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Modeling Satellite Orbital Segments using Orbit-Attitude Models

Taejung Kim[†]

Dept. of Geoinformatic Engineering, Inha University

Abstract : Currently, in order to achieve accurate geolocation of satellite images we need to generate control points from individual scenes. This requirement increases the cost and processing time of satellite mapping greatly. In this paper we investigate the feasibility of modeling entire image strips that has been acquired from the same orbital segments. We tested sensor models based on satellite orbit and attitude with different sets of unknowns. We checked the accuracy of orbit modeling by establishing sensor models of one scene using control points extracted from the scene and by applying the models to adjacent scenes within the same orbital segments. Results indicated that modeling of individual scenes with 2nd order unknowns was recommended. In this case, unknown parameters were position biases, drifts, accelerations and attitude biases. Results also indicated that modeling of orbital segments with zero-degree unknowns was recommended. In this case, unknown parameters were attitude biases.

Key Words : Sensor model, Orbit modeling, SPOT, Orbit-attitude model.

1. Introduction

Sensor models refer to mathematical expressions of geometric relationship between sensor (or image) coordinates and their corresponding ground coordinates. An accurate sensor model is a prerequisite to generation of accurate geometric information from any types of image data. For accurate sensor models one usually utilizes control points, whose image coordinates and ground coordinates are known very precisely, and estimates parameters of sensor models to best fit the given control points.

In aerial photogrammetry sensor models are expressed with respect to photographs, i.e., for each aerial photograph, there exists one unique sensor model. Since one typical aerial surveying consists of a few hundreds of photographs, it possesses as many sensor models. It would be very troublesome if one has to gather control points for each photograph and establish sensor models separately. However, there exist techniques to relate the relationship between sensor models and to use only a few control points to build a set of sensor models for the whole block of photographs (ASPRS, 2004). This process is so-called block adjustments.

For satellite images we use the term, “scene”, as a unit to describe “one image”. However, the definition

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[†] Corresponding Author: T. Kim (tezid@inha.ac.kr)

of "scene" is somewhat arbitrary and varies satellites by satellites. For example, one SPOT panchromatic scene is defined as 6000 lines whereas for KOMPSAT-1 EOC 2796 lines. In reality, as the satellite flies along its orbit, it continuously takes a series of one dimensional image lines. There is no physical mechanism to distinguish one scene from another. The image lines collected during one orbital segment are cut into a number of scenes according to predefined methods (SPOT Image, 1997).

For satellite images sensor models are usually expressed with respect to individual scenes, i.e., for each scene there exists one unique sensor model. As a satellite takes a number of image scenes along its orbit, there would be as many sensor models. If one establishes sensor models for the scenes individually, one requires a set of control points per scene. This requirement greatly increases the cost and processing time for satellite mapping. This requirement also limits wider applications of satellite images. One cannot achieve precise geolocation of satellite images over those areas where there are no means to extract control points. For these reasons, previous researches investigated the possibility of modelling entire image strips, instead of individual scenes, that have been acquired from the same orbital pass through bundle adjustments. Dowman (1991) reported test results of eight institutes for modelling SPOT image strips with various control point configurations. All eight institutes reported that when control points were well spread over an entire strip, they achieved accurate strip models.

In this paper, we also investigate the feasibility of modeling entire image strips. Our purpose is to check this feasibility with a particular control point configuration, where control points are available only on one end of the strip. This configuration has not been thoroughly studied (Dowman, 1991). If feasible, techniques for modeling satellite orbital segments with such control point configuration can offer many benefits over the current practice of satellite mapping. They may provide partial solutions of mapping where no control points are available.

So far many types of sensor models have been proposed. The model based modified collinearity equations is one of the most widely used models (Konecny *et al.*, 1987; Gugan and Dowman, 1988; Orun and Natarajan, 1994). A modified direct linear transformation model was also proposed for satellite images (Gupta and Hartley, 1997). Recently a model based on rational polynomials gained interests (Dial and Grodecki, 2002). In this paper, we limit our investigation into one particular type of sensor model, which is based on satellite orbit and attitude angles (Orbit-Attitude Models) (Salamonowizc, 1986; Radhadevi *et al.*, 1998; Emery *et al.*, 2003). The major reason is because earlier investigation showed that this model showed better accuracy of estimating exterior orientation parameters over the modified collinearity equations for KOMPSAT-1 EOC images (Kim, 2005). Besides, the possibilities of orbit modeling with the collinearity equations were studied before (Bang and Cho, 2001). However we do not preclude other models than the one tested here on their possibilities of orbit modeling. A thorough investigation among different models may be required later.

2. Orbit-Attitude Model

The Orbit-Attitude model is a model close to physical configuration of imaging geometry of the spacecraft, its orbit and the sensor. Usually, a satellite is "physically" controlled to align with the orbit reference system (Wertz, 1978; Radhadevi *et al.*, 1998; SPOT Image, 2002). The orbital reference system is a time-varying system as a satellite is continuously circulating along its orbit. The orbit can be represented by orbital parameters either such as inclination, right ascension of ascending node and mean anomaly (Wertz, 1978; Radhadevi *et al.*, 1998) or such as satellite position and velocity vectors (SPOTImage, 2002). Figure 1 represents an example of the definition of the orbit reference system (the dashed coordinate system in the figure) based on the satellite position vector **P** and the velocity vector **V**. In this example, yaw axis is defined as the direction of position vector **P**. Sometimes yaw axis is defined as opposite to the direction of position vector. Roll axis is defined as the direction of the velocity vector **V**. These two axes define the third axis, pitch axis.

The misalignment angles between the satellite coordinate system (the solid coordinate system in the figure) and the orbit reference system are given as attitude angles of roll, pitch and yaw angle (Wertz, 1978; Radhadevi *et al.*, 1998; SPOT Image, 2002). In attitude angles, the order of rotation is important since a different order produces different rotation matrix. The definition of the orbit reference system, attitude angles and the order of rotation varies satellites by satellites.

This "physical" properties lead to a conclusion that the orientation of the satellite with respect to the ground reference coordinate system is a non-linear function of satellite position, velocity and attitude angles. It has been addressed that the modified collinearity models, which use position and rotation angles as parameters, may over-simplify this relationship (Kim, 2005).

It is not trivial to formulate mathematical representation of sensor models based on satellite orbit and attitude. The formulation may appear very complicated because there are a good number of parameters involved. Often it appears differently for different satellites because each satellite has its own formulation of the orbit reference coordinate system (Wertz, 1978). Many previous sensor models based on satellite orbit and attitude were presented in different fashions (Wolff, 1985; Salamonowizc, 1986; Westin, 1990; Radhadevi *et al.*, 1998; SPOT Image, 2002). In this paper, we attempt to generalize them with the following simple matrix equation. There exist other ways of matrix representation, for example, based on elevation and azimuth angle of image look vectors (Salamonowizc, 1986; Emery *et al.*, 2003).

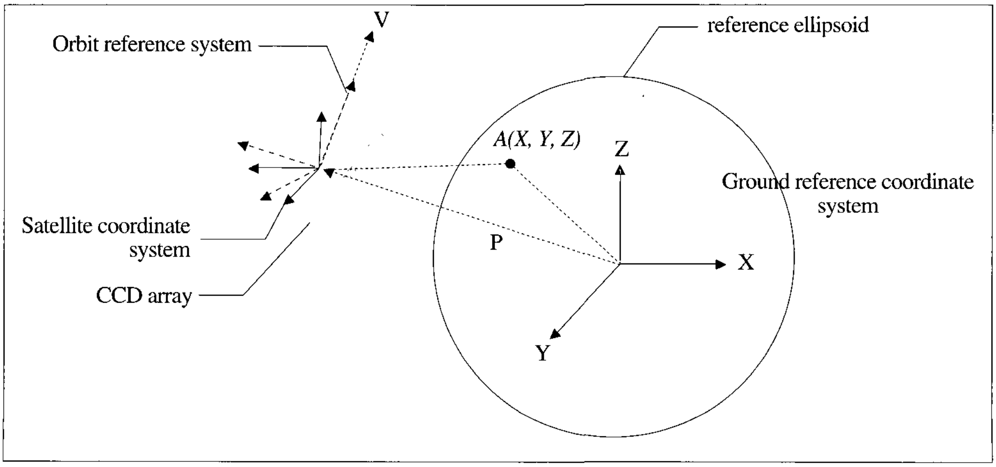

Figure 1. An example of imaging geometry based on orbit reference system.

$$\begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \lambda \mathbf{R}_{rpy}^{\mathbf{T}} \mathbf{R}_{orbit}^{\mathbf{T}} \begin{pmatrix} X - X_S \\ Y - Y_S \\ Z - Z_S \end{pmatrix} \qquad (1)$$

where (x, y) are sensor coordinates, f focal length, (X, Y, Z) the ground coordinates, (X_S, Y_S, Z_S) coordinates for satellite position. $\mathbf{R}_{rpy}$ represents the rotation matrix determined by the attitude angles, which makes satellite coordinate system aligned with the orbit reference coordinate system. $\mathbf{R}_{orbit}$ represents the rotation matrix determined by the orbit reference coordinate system, which makes the orbit reference coordinate system aligned with the ground reference coordinate system.

As mentioned before, the exact mathematical representation of $\mathbf{R}_{rpy}$ and $\mathbf{R}_{orbit}$ may vary between different satellites. It can be expressed as below for SPOT

$$\mathbf{R}_{rpy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos P & \sin P \\ 0 & -\sin P & \cos P \end{pmatrix} \begin{pmatrix} \cos R & 0 & -\sin R \\ 0 & 1 & 0 \\ \sin R & 0 & \cos R \end{pmatrix} \begin{pmatrix} \cos \Psi & -\sin \Psi & 0 \\ \sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where R, P and Ψ represents roll, pitch and yaw angles derived from the metadata of SPOT scenes (SPOT Image, 2002). Note that for a historical reason, pitch and roll angles given in the metadata are negated and hence we apply $-P$ and $-R$ in the above rotation matrix (SPOT Image, 2002). Similarly, $\mathbf{R}_{orbit}$ depends on the exact definition of orbit reference system. Radhadevi *et al.* (1998) represented this matrix for IRS-1C/1D with orbital parameters of inclination, right ascension of ascending node and mean anomaly. SPOT Image (2002) represented this matrix for SPOT satellites with satellite position and velocity vectors as below

$$\mathbf{R}_{orbit} = \begin{pmatrix} X_X & Y_X & Z_X \\ X_Y & Y_Y & Z_Y \\ X_Z & Y_Z & Z_Z \end{pmatrix}, \begin{pmatrix} Z_X \\ Z_Y \\ Z_Z \end{pmatrix} = \frac{\mathrm{P}}{\|\mathrm{P}\|}, \begin{pmatrix} X_X \\ X_Y \\ X_Z \end{pmatrix} = \frac{\mathrm{V}}{\|\mathrm{V}\|} \times \begin{pmatrix} Z_X \\ Z_Y \\ Z_Z \end{pmatrix},$$

$$\begin{pmatrix} Y_X \\ Y_Y \\ Y_Z \end{pmatrix} = \begin{pmatrix} Z_X \\ Z_Y \\ Z_Z \end{pmatrix} \times \begin{pmatrix} X_X \\ X_Y \\ X_Z \end{pmatrix}$$

where $\| \ \|$ denotes the magnitude of a vector.

We can summarize that the model in equation (1) can be represented by position and velocity (or orbital parameters) and attitude angles. Therefore we can call this model an "Orbit-Attitude" (OA) model. We regard this model has nine exterior orientation parameters (three in the position vector, three in the velocity vector and three in attitude angles). For the sake of computational convenience we will model the position and velocity as the following 2nd-order polynomials of time t (or image coordinate x).

$$\begin{aligned} X_S &= X_0 + a_1 t + b_1 t^2 \\ Y_S &= Y_0 + a_2 t + b_2 t^2 \\ Z_S &= Z_0 + a_3 t + b_3 t^2 \\ V_X &= V_{X0} + a_4 t + b_4 t^2 \\ V_Y &= V_{Y0} + a_5 t + b_5 t^2 \\ V_Z &= V_{Z0} + a_6 t + b_6 t^2 \end{aligned}$$

where (V_X, V_Y, V_Z) is the velocity vector of the spacecraft. And we will model attitude angles as piecewise-linear functions of time, whose shapes can be derived from the satellite attitude rate information provided in the metadata (SPOT Image, 2002).

Traditionally, Orbit-Attitude models have been developed and used among satellite communities where access to the information on satellite orbit and attitude control system is available. The choice of unknowns that to be adjusted through bundle adjustment process differed by authors. Salamonowicz (1988) selected as unknowns the zero-order terms of satellite orbital parameters (or position biases) and the zero- and first-order terms of satellite attitude (or attitude biases and drifts). Radhadevi *et al.* (1998) chose the zero-order terms for the orbit and attitude (or position and attitude biases) as unknowns. In this paper, we implemented seven orbit-attitude models with difference sets of unknowns as specified in table 1. OA-1 models the 2nd order coefficients of satellite position equations (or position biases, drifts and accelerations) and the attitude biases (R_0 for roll, P_0 for pitch, Ψ_0 for yaw) as unknowns. OA-2 differs from OA-1 in that it models roll bias, drift $\dot{R}$ and

Table 1. List of unknowns for each sensor model.

ID	Unknowns
OA-1	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3, R_0, P_0, \Psi_0$
OA-2	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3, R_0, P_0, \dot{R}, \ddot{R}$
OA-3	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3$
OA-4	$R_0, \dot{R}, \ddot{R}, P_0, \dot{P}, \ddot{P}, \Psi_0, \dot{\Psi}, \ddot{\Psi}$
OA-5	$X_0, Y_0, Z_0, R_0, P_0, \Psi_0$
OA-6	X_0, Y_0, Z_0
OA-7	R_0, P_0, Ψ_0

acceleration $\ddot{R}$ as attitude unknowns. OA-3 models the biases, drifts and accelerations of position as unknowns and OA-4 the biases, drifts and accelerations of attitude angles. OA-5 to OA-7 have much simpler unknowns. OA-5 models position biases and attitude biases as unknowns, which are the same unknowns as in Radhadevi *et al.* (1998). OA-6 uses position biases as unknowns. OA-7 uses attitude biases only as unknowns. These models were tested to check the possibility of orbit modeling.

3. Results and Discussions

1) Dataset and Test Methods

We used two strips that were taken by SPOT-3. Each strip consists of three scenes. The top scene is over Daejeon area, the middle over Junju area and the bottom over Kwangju area. The lengths of the strips are approximately 180 km. Figure 2 shows the extents of the strips and individual scenes. The two strips constitute a stereo coverage. There are slight overlaps among the three scenes within each strip. Table 2 summaries the properties of the strips. Strip-1 was acquired by tilting the spacecraft by +19.8° (tilt to the east) and Strip-2 by -23.4° (tilt to the west). For experiments, a total of 75 control points were prepared for each strip by GPS surveying. Figure 3 shows the location of control points within the three scenes of Strip-1. 27 control points were extracted from Daejeon scene (top image in figure 3). Among them 14 control points were used as "model" points, i.e., the points to set up camera models of Daejeon scene. These points were indicated as black crosses in the top image of figure 3. The rest 13 control points were used as "check" points, i.e., the points to check the accuracy of sensor models established. These were indicated as white crosses. For Junju scene 25 control points were extracted and for Kwangju scene 23 control points. These control points were all used to check the accuracy of sensor models. In the middle and bottom images of figure 3 these were represented as white crosses.

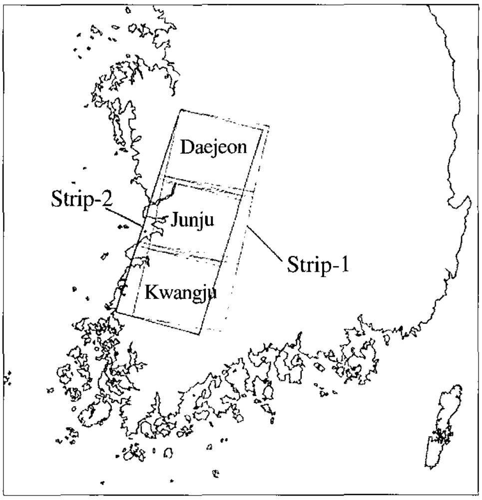

Figure 2. Extents of image strips and individual scenes.

Table 2. Properties of image strips used.

ID	Strip-1	Strip-2
Satellite	SPOT-3	SPOT-3
Date of Acquisition	4 April 1995	28 Jan 1995
Tilt Angle	+19.8°	-23.4°
No of GCPs (Daejeon)	27	27
No of GCPs (Junju)	25	25
No of GCPs (Kwangju)	23	23

We performed experiments as follows. First, we established sensor models of Daejeon scenes with the

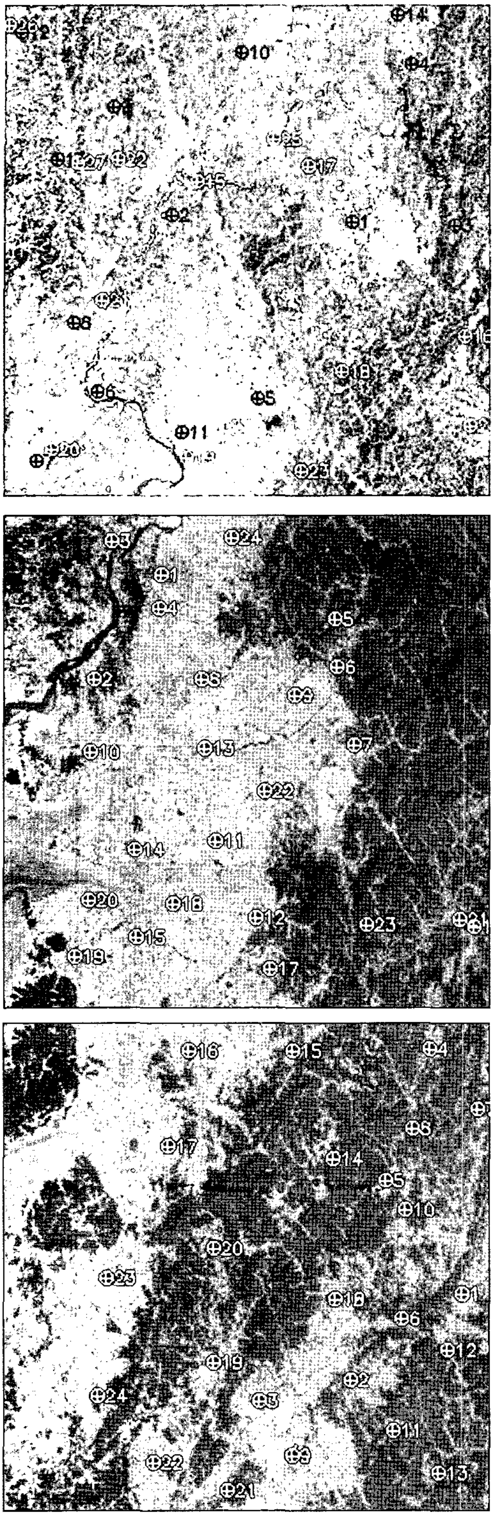

Figure 3. Location of control points within Daejeon (Top), Junju (Middle) and Kwangju (Bottom) scenes.

14 model points. The accuracy of sensor models for Daejeon scene was checked against the 13 check points. We then tested the possibilities of orbit modeling by extending sensor models of Daejeon scene to Junju and Kwangju scenes. The accuracy of orbit modeling was checked against the control points over Junju and Kwangju scenes. For this we converted row coordinates of control points over Junju and Kwangju scenes as if the two scenes were parts of one long scene whose upper part was the Daejeon scene.

2) Results of Modeling Single Scenes

Table 3 summarizes the results of modeling single scenes. The 14 model points of Daejeon scene were used to establish sensor models for Daejeon scene and the 13 check points of Daejeon scene was used to check the accuracy. In the table, "Model Error" means the error of sensor models against the 14 model points. This error indicates how well sensor models were fitted to given model points. "Check Error" means the error sensor models against the 13 check points. This error indicates the accuracy of sensor models within the extent of Daejeon scene.

Table 3 shows that the models with higher degree unknowns (OA-1 and OA-2) produced the most accurate model errors. This indicates that OA-1 and OA-2 could adjust parameters to make models best fit

Table 3. Results of modeling single scenes.

Model ID	Daejeon scene in Strip-1		Daejeon scene in Strip-2	
	Model Error (rms, pixels)	Check Error (rms, pixels)	Model Error (rms, pixels)	Check Error (rms, pixels)
OA-1	0.785	1.189	1.117	1.790
OA-2	0.775	1.354	1.161	1.738
OA-3	1.604	2.388	1.965	2.462
OA-4	1.246	1.744	1.329	1.687
OA-5	0.926	1.239	1.449	1.728
OA-6	1.771	2.252	2.234	2.415
OA-7	1.377	1.486	1.715	1.762

the given model points. They also produced small check errors. This result agreed with previous publications, where models with 2^{nd} order unknowns were proposed to model single scenes (Gugan and Dowman, 1988; Orun and Natarajan, 1994).

OA-4 (that models attitude biases, drifts and accerlerations as unknowns), OA-5 (that models position and attitude biases), and OA-7 (that models attitude biases) had slightly less accurate model errors than OA-1 and OA-2 but their check errors were comparable to OA-1 and OA-2. The models with only positional unknowns (OA-3 for position biases, drifts and accelerations and OA-6 for position biases) had largest model and check errors.

3) Results of Modeling Orbital Segments

Table 4 and 5 show the results of orbit modeling, i.e., the accuracy of sensor models of Daejeon scenes when applied to other scenes. In the tables, “Orbit Error for Daejeon” means the accuracy of sensor models of Daejeon scenes against the 13 check points of Daejeon scenes. These figures are identical to “Check Error” in table 3. “Orbit Error for Junju” refers to the error of sensor models over Junju scenes. This was estimated against the 25 control points of Junju scene. “Orbit Error for Kwangju” refers to the error of sensor models over Kwangju scenes. This was estimated against the 23 control points of Kwangju scene.

The two tables contain very interesting results. The accuracy of orbit modeling appeared quite different from that of modeling individual scenes. For both Strip-1 and Strip-2 any models that used higher degree unknowns produced far much worse accuracy than those with simpler unknowns. Figure 4 shows the magnitude of errors for Strip-1 as the function of row number. The row number between 0 and 6000 corresponds to Daejeon scene (roughly, if we ignore the overlap), between 6000 and 12000 Junju scene and between 12000 and 18000 Kwangju scene. Note that this row number is proportional to the distance from the top of Daejeon scene. In all models errors were small between 0 and 6000 since these extents are where model points were extracted. Those models with high degree unknowns (OA-1~OA-4) errors increased exponentially. It is interesting to see that OA5, OA6 and OA7 produced much accurate results and errors did not grow fast.

Figure 5 shows the accuracy of orbit modeling for Strip-2. In this time, only plots for OA-5, OA-6 and OA-7 are shown. Although OA-5 and OA-6 produced much accurate results than OA-1~OA-4, their errors also increased exponentially as distance increased. The plot for OA7 shows that the magnitude of errors did not have any correlation with the image row coordinates. With this model we have

Table 4. Results of orbit modeling of strip-1.

Model ID	Orbit Error for Daejeon (rms, pixels)	Orbit Error for Junju (rms, pixels)	Orbit Error for Kwangju (rms, pixels)
OA1	1.189	8.186	27.052
OA2	1.354	9.855	34.343
OA3	2.388	12.329	38.431
OA4	1.175	11.440	39.833
OA5	1.239	1.782	3.534
OA6	2.252	2.429	3.892
OA7	1.486	1.906	2.233

Table 5. Results of orbit modeling of strip-2.

Model ID	Orbit Error for Daejeon (rms, pixels)	Orbit Error for Junju (rms, pixels)	Orbit Error for Kwangju (rms, pixels)
OA1	1.790	8.285	31.065
OA2	1.738	12.381	51.408
OA3	2.462	14.580	57.086
OA4	1.687	8.885	37.162
OA5	1.728	3.874	12.039
OA6	2.415	4.648	13.881
OA7	1.762	1.551	2.057

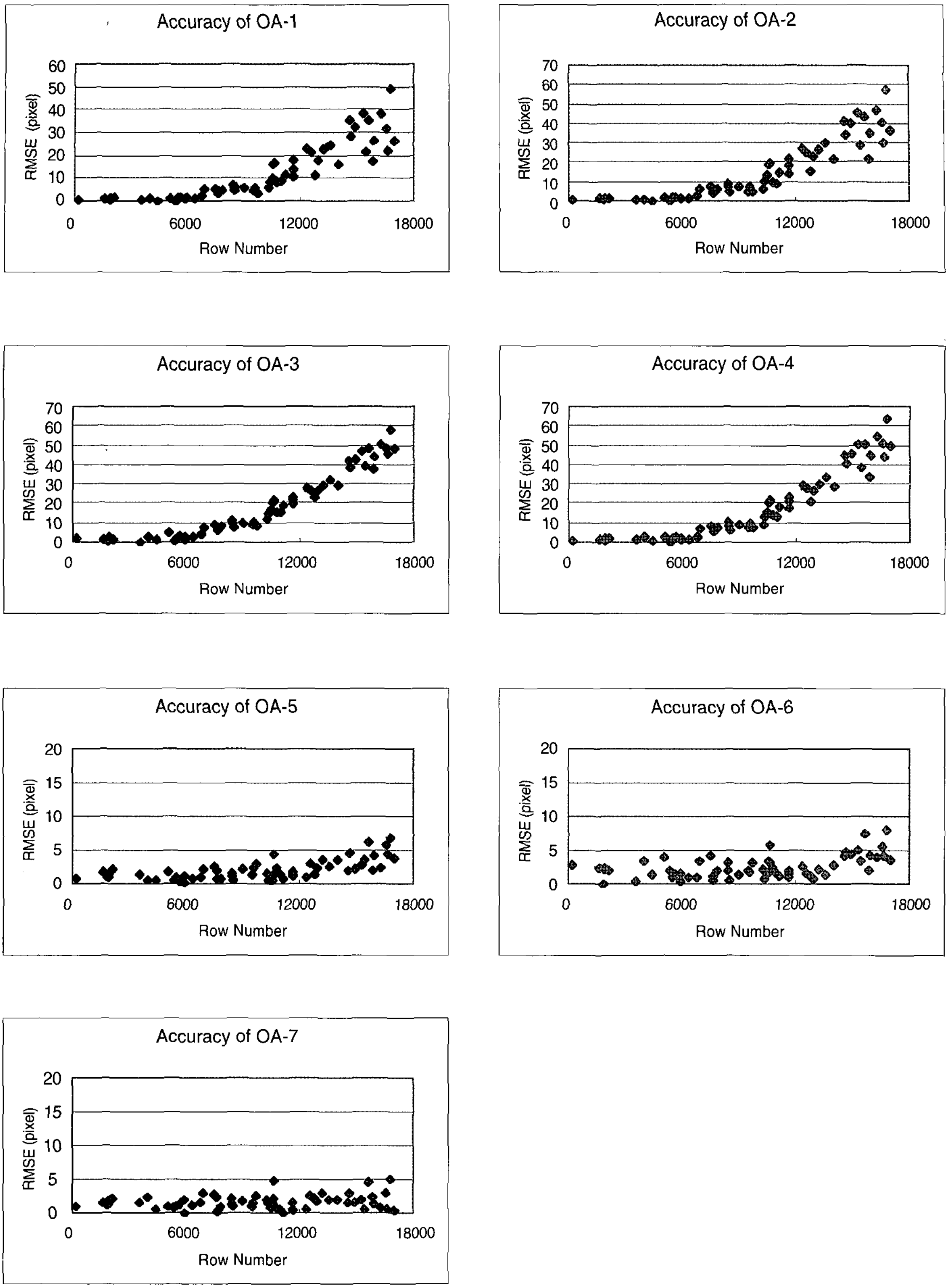

Figure 4. Accuracy of orbit modeling for Strip-1.

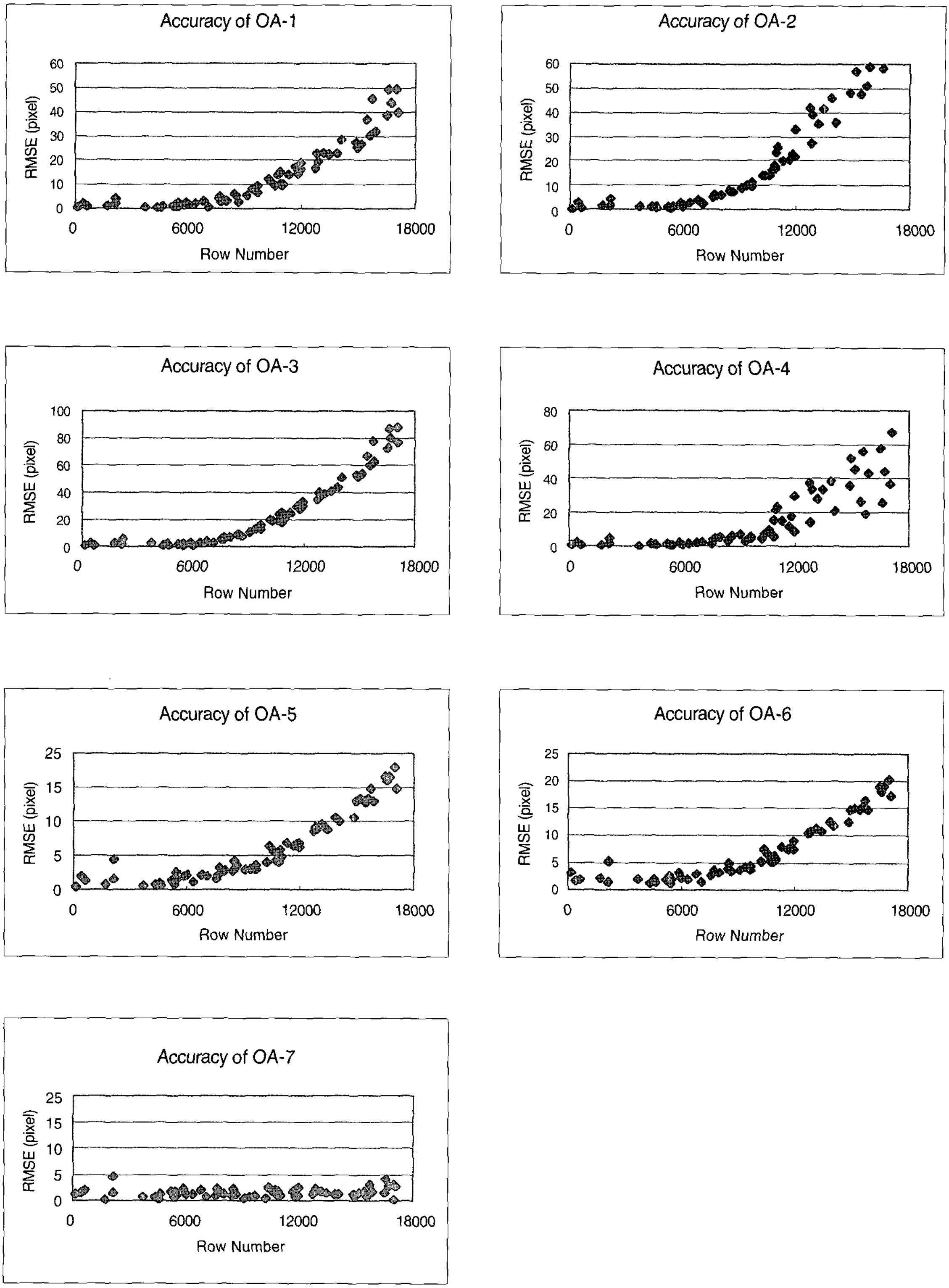

Figure 5. Accuracy of orbit modeling for Strip-2.

successfully modeled the orbital segments of 180 km in length. This result may indicate that although the models with high degree unknowns produced good accuracy of modeling individual scenes, they distorted true exterior orientations of sensor models so that they could not be applied beyond the extent of single scenes.

4. Conclusions

In this paper, we discussed the possibility of modeling orbital segments instead of individual scenes. We tested sensor models based on satellite orbit and attitude with different sets of unknowns. Results indicated the followings.

For modeling individual scenes accurately, models with high degree (or 2^{nd} order) unknowns were recommended. The models that used position biases, drifts and accelerations and attitude biases produced most accurate results. This recommendation coincides with previous researches. Models with high degree unknowns should have high degree of freedom and they could adjust model parameters so that the models could best fit the given control points.

To model orbital segments, it is better to use models with zero-degree unknowns. The models that used attitude biases as unknowns were recommended. This recommendation does not coincide with previous researches nor results of modeling individual scenes. Models with high degree unknowns, although they produced most accurate results for single scenes, introduced distortions on the estimated parameters. Their parameters were only valid within the extent where control points were available. On the other hands, the results of modeling orbital segments indicated that models with zero-degree unknowns estimated more physically meaningful parameters.

In order to generalize these findings, tests with longer length of strips are required. Tests with other types of satellite data are also required. We may need to include other types of unknowns such as time inaccuracy. These issues will be studied in near future.

Acknowledgments

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References

ASPRS, 2004. “Manual of Photogrammetry (5th Edition)”, J.C. McGlone (Editor).

Bang, K-I. and Cho, W-S., 2001. Pseudo Image Composition and Sensor Models Analysis of SPOT Satellite Imagery for Inaccessibe Area, Korean Journal of Remote Sensing, 17(1): 33-44.

Dial, G. and Grodecki, J., 2002. “Block Adjustment with Rational Polynomial Camera Models”, Proc. of ACSM-ASPRS 2002 Annual Conference.

Dowman, I.(Editor), 1991. Test of Triangulation of SPOT Data, European Organization for Experimental Photogrammetric Research, 26.

Emery, W. J., Baldwin, D., and Matthews, D., 2003. “Maximum Cross Correlation Automatic Satellite Image Navigation and Attitude Corrections for Open-Ocean Image Navigation”, IEEE Trans. Geoscience and Remote Sensing, 41(1): 33-42.

Gugan, D. J. and Dowman, I. J., 1988. “Accuracy and completeness of topographic mapping from SPOT imagery”, Photogrammetric Record, 12(72): 787-796.

Gupta, R. and Hartley, R., 1997. "Linear pushbroom cameras", IEEE Trans. PAMI, 19(9): 963-975.

Kim, T., 2005. "Investigation on the Accuracy of bundle Adjustments and Exterior Orientation Parameter Estimation of Linear Pushbroom Sensor Models", Journal of Korean Surveying Society, 23(2): 137-145.

Konecny, G., Lohmann, P., Engel, H., and Kruck, E., 1987. "Evaluation of SPOT imagery on analytical photogrammetric instruments", Photogrammetric Engineering and Remote Sensing, 53(9): 1223-1230.

Orun, A. B. and Natarajan, K., 1994. "A modified bundle adjustment software for SPOT imagery and photography: Tradeoff", Photogrammetric Engineering and Remote Sensing, 60(12): 1431-143.

Radhadevi, P. V., Ramachandran, R., and Mohan, M., 1998. "Restitution of IRS-1C PAN data using an orbit attitude model and minimum control", ISPRS Journal of Photogrammetry and Remote Sensing, 53(1998): 262-271.

Salamonowicz, P. H., 1986. "Satellite orientation and position for geometric correction of scanner imagery", Photogrammetric Engineering and Remote Sensing, 52(4): 491-499.

SPOT Image, 1997. "The SPOT Scene Standard Digital Product Format", S4-ST-73-01-SI.

SPOT Image, 2002. "SPOT Satellite Geomtry Handbook", S-NT-73-12-SI.

Wertz, J. R. (Editor), 1978. Spacecraft Attitude Determination and Control, Kluwer Academic Publishers.

Westin, T., 1990. "Precision Rectification of SPOT Imagery", Photogrammetric Engineering and Remote Sensing, 56(2): 247-253.

Wolff, T., 1985. "An image geometry model for METEOSAT", Int. J. Remote Sensing, 6(10): 1599-1606.