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LINEAR θ–DERIVATIONS ON JB^*–TRIPLES

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Abstract. In [1], the concept of generalized (θ, ϕ)-derivations on rings was introduced. We introduce the concept of linear θ-derivations on JB^*-triples, and prove the Cauchy–Rassias stability of linear θ-derivations on JB^*-triples.

1. Introduction

The original motivation to introduce the class of nonassociative algebras known as Jordan algebras came from quantum mechanics (see [24]). Let $\mathcal{H}$ be a complex Hilbert space, regarded as the "state space" of a quantum mechanical system. Let $\mathcal{L}(\mathcal{H})$ be the real vector space of all bounded self-adjoint linear operators on $\mathcal{H}$, interpreted as the (bounded) *observables* of the system. In 1932, Jordan observed that $\mathcal{L}(\mathcal{H})$ is a (nonassociative) algebra via the *anticommutator product* $x \circ y := \frac{xy+yx}{2}$. A commutative algebra X with product $x \circ y$ (not necessarily given by an anticommutator) is called a *Jordan algebra* if $x^2 \circ (x \circ y) = x \circ (x^2 \circ y)$ holds.

A complex Jordan algebra $\mathcal{B}$ with product $x \circ y$, unit element e and involution $x \mapsto x^*$ is called a JB^*-algebra if $\mathcal{B}$ carries a Banach space norm $\| \cdot \|$ satisfying $\|x \circ y\| \le \|x\| \cdot \|y\|$ and $\|\{xx^*x\}\| = \|x\|^3$. Here $\{xy^*z\} := x \circ (y^* \circ z) - y^* \circ (z \circ x) + z \circ (x \circ y^*)$ denotes the *Jordan triple product* of $x, y, z \in \mathcal{B}$.

The Jordan triple product of a JB^*-algebra leads to a more general algebraic structure, which turns out to be appropriate for most applications to analysis: the so-called Jordan triple systems. Suppose $\mathcal{J}$ is a complex vector space endowed with a real trilinear composition

$$\mathcal{J} \times \mathcal{J} \times \mathcal{J} \ni (x, y, z) \mapsto \{xy^*z\} \in \mathcal{J}$$

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which is complex bilinear in (x, z) and conjugate linear in y. Then $\mathcal{J}$ is called a *Jordan triple system* if $\{xy^*z\} = \{zy^*x\}$ and

$$\{\{xy^*z\}u^*v\} + \{\{xy^*v\}u^*z\} - \{xy^*\{zu^*v\}\} = \{z\{yx^*u\}^*v\}$$

hold.

We are interested in Jordan triple systems having a Banach space structure. A complex Jordan triple system $\mathcal{J}$ with a Banach space norm $\|\cdot\|$ is called a J^*-*triple* if, for every $x \in \mathcal{J}$, the operator $x\Box x^*$ is hermitian in the sense of Banach algebra theory. Here the operator $x\Box x^*$ on $\mathcal{J}$ is defined by $(x\Box x^*)y := \{xx^*y\}$. This implies that $x\Box x^*$ has real spectrum $\sigma(x\Box x^*) \subset \mathbb{R}$. A J^*-triple $\mathcal{J}$ is called a JB^*-triple if every $x \in \mathcal{J}$ satisfies $\sigma(x\Box x^*) \geq 0$ and $\|x\Box x^*\| = \|x\|^2$.

Our knowledge concerning the continuity properties of epimorphisms on Banach algebras, Jordan-Banach algebras, and, more generally, nonassociative complete normed algebras, is now fairly complete and satisfactory (see [6], [25]). A basic continuity problem consists in determining algebraic conditions on a Banach algebra A which ensure that derivations on A are continuous. In 1996, Villena [25] proved that derivations on semisimple Jordan-Banach algebras are continuous.

Let X and Y be Banach spaces with norms $||\cdot||$ and $\|\cdot\|$, respectively. Consider $f : X \to Y$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$. Th.M. Rassias [19] introduced the following inequality, that we call *Cauchy–Rassias inequality* : Assume that there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(||x||^p + ||y||^p)$$

for all $x, y \in X$. Th.M. Rassias [19] showed that there exists a unique $\mathbb{R}$-linear mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p}||x||^p$$

for all $x \in X$. Beginning around the year 1980 the topic of approximate homomorphisms, or the stability of the equation of homomorphism, was studied by a number of mathematicians. Găvruta [3] generalized the Rassias' result in the following form: Let G be an abelian group and X a Banach space. Denote by $\varphi : G \times G \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{k=0}^{\infty} \frac{1}{2^k} \varphi(2^k x, 2^k y) < \infty$$

for all $x, y \in G$. Suppose that $f : G \to X$ is a mapping satisfying

$$\|f(x + y) - f(x) - f(y)\| \leq \varphi(x, y)$$

for all $x, y \in G$. Then there exists a unique additive mapping $T : G \to X$ such that

$$\|f(x) - T(x)\| \leq \frac{1}{2} \widetilde{\varphi}(x, x)$$

for all $x \in G$.

Jun and Lee [5] proved the following: Denote by $\varphi : X \setminus \{0\} \times X \setminus \{0\} \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{j=0}^{\infty} \frac{1}{3^j} \varphi(3^j x, 3^j y) < \infty$$

for all $x, y \in X \setminus \{0\}$. Suppose that $f : X \to Y$ is a mapping satisfying

$$\|2f(\frac{x + y}{2}) - f(x) - f(y)\| \leq \varphi(x, y)$$

for all $x, y \in X \setminus \{0\}$. Then there exists a unique additive mapping $T : X \to Y$ such that

$$\|f(x) - f(0) - T(x)\| \leq \frac{1}{3} \big(\widetilde{\varphi}(x, -x) + \widetilde{\varphi}(-x, 3x)\big)$$

for all $x \in X \setminus \{0\}$. The stability problem of functional equations has been investigated in several papers (see [7]–[23]).

Let θ, ϕ be endomorphisms of a ring R. An additive mapping $D : R \to R$ is called a (θ, ϕ)-*derivation* on R if $D(xy) = D(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in R$. An additive mapping $U : R \to R$ is called a *generalized* (θ, ϕ)-*derivation* on R if there exists a (θ, ϕ)-derivation $D : R \to R$ such that $U(xy) = U(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in R$ (see [1], [2], [4]).

In this paper, we introduce the concept of linear θ-derivations on JB^*-triples, and prove the Cauchy–Rassias stability of linear θ-derivations on JB^*-triples.

2. Linear θ-derivations on JB^*-triples

Throughout this section, let B be a complex JB^*-triple with norm $\|\cdot\|$.

DEFINITION 2.1. Let $\theta : B \to B$ be an involutive $\mathbb{C}$-linear mapping. An involutive $\mathbb{C}$-linear mapping $D : B \to B$ is called a *linear θ-derivation* on B if

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

holds for all $x, y, z \in B$.

THEOREM 2.1. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} \frac{1}{2^j}\varphi(2^j x, 2^j y, 2^j z) < \infty, \tag{2.1}$$

$$\|f(\mu x + \mu y + z^*) - \mu f(x) - \mu f(y) - f(z)^*\| \leq \varphi(x, y, z), \tag{2.2}$$

$$\|h(\mu x + \mu y + z^*) - \mu h(x) - \mu h(y) - h(z)^*\| \leq \varphi(x, y, z), \tag{2.3}$$

$$\begin{aligned} &\|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ &\quad -\{h(x)h(y)^*f(z)\}\| \leq \varphi(x, y, z), \end{aligned} \tag{2.4}$$

for all $x, y, z \in B$ and all $\mu \in S^1 := \{\lambda \in \mathbb{C} \mid |\lambda| = 1\}$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x, 0), \tag{2.5}$$

$$\|h(x) - \theta(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x, 0) \tag{2.6}$$

for all $x \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. Let $\mu = 1 \in S^1$ and $z = 0$ in (2.2) and (2.3). It follows from the Găvruta's theorem [3] that there exist unique additive mappings $D, \theta : B \to B$ satisfying (2.5) and (2.6). The additive mappings $D, \theta : B \to B$ are given by

$$D(x) = \lim_{l\to\infty} \frac{1}{2^l} f(2^l x), \tag{2.7}$$

$$\theta(x) = \lim_{l\to\infty} \frac{1}{2^l} h(2^l x) \tag{2.8}$$

for all $x \in B$.

By the same reasoning as in the proof of Theorem 2.1 in [12], the additive mappings $D, \theta : B \to B$ are $\mathbb{C}$-linear.

Letting $x = y = 0$ in (2.2), we get

$$D(x^*) = \lim_{l\to\infty} \frac{f(2^l x^*)}{2^l} = \lim_{l\to\infty} \frac{(f(2^l x))^*}{2^l} = (\lim_{l\to\infty} \frac{f(2^l x)}{2^l})^* = D(x)^*$$

for all $x \in B$.

Similarly, one can show that $\theta : B \to B$ is an involutive mapping.

It follows from (2.4) that

$$\frac{1}{2^{3l}}\|f(2^{3l}\{xy^*z\}) - \{f(2^l x)h(2^l y)^*h(2^l z)\} - \{h(2^l x)f(2^l y)^*h(2^l z)\}$$
$$-\{h(2^l x)h(2^l y)^*f(2^l z)\}\| \le \frac{1}{2^{3l}}\varphi(2^l x, 2^l y, 2^l z) \le \frac{1}{2^l}\varphi(2^l x, 2^l y, 2^l z),$$

which tends to zero as $l \to \infty$ for all $x, y, z \in B$ by (2.1). By (2.7) and (2.8),

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

for all $x, y, z \in B$. So the additive mapping $D : B \to B$ is a linear θ-derivation on B, as desired. □

Corollary 2.2. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \ge 0$ and $p \in [0, 1)$ such that*

$$\|f(\mu x + \mu y + z^*) - \mu f(x) - \mu f(y) - f(z)^*\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$
$$\|h(\mu x + \mu y + z^*) - \mu h(x) - \mu h(y) - h(z)^*\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$
$$\|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\}$$
$$-\{h(x)h(y)^*f(z)\}\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \le \frac{2\epsilon}{2 - 2^p}\|x\|^p,$$
$$\|h(x) - \theta(x)\| \le \frac{2\epsilon}{2 - 2^p}\|x\|^p$$

for all $x \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$ to be Th.M. Rassias upper bound in the Cauchy–Rassias inequality, and apply Theorem 2.1. □

THEOREM 2.3. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \to [0, \infty)$ satisfying (2.4) such that*

$$\widetilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} \frac{1}{3^j} \varphi(3^j x, 3^j y, 3^j z) < \infty, \tag{2.9}$$

$$\|2f(\frac{\mu x + \mu y + z^*}{2}) - \mu f(x) - \mu f(y) - f(z)^*\| \le \varphi(x, y, z), \tag{2.10}$$

$$\|2h(\frac{\mu x + \mu y + z^*}{2}) - \mu h(x) - \mu h(y) - h(z)^*\| \le \varphi(x, y, z) \tag{2.11}$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x, -x, 0) + \widetilde{\varphi}(-x, 3x, 0)\big), \tag{2.12}$$

$$\|h(x) - \theta(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x, -x, 0) + \widetilde{\varphi}(-x, 3x, 0)\big) \tag{2.13}$$

for all $x \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. Let $z = 0$ in (2.10) and (2.11). It follows from the Jun and Lee's theorem [5, Theorem 1] that there exist unique additive mappings $D, \theta : B \to B$ satisfying (2.12) and (2.13). The additive mappings $D, \theta : B \to B$ are given by

$$D(x) = \lim_{l \to \infty} \frac{1}{3^l} f(3^l x), \tag{2.14}$$

$$\theta(x) = \lim_{l \to \infty} \frac{1}{3^l} h(3^l x) \tag{2.15}$$

for all $x \in B$.

The rest of the proof is similar to the proof of Theorem 2.1. □

COROLLARY 2.4. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \ge 0$ and $p \in [0, 1)$ such that*

$$\|2f(\frac{\mu x + \mu y + z^*}{2}) - \mu f(x) - \mu f(y) - f(z)^*\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$

$$\|2h(\frac{\mu x + \mu y + z^*}{2}) - \mu h(x) - \mu h(y) - h(z)^*\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$

$$\|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ - \{h(x)h(y)^*f(z)\}\| \le \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \le \frac{3+3^p}{3-3^p}\epsilon\|x\|^p,$$
$$\|h(x) - \theta(x)\| \le \frac{3+3^p}{3-3^p}\epsilon\|x\|^p$$

for all $x \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$, and apply Theorem 2.3, as desired. □

THEOREM 2.5. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \to [0, \infty)$ satisfying* (2.10), (2.11) *and* (2.4) *such that*

$$\sum_{j=0}^{\infty} 3^{3j}\varphi(\frac{x}{3^j}, \frac{y}{3^j}, \frac{z}{3^j}) < \infty \tag{2.16}$$

for all $x, y, z \in B$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \le \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}, 0) + \widetilde{\varphi}(-\frac{x}{3}, x, 0), \tag{2.17}$$
$$\|h(x) - \theta(x)\| \le \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}, 0) + \widetilde{\varphi}(-\frac{x}{3}, x, 0) \tag{2.18}$$

for all $x \in B$, where

$$\widetilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} 3^j\varphi(\frac{x}{3^j}, \frac{y}{3^j}, \frac{z}{3^j})$$

for all $x, y, z \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. By the Jun and Lee's theorem [5, Theorem 6], it follows from (2.16), (2.10) and (2.11) that there exist unique additive mappings $D, \theta : B \to B$ satisfying (2.17) and (2.18). The additive mappings $D, \theta : B \to B$ are given by

$$D(x) = \lim_{l\to\infty} 3^l f(\frac{x}{3^l}), \tag{2.19}$$
$$\theta(x) = \lim_{l\to\infty} 3^l h(\frac{x}{3^l}) \tag{2.20}$$

for all $x \in B$.

By a similar method to the proof of Theorem 2.1, one can show that $D, \theta : B \to B$ are involutive $\mathbb{C}$-linear mappings.

It follows from (2.4) that

$$3^{3l}\|f(\frac{\{xy^*z\}}{3^{3l}}) - \{f(\frac{x}{3^l})h(\frac{y}{3^l})^*h(\frac{z}{3^l})\} - \{h(\frac{x}{3^l})f(\frac{y}{3^l})^*h(\frac{z}{3^l})\} \\ -\{h(\frac{x}{3^l})h(\frac{y}{3^l})^*f(\frac{z}{3^l})\}\| \leq 3^{3l}\varphi(\frac{x}{3^l}, \frac{y}{3^l}, \frac{z}{3^l}),$$

which tends to zero as $l \to \infty$ for all $x, y, z \in B$ by (2.16). By (2.19) and (2.20),

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

for all $x, y, z \in B$. So the additive mapping $D : B \to B$ is a linear θ-derivation on B, as desired. □ □

Corollary 2.6. *Let $f, h : B \to B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p \in (3, \infty)$ such that*

$$\|2f(\frac{\mu x + \mu y + z^*}{2}) - \mu f(x) - \mu f(y) - f(z)^*\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$
$$\|2h(\frac{\mu x + \mu y + z^*}{2}) - \mu h(x) - \mu h(y) - h(z)^*\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p),$$
$$\|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ -\{h(x)h(y)^*f(z)\}\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$-linear mappings $D, \theta : B \to B$ such that

$$\|f(x) - D(x)\| \leq \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p,$$
$$\|h(x) - \theta(x)\| \leq \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p$$

for all $x \in B$. Moreover, $D : B \to B$ is a linear θ-derivation on B.

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$, and apply Theorem 2.5, as desired. □

DEFINITION 2.2. Let $\theta : B \to B$ be an involutive $\mathbb{C}$-linear mapping. An involutive $\mathbb{C}$-linear mapping $D : B \to B$ is called a *Jordan linear θ-derivation* on B if

$$D(\{xx^*x\}) = \{D(x)\theta(x)^*\theta(x)\} + \{\theta(x)D(x)^*\theta(x)\} + \{\theta(x)\theta(x)^*D(x)\}$$

holds for all $x \in B$.

Problem 2.1. Is every Jordan linear θ-derivation a linear θ-derivation?

REFERENCES

1. M. Ashraf, A. Ali, and S. Ali, *On Lie ideals and generalized (θ, ϕ)-derivations in prime rings*, Commun. Algebra **32** (2004), 2977–2985.
2. M. Ashraf, S.M. Wafa, and A. AlShammakh, *On generalized (θ, ϕ)-derivations in rings*, Internat. J. Math. Game Theo. Algebra **12** (2002), 295–300.
3. P. Găvruta, *A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.
4. B. Hvala, *Generalized derivations in rings*, Commun. Algebra **26** (1998), 1147–1166.
5. K. Jun, and Y. Lee, *A generalization of the Hyers–Ulam–Rassias stability of Jensen's equation*, J. Math. Anal. Appl. **238** (1999), 305–315.
6. A.F. López, H. Marhnine and C. Zarhouti, *Derivations on Banach–Jordan pairs*, Quart. J. Math. **52** (2001), 269–283.
7. M. Noor and Th.M. Rassias, *Resolvent equations for set-valued mixed variational inequalities*, Nonlinear Analysis–TMA **42** (2000), 71–83.
8. C. Park, *On the stability of the linear mapping in Banach modules*, J. Math. Anal. Appl. **275** (2002), 711–720.
9. C. Park, *Linear functional equations in Banach modules over a C^*-algebra*, Acta Appl. Math. **77** (2003), 125–161.
10. C. Park, *Generalized quadratic mappings in several variables*, Nonlinear Analysis–TMA **57** (2004), 713–722.
11. C. Park, *Lie $*$-homomorphisms between Lie C^*-algebras and Lie $*$-derivations on Lie C^*-algebras*, J. Math. Anal. Appl. **293** (2004), 419–434.
12. C. Park, *Homomorphisms between Lie JC^*-algebras and Cauchy–Rassias stability of Lie JC^*-algebra derivations*, J. Lie Theory **15** (2005), 393–414.
13. C. Park, *Generalized Hyers–Ulam–Rassias stability of n-sesquilinear-quadratic mappings on Banach modules over C^*-algebras*, J. Comput. Appl. Math. **180** (2005), 279–291.
14. C. Park, *Approximate homomorphisms on JB^*-triples*, J. Math. Anal. Appl. **306** (2005), 375–381.
15. C. Park, *Homomorphisms between Poisson JC^*-algebras*, Bull. Braz. Math. Soc. **36** (2005), 79–97.
16. C. Park and J. Hou, *Homomorphisms between C^*-algebras associated with the Trif functional equation and linear derivations on C^*-algebras*, J. Korean Math. Soc. **41** (2004), 461–477.
17. C. Park, J. Hou and S. Oh, *Homomorphisms between JC^*-algebras and between Lie C^*-algebras*, Acta Math. Sinica **21** (2005), 1391–1398.

18. C. Park and W. Park, *On the Jensen's equation in Banach modules*, Taiwanese J. Math. **6** (2002), 523–532.
19. Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.
20. Th.M. Rassias, *The problem of S.M. Ulam for approximately multiplicative mappings*, J. Math. Anal. Appl. **246** (2000), 352–378.
21. Th.M. Rassias, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl. **251** (2000), 264–284.
22. Th.M. Rassias and P. Šemrl, *On the behavior of mappings which do not satisfy Hyers–Ulam stability*, Proc. Amer. Math. Soc. **114** (1992), 989–993.
23. Th.M. Rassias and P. Šemrl, *On the Hyers–Ulam stability of linear mappings*, J. Math. Anal. Appl. **173** (1993), 325–338.
24. H. Upmeier, *Jordan Algebras in Analysis, Operator Theory, and Quantum Mechanics*, Regional Conference Series in Mathematics No. 67, Amer. Math. Soc., Providence, 1987.
25. A.R. Villena, *Derivations on Jordan–Banach algebras*, Studia Math. **118** (1996), 205–229.

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