stiffness changes in the run of LS7.

Seg No.	f_i (Hz)	f_i/f_0	K_i/K_0
1	12.909	1.000	1.000
2	11.127	0.862	0.743
3	6.784	0.526	0.276
4	5.320	0.412	0.170
5	5.539	0.429	0.184

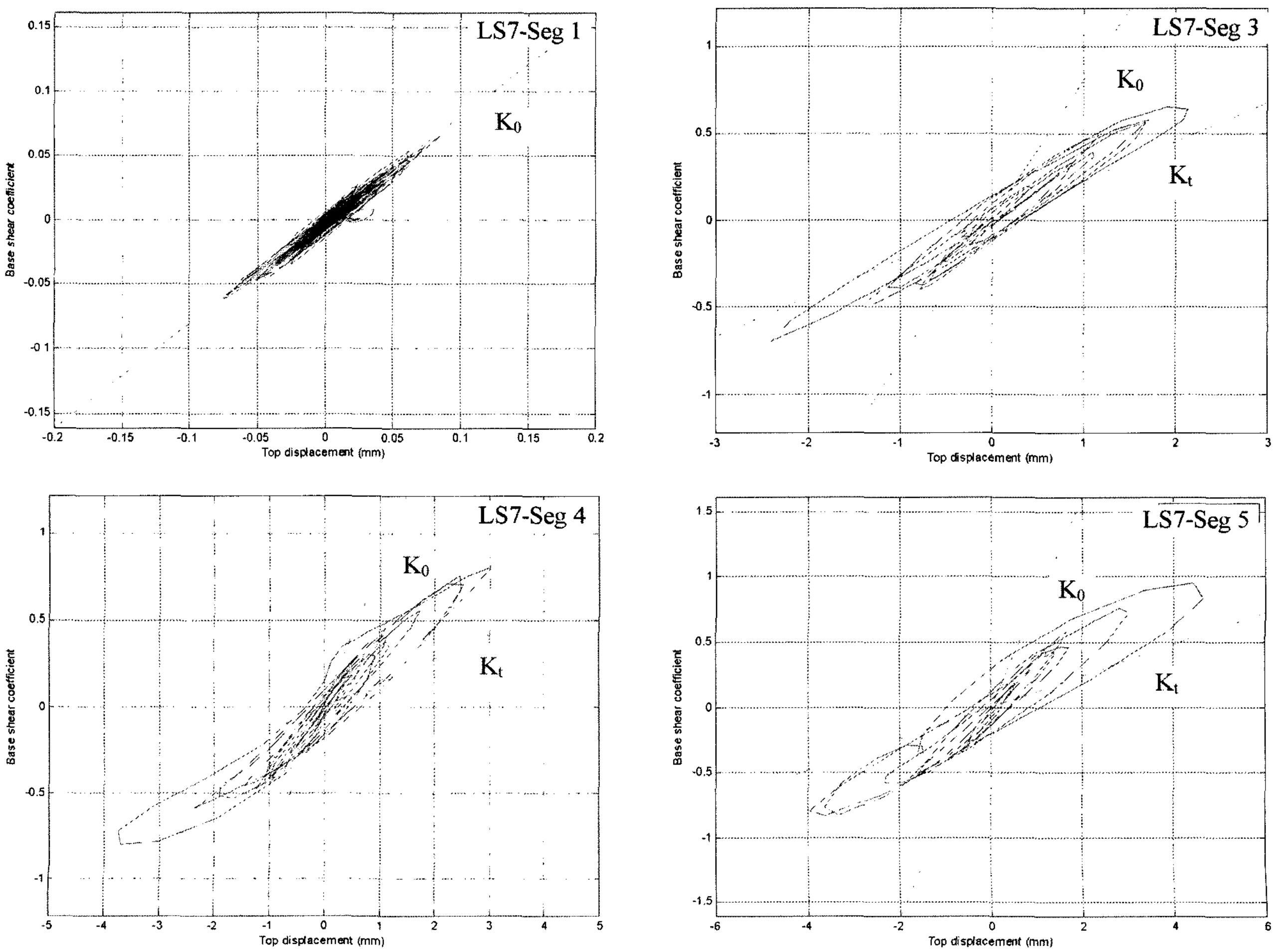
can therefore be evaluated as follows:

$$K_0=M(2\pi f_0)^2 \qquad (3)$$

Combining equations of (2) and (3) yields the following expression:

$$K=K_0(f/f_0)^2 \qquad (4)$$

If the stiffness changes are evaluated using the equations described above, then the stiffness changes can be given in the 3rd and the 4th columns of Table 3 respectively. Figure 6 plots the average stiffness for each segment in comparison with the initial stiffness obtained for the first segment Seg 1. It seems that the slope of the average stiffness obtained by transforming the frequency value is very close to the average slope of force-displacement curve ranging from the maximum values of displacements and accelerations in the previous segment to the maximum values in the current segment. Note that the ratio of the



⟨Figure 6⟩ Stiffness changes during the run of LS7.

average stiffness is closer to that of the tangential values. Therefore, it is possible to state that the average stiffness represents the tangential stiffness of the concerned segment not the stiffness of the whole time duration.

One of the most important parameters for the development of a bi-linear curve is the value of tangential stiffness K_t. Based on the previous observation in Figure 6, the frequency values obtained by the windowed FFT analysis can be used for the calculation of tangential stiffness K_t. To verify this finding in the load-displacement time histories, two different values are prepared for each time segment: (1) the linear regression analysis results for the data of displacement and acceleration and (2) the tangential values for the same data. The comparisons of these values are made in Table 4. Note that the ratio of the average stiffness change in the 2nd column is more close to that of the tangential values in the 4th column. Therefore, it can be clearly stated that the average stiffness represents the tangential stiffness of the concerned segment not for whole time duration.

5. Approximation of Bi-linear Curve

Upon the observation on that measured deflected shapes were nearly in phase despite the amplitude of motion, the development of an equivalent SDOF system of the URM structure was motivated for the nonlinear dynamic response analysis. The development of a bi-linear curve needs of defining its yield resistance R_y, its initial elastic stiffness K_o, the maximum displacement, dmax $d_{\max}$, (not necessarily the maximum displacement during the interval under consideration), and its tangential stiffness K_t, as shown in Figure 6. This study determines the parametric values for the bi-linear curve as follows:

Step 1: The initial elastic stiffness K_0 was determined by using the equation (3).

Step 2: The maximum displacement $d_{\max}$ takes the maximum top displacement occurred during Seg 3 after which the frequency value was greatly reduced in the case of LS7.

Step 3: Given that the elastic stiffness and the maximum displacement values are known, the yield resistance R_y is then determined by multiplying the elastic stiffness by the maximum displacement as in the following equation.

$$R_y = K_0 \cdot d_{\max} \tag{5}$$

Step 4: The slope of the tangential stiffness may be determined grounding on the frequency change, as investigated in Table 5. Note that the tangential stiffness K_t is determined by the representative tangential stiffness for the ranges where the frequency values had been dramatically reduced. Finally, the parameter values for the bilinear curve for URM tested in this study are summarized in Table 6. Also, K_{sec} can be determined as shown in Figure 7.

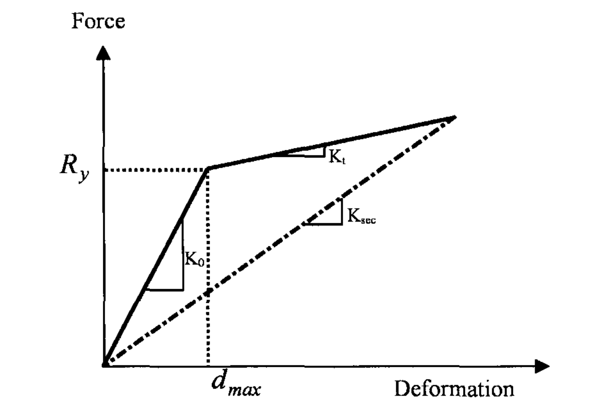
Figure 8 plots the bi-linear curves for each load step from LS1 thru LS7. Note that the initial stiffness has been decreased continuously as the extent of the damage of the test structure has been increased. Also, the bi-linear

⟨Table 4⟩ Comparison of average stiffness changes with the regression analysis results.

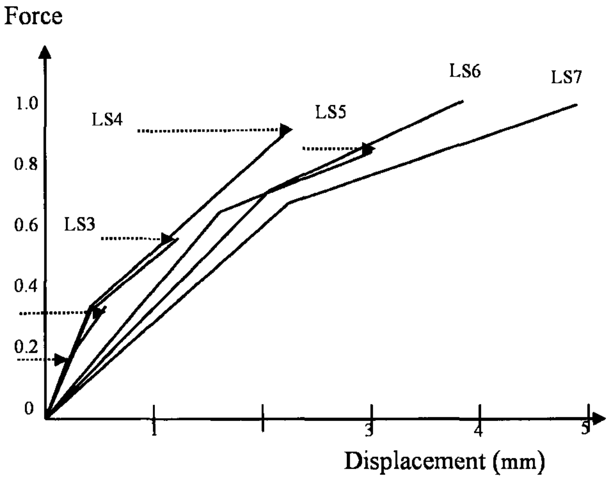
Seg No.	K_i/K_0	$K_{reg,i}/K_{reg,0}$	$K_{\tan,i}/K_{\tan,0}$
1	1.000	1	1
2	0.743	0.607	0.538468
3	0.276	0.415	0.30503
4	0.170	0.359	0.222979
5	0.184	0.297	0.121814

⟨Table 5⟩ Parameter values for the bi-linear curve of URM.

	Parameter	Value	Remarks
Step 1	K_0	7719 kN/mm	$K_0 = M(2\pi f_0)^2$ where, $M = 15.6 kN-\sec/mm,\ f_0 = 11Hz$
Step 2	K_t	1543.8 k/mm	$K_t = 0.2K_0$
Step 3	$d_{\max}$	1.2 mm	Arbitrarily selected from the data of LS3
Step 4	R_y	9263 kN	$R_y = K_0 \cdot d_{\max}$



〈Figure 7〉 A bi-linear model for an equivalent SDOF system.



〈Figure 8〉 Bi-linear curves.

curves represent the structural behavior for the structure which was continuously damaged not for the undamaged. Therefore, the development of numerical analytic model will be different depending on the experimental test results to be used. For example, if the development of the numerical model begins with the test results of LS1, then the pushover curve may follow the dotted line of Figure 8 which accompanies large differences from the other test results.

6. Concluding Remarks

When the1/3-scale URM test structure on the shaking table subjected to a series of successive ground motions with varying shaking intensities, the experimental test results revealed that the structural performance included a considerable amount of residual damage. However, the investigation into nonlinear dynamic responses of URM structures was difficult due to their nonlinearity of non-homogeneous material. Considering the above issues related to the experimental research on URM structure, this paper has proposed the followings: (a) a simplified way of investigating the evolution of the deformation and damage of the structure tested on shaking table; and (2) an equivalent single degree of freedom (SDOF) model using a bi-linear curve. For comprehensive investigation of seismic behavior of URM structures, the most simple but useful algorithm of Fast Fourier Transformation (FFT) was adopted. Then the vibration analysis was further used in the development of an equivalent SDOF system of the URM structure by exploiting the frequency and stiffness relationships. Some important findings of inelastic seismic behavior of the URM structures that were obtained by using the proposed windowed FFT analysis and the simplified bi-linear curves are as follows:

- The windowed FFT analysis provides more beneficial tools to investigate inelastic dynamic responses of the URM structure.
- The frequency values that are obtained using the windowed FFT analysis are more closely related to the tangential stiffness values of the force-displacement under consideration.
- The URM behaved well after the crack was closed to the original position. When the test structure that was damaged but restored to the original shape in the previous loading step subjected to the next level of loading, the structure was able to reserve almost about the same capacity up to the previous load level. That is, the envelope curves in early parts of each loading step were very close to each other.
- Prior to cracking, natural frequency values remained at a constant level while, after cracking, natural frequencies decreased as structural damage, in the form of cracking, increased.
- Post-cracking force-displacement curves were bilinear in shape which is indicative of rocking.
- The development of the bilinear curve for an equivalent SDOF system for the URM was possible using the natural frequency changes during the excitation of the structure.
- A comparison of bi-linear curves of LS4 and LS5

showed a considerable amount of residual damage that remained after LS4 when the severe cracking occurred. However, there still existed substantial strength and deformation capacity after the cracking with some stiffness degradation.

It is expected that the equivalent single degree of freedom system resulted from this study will provide a good starting point toward the development of fragility curves for URM structures consisting of shear walls and rigid diaphragms.

Acknowledgement

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