Bull. Korean Math. Soc. **43** (2006), No. 1, pp. 43–52

WEAK INVERSE SHADOWING AND GENERICITY

TAEYOUNG CHOI, SUNGSOOK KIM AND KEONHEE LEE

ABSTRACT. We study the genericity of the first weak inverse shadowing property and the second weak inverse shadowing property in the space of homeomorphisms on a compact metric space, and show that every shift homeomorphism does not have the first weak inverse shadowing property but it has the second weak inverse shadowing property.

1. Introduction

Simulating the behavior of a dynamical system we often encounter the following problems.

Does the orbit displayed on the computer screen actually correspond to some true orbit?

Can every true orbit be recovered, at least with a given accuracy?

The first problem is in fact a question about the shadowing property of the system while the second corresponds to the property known as inverse shadowing. Shadowing, or the pseudo orbit tracing property is one of the interesting concepts in the qualitative theory of smooth dynamical systems. It says that any δ-pseudo orbit can be uniformly approximated by a true orbit with a given accuracy if $\delta > 0$ is sufficiently small. The concept of inverse shadowing property was established by Corless and Pilyugin in [1] and redefined by Kloeden and Ombach in [3] using the notion of δ-method. Generally speaking, a dynamical system has the inverse shadowing property with respect to a class of methods if any true orbit can be uniformly approximated with given accuracy by a δ-pseudo orbit generated by a method from the chosen class if $\delta > 0$

Received August 23, 2004.

2000 Mathematics Subject Classification: Primary 37C20; Secondary 37C50.

Key words and phrases: shadowing, inverse shadowing, weak inverse shadowing, generic.

This work was supported by grant Korea Research Foundation Grant (KRF-2003-042-C00008).

is sufficiently small. An appropriate choice of the class of admissible pseudo orbits is crucial here (see [2, 3, 4, 7]).

When we study the geometric pattern of the set of orbits of a system under investigation, the main objects of interest are just the real orbit sets while the orbit behavior of the system under special time is irrelevant.

To develop the approach, Corless and Pilyugin introduced the notion of the weak shadowing property which is weaker than the shadowing property, and they proved that the weak shadowing property is generic in the space of all homeomorphisms on a compact smooth manifold. In the proof, the smoothness condition of the phase space could not be removed (see [1]). Moreover, Sakai [9] showed that every element of the C^1 interior of the set of all diffeomorphisms on a C^∞ closed surface having the weak shadowing property is structurally stable.

Recently, Pilyugin, Rodionova, and Sakai [8] considered various weak shadowing properties of dynamical systems: the first weak shadowing property, the second weak shadowing property and the orbital shadowing property, and studied the qualitative theory of dynamical systems with the properties.

In this paper, we introduce a notion of the first weak inverse shadowing property (resp. the second weak inverse shadowing property) for homeomorphism as an inverse form of the first weak shadowing property (resp. the second weak shadowing property), and prove that the first weak inverse shadowing property is generic in the space of all homeomorphisms on a compact metric space. We show that every homeomorphism on a compact metric space has the second weak inverse shadowing property. Furthermore, we claim that every shift homeomorphism does not have the first weak inverse shadowing property but it has the second weak inverse shadowing property.

2. Weak shadowing and weak inverse shadowing

Consider a dynamical system generated by a homeomorphism f of a metric space X with a metric d. For a point $x \in X$, we denote by $O(x, f)$ its orbit in the system f, i.e., the set

$$O(x, f) = \{f^n(x) : n \in \mathbb{Z}\}.$$

We say that a sequence $\xi = \{x_n \in X : n \in \mathbb{Z}\}$ is a *δ-pseudo orbit* of f if the inequalities

$$d(f(x_n), x_{n+1}) < \delta, \ n \in \mathbb{Z},$$

hold.

A δ-pseudo orbit is a natural model of computer output in a process of numerical investigation of the system f. In this case, the value δ measures errors of the method, round-off errors, etc.

Recall that f has the *shadowing property* if given $\varepsilon > 0$ there exists $\delta > 0$ such that for any δ-pseudo orbit $\xi = \{x_n\}$ of f we can find a point $y \in X$ with the property

$$d(f^n(y), x_n) < \varepsilon, \ n \in \mathbb{Z}.$$

Of course, if f has the shadowing property formulated above, then the results of its numerical study with a proper accuracy reflect its qualitative structure.

We say that f has the *first weak shadowing property* (1WSP)(resp. the *second weak shadowing property* (2WSP)) if given $\varepsilon > 0$ there exists $\delta > 0$ such that for any δ-pseudo orbit $\xi = \{x_n\}$ of f we can choose a point $y \in X$ with the property

$$\xi \subset N_\varepsilon(O(y, f)) \text{ [resp. } O(y, f) \subset N_\varepsilon(\xi)].$$

The first weak shadowing property was introduced in [1] and called there the *weak shadowing property.*

Pilyugin, Rodionova, and Sakai characterized the 1WSP and the 2WSP for linear diffeomorphisms and studied some C^1 open sets of diffeomorphisms defined in terms of these properties (see [8]).

Let $X^{\mathbb{Z}}$ be the space of all two sided sequences $\xi = \{x_n : n \in \mathbb{Z}\}$ with elements $x_n \in X$, endowed with the product topology. For $\delta > 0$, let $\Phi_f(\delta)$ denote the set of all δ-pseudo orbits of f.

A mapping $\varphi : X \to \Phi_f(\delta) \subset X^{\mathbb{Z}}$ is said to be a *δ-method* for f if $\varphi(x)_0 = x$, where $\varphi(x)_0$ is the 0-component of $\varphi(x)$. Then each $\varphi(x)$ is a δ-pseudo orbit of f through x. For convenience, write $\varphi(x)$ for $\{\varphi(x)_k\}_{k\in\mathbb{Z}}$. Say that φ is a *continuous δ-method* for f if the map φ is continuous. The set of all δ-methods (resp. continuous δ-methods) for f will be denoted by $\mathcal{T}_0(f, \delta)$ (resp. $\mathcal{T}_c(f, \delta)$).

We denote $Z(X)$ the space of all homeomorphisms on X with the C^0 topology induced by the C^0 metric d_0 : for any $f, g \in Z(X)$,

$$d_0(f, g) = \sup_{x\in X}\{d(f(x), g(x)), d(f^{-1}(x), g^{-1}(x))\}.$$

If $g : X \to X$ is a homeomorphism with $d_0(f, g) < \delta$, then g induces a continuous δ-method φ_g for f by defining

$$\varphi_g(x) = \{g^n(x) : n \in \mathbb{Z}\}.$$

Let $\mathcal{T}_h(f,\delta)$ denote the set of all continuous δ-methods φ_g for f which are induced by $g \in Z(X)$ with $d_0(f,g) < \delta$. We define $\mathcal{T}_\alpha w(f)$ by

$$\mathcal{T}_\alpha(f) = \bigcup_{\delta>0} \mathcal{T}_\alpha(f,\delta),$$

where $\alpha = 0, c, h$. Clearly,

$$\mathcal{T}_h(f) \subset \mathcal{T}_c(f) \subset \mathcal{T}_0(f).$$

Note that a method in $\mathcal{T}_c(f)$ need not be generated by a single mapping.

We say that f has the *inverse shadowing property* with respect to the class $\mathcal{T}_\alpha$, $\alpha = 0, c, h$, if for any $\varepsilon > 0$ there is $\delta > 0$ such that for any δ-method φ in $\mathcal{T}_\alpha(f,\delta)$ and any point $x \in X$ there exists a point $y \in X$ for which

$$d(f^n(x), \varphi(y)_n) < \varepsilon, \ n \in \mathbb{Z}.$$

Now let us introduce two notions of weak inverse shadowing properties which are really weaker than the inverse shadowing property.

DEFINITION 2.1. We say that f has the *first weak inverse shadowing property* ($1\mathrm{WISP}_\alpha$) (resp. the *second weak inverse shadowing property* ($2\mathrm{WISP}_\alpha$)) with respect to the class $\mathcal{T}_\alpha$, $\alpha = 0, c, h$, if for any $\varepsilon > 0$ there exists $\delta > 0$ such that for any δ-method $\varphi \in \mathcal{T}_\alpha(f)$ and any point $x \in X$ there is a point $y \in X$ for which

$$\varphi(y) \subset N_\varepsilon(O(x,f)) \ [\text{resp. } O(x,f) \subset N_\varepsilon(\varphi(y))].$$

It is clear that if $\mathcal{T}_a \subset \mathcal{T}_b$, then the $1\mathrm{WISP}_a$ (resp. $2\mathrm{WISP}_a$) is weaker than the $1\mathrm{WISP}_b$ (resp. $2\mathrm{WISP}_b$), where $a, b \in \{0, c, h\}$.

REMARK 2.2. Every homeomorphism having the inverse shadowing property has the weak inverse shadowing property but the converse is not true. It is clear that every irrational rotation f on the unit circle S^1 has the $1\mathrm{WISP}_c$ and the $2\mathrm{WISP}_c$. But it does not have the inverse shadowing property with respect the class $\mathcal{T}_c$. In fact, it is known that f does not have the shadowing property (see [6]). Since the shadowing property and the inverse shadowing property with respect to the class $\mathcal{T}_c$ (or $\mathcal{T}_h$) for homeomorphisms on S^1 are pairwise equivalent (see [5]), we can see that f does not have the inverse shadowing property with respect to the class $\mathcal{T}_c$. Furthermore, we can show that every rational rotation on the unit circle does not have the $1\mathrm{WISP}_c$.

It is well known that every shift homeomorphism is expansive and has the shadowing property (see [6]). However, in the following example, we can know that every shift homeomorphism does not have the $1\mathrm{WISP}_h$

but it has the $2\mathrm{WISP}_h$ as we can see in Theorem 3.2. In fact, we can see that the shift homeomorphism does not have the $2\mathrm{WISP}_c$.

EXAMPLE 2.3. Let $\{0,1\}^{\mathbb{Z}}$ be the space of all two sided sequences $\mathbf{x} = \{\mathbf{x}_i : i \in \mathbb{Z}\}$ with elements $\mathbf{x}_i \in \{0,1\}$, endowed with a metric D defined by

$$D(\mathbf{x},\mathbf{y}) = \sup_{i\in\mathbb{Z}}\left\{\frac{|\mathbf{x}_i - \mathbf{y}_i|}{2^{|i|}}\right\},$$

where $\mathbf{x},\mathbf{y} \in \{0,1\}^{\mathbb{Z}}$. We also write this space as $\sum_2$ to shorten the notation. Define a shift map $\sigma : \sum_2 \longrightarrow \sum_2$ by

$$\sigma(\mathbf{x})_i = \mathbf{x}_{i+1} \quad (i \in \mathbb{Z}),$$

where $\mathbf{x} \in \sum_2$. We know that σ is an expansive homeomorphism with the shadowing property.

Now we show that the shift homeomorphism does not have the $1\mathrm{WISP}_h$. Let $\varepsilon = \frac{1}{2}$ and $\delta > 0$ be arbitrary. Choose a natural number $n \in \mathbb{N}$ with $2^{-n} < \delta$. Define a map $g_n : \sum_2 \longrightarrow \sum_2$ by

$$g_n(\mathbf{x})_i = \begin{cases} \mathbf{x}_i & \text{if } |i| > n, \\ \mathbf{x}_{i+1} & \text{if } i \in \{-n,\ldots,0,\ldots,n-1\}, \\ \mathbf{x}_{-i} + 1 \pmod 2 & \text{if } i = n, \end{cases}$$

where $\mathbf{x} \in \sum_2$. Then it is easy to show that g_n is a homeomorphism such that

$$g_n^{2(2n+1)} = \mathrm{id}_{\sum_2} \quad \text{and} \quad g_n^{(2n+1)}(\mathbf{x})_0 = \mathbf{x}_0 + 1 \pmod 2,$$

where $\mathrm{id}_{\sum_2}$ is the identity map on $\sum_2$. Moreover, each orbit $\{g_n^k(\mathbf{x}) : k \in \mathbb{Z}\}$ of g_n is a 2^{-n}-pseudo orbit of σ. Let φ_n be the continuous 2^{-n}-method for σ induced by the homeomorphism g_n. Define $\mathbf{x} \in \sum_2$ by $\mathbf{x}_i = 0$ for all $i \in \mathbb{Z}$. We can easily see that for any $\mathbf{y} \in \sum_2$ there is $k(\mathbf{y}) \in \mathbb{Z}$ such that

$$(*) \qquad D(\varphi_n(\mathbf{y})_{k(\mathbf{y})}, \mathbf{x}) > \frac{1}{2}.$$

In fact, if $\mathbf{y}_0 = 1$, then the inequality $(*)$ holds for $k(\mathbf{y}) = 0$. On the other hand, if $\mathbf{y}_0 = 0$, then the inequality $(*)$ holds for $k(\mathbf{y}) = 2n+1$. This implies that

$$\varphi_n(\mathbf{y}) \not\subset N_{\frac{1}{2}}(O(\mathbf{x},\sigma))$$

for any $\mathbf{y} \in \sum_2$, and so completes the proof.

3. Genericity of weak inverse shadowing

In the following, let M be a compact metric space. In this section, we will study the problem of genericity of the first weak inverse shadowing property and the second weak inverse shadowing property in the space $Z(M)$.

We recall that a property P of elements of topological space X is said to be *generic* if the set of all $x \in X$ satisfying P is *residual*, i.e., it includes a countable intersection of open and dense subsets of X.

The aim of this paper is to prove the following two theorems.

THEOREM 3.1. *The first weak inverse shadowing property* ($1\mathrm{WISP}_h$) *with respect to the class* $\mathcal{T}_h$ *is generic in* $Z(M)$.

THEOREM 3.2. *Every homeomorphism on* M *has the second weak inverse shadowing property* ($2\mathrm{WISP}_c$) *with respect to the class* $\mathcal{T}_c$.

To prove our theorems, some further notations and Lemma 3.3 are required.

Let X and Y denote a topological space and a metric space, respectively. We will write $C(Y)$ for the space of nonempty compact subsets of Y with a Hausdorff metric ϱ_H. A mapping $F : X \to C(Y)$ is called *upper semi-continuous* if for every $x \in X$ and $\varepsilon > 0$ there exists a neighborhood U of x such that $F(z) \subset N_\varepsilon(F(x))$ for all $z \in U$.

LEMMA 3.3. [10] *If* F *is upper semi-continuous, then for any* $\varepsilon > 0$ *the set*

$$\left\{ \begin{array}{ll} x \in X : & \text{there exists a neighborhood } U \text{ of } x \text{ such that} \\ & \varrho_H(F(x), F(z)) \leq \varepsilon \text{ for all } z \in U \end{array} \right\}$$

is open and dense in X.

Proof of Theorem 3.1. Let $\varepsilon > 0$ be arbitrary, and let $\mathcal{A} = \{U_1, \ldots, U_k\}$ be a finite covering of M by closed sets with diameter not greater than $\frac{\varepsilon}{2}$. Consider the set $\mathcal{K} = \{1, 2, \ldots, k\}$ as a compact metric space with a discrete metric. Define a map $\Psi : Z(M) \to C(C(\mathcal{K}))$ by

$$\Psi(f) = \left\{ \begin{array}{ll} L \subset \mathcal{K} : & \text{there is } x \in M \text{ such that } O(x, f) \subset \bigcup_{i \in L} U_i \\ & \text{and } O(x, f) \bigcap U_i \neq \emptyset \text{ for any } i \in L \end{array} \right\}.$$

First we show that the map Ψ is upper semi-continuous. To prove this, it is enough to show that for any $f \in Z(M)$ there is a neighborhood W_f of f in $Z(M)$ satisfying $\Psi(g) \subset \Psi(f)$ for any $g \in W_f$. Let $L \in$

$C(C(\mathcal{K})) - \Psi(f) := \Psi(f)^c$. Then for any $x \in M$ there exists $n(x) \in \mathbb{Z}$ satisfying

$$f^{n(x)} \notin \bigcup_{i \in L} U_i.$$

Since M is compact, we can choose $T > 0$ and $\alpha > 0$ such that for any $x \in M$ there is $n(x) \in \mathbb{Z}$ satisfying

$$|n(x)| \leq T \quad \text{and} \quad d(f^{n(x)}, \cup_{i \in L} U_i) > \alpha.$$

Choose $0 < \alpha_L < \frac{\alpha}{2}$ such that if $d_0(f, g) < \alpha_L$ for any $f, g \in Z(M)$, then $d_0(f^k, g^k) < \frac{\alpha}{2}$ for all $|k| \leq T$. Put $W_L(f) = B_{d_0}(f, \alpha_L)$. For any $g \in W_L(f)$ and $x \in M$, we have

$$d(f^{n(x)}(x), \bigcup_{i \in L} U_i) > \alpha \quad \text{and} \quad d(f^{n(x)}(x), g^{n(x)}(x)) < \frac{\alpha}{2}.$$

for some $|n(x)| < T$. Since

$$d(g^{n(x)}(x), \bigcup_{i \in L} U_i) > \frac{\alpha}{2},$$

we obtain $L \notin \Psi(g)$. Since $\Psi(f)^c$ is finite, the set

$$W_f = \bigcap_{L \in \Psi(f)^c} W_L(f)$$

is also a neighborhood of f such that

$$\Psi(f)^c \subset \Psi(g)^c$$

for all $g \in W_f$.

For any $\varepsilon > 0$, we let

$$R_\varepsilon = \left\{ \begin{array}{ll} f \in Z(M): & \text{there exists a neighborhood } W_f \text{ of } f \text{ such that} \\ & \varrho_H(\Psi(g_1), \Psi(g_2)) \leq \varepsilon \;\text{ for }\; g_1, g_2 \in W_f \end{array} \right\}.$$

Then the set R_ε is open and dense in $Z(M)$ by Lemma 3.3. Moreover, the map Ψ is locally constant on the set R_ε if $\varepsilon < 1$, i.e., for any $f \in R_\varepsilon$ there is a neighborhood W_f of f satisfying $\Psi(f) = \Psi(g)$ for all $g \in W_f$.

To complete the proof, it remains to show that the set

$$R := \bigcap_{n=1}^{\infty} R_{\frac{1}{n}}$$

is residual in the space $Z(M)$ such that each $f \in R$ has the 1WISP$_h$. Let $0 < \varepsilon < 1$ be arbitrary and $f \in R$. Then there is $\delta > 0$ such that the map Ψ is locally constant on the set $B_{d_0}(f, \delta)$. Let $\varphi \in \mathcal{T}_h(f, \delta)$ be a continuous δ-method for f and $x \in M$. Then there exists $g \in B_{d_0}(f, \delta)$ satisfying $\varphi(x)_k = g^k(x)$ for all $k \in \mathbb{Z}$. Choose $L \in \Psi(f)$ such that

$$O(x, f) \subset \bigcup_{i \in L} U_i \quad \text{and} \quad O(x, f) \cap U_i \neq \emptyset$$

for all $i \in L$. Since $\Psi(f) = \Psi(g)$, there exists $y \in M$ satisfying

$$O(y, g) \subset \bigcup_{i \in L} U_i \quad \text{and} \quad O(y, g) \cap U_i \neq \emptyset$$

for all $i \in L$. This implies that

$$\varphi(y) \subset N_\varepsilon(O(x, f)),$$

and so completes the proof. □

Proof of Theorem 3.2. Let f be a homeomorphism of M, let $\varepsilon > 0$ be arbitrary, and let $\mathcal{A} = \{U_1, \ldots, U_k\}$ be a finite open covering of M with diam$U_i < \frac{\varepsilon}{2}$ for all $i = 1, \ldots, k$. Put $\mathcal{K} = \{1, \ldots, k\}$. For each $x \in M$, we choose a subset L_x of $\mathcal{K}$ satisfying the following conditions :

(i) $O(x, f) \subset \bigcup_{i \in L_x} U_i$;
(ii) $O(x, f) \cap U_i \neq \emptyset$ for $i \in L_x$.

Put $\mathcal{L} = \{L_x : x \in M\}$. Since $\mathcal{L}$ is finite, we can choose a finite subset $\mathcal{P} = \{x_1, \ldots, x_n\}$ of M such that

$$\mathcal{L} = \{L_{x_i} : x_i \in \mathcal{P}\}.$$

For each $x_i \in \mathcal{P}$, $i = 1, \ldots, n$, we let

$$L_{x_i} = \{i_1, \ldots, i_m\}.$$

Then there are $k_1, \ldots, k_m \in \mathbb{Z}$ such that

$$f^{k_1}(x_i) \in U_{i_1}, \ldots, f^{k_m}(x_i) \in U_{i_m} \quad \text{and} \quad O(x_i, f) \subset \bigcup_{j=1}^{m} U_{i_j}.$$

Let

$$T_i = \max\{|k_j| : j = 1, \ldots, m\} \quad \text{and} \quad T = \max\{T_1, \ldots, T_n\}.$$

Choose $\delta > 0$ such that if $d(x, y) < \delta$ for $x, y \in M$, then $d(f^i(x), f^i(y)) < \frac{\varepsilon}{2T}$ for all $|i| \leq T$. Let $\varphi \in \mathcal{T}_c(f, \delta)$ be a continuous δ-method for f. Then we can easily show that

$$d(\varphi(x)_i, f^i(x)) < \frac{\varepsilon}{2}$$

for any $x \in M$ and $|i| \leq T$. For any $x \in M$, choose $x_j \in \mathcal{P}$ with $L_x = L_{x_j}$. Then we obtain

$$O(x,f) \subset \bigcup_{i \in L_{x_j}} U_i \quad \text{and} \quad O(x,f) \cap U_i \neq \emptyset$$

for all $i \in L_{x_j}$. For any $n \in \mathbb{Z}$, we can choose $i(n) \in L_{x_j}$ and $l(n) \in \mathbb{Z}$ such that

$$f^n(x) \in U_{i(n)} \quad \text{and} \quad f^{l(n)}(x_j) \in U_{i(n)},$$

where $|l(n)| < T$. Then we have

$$\begin{aligned} &d(f^n(x), \varphi(x_j)_{l(n)}) \\ &\leq d(f^n(x), f^{l(n)}(x_j)) + d(f^{l(n)}(x_j), \varphi(x_j)_{l(n)}) \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \end{aligned}$$

for all $n \in \mathbb{Z}$. This implies that

$$O(x,f) \subset N_\varepsilon(\varphi(x_j)),$$

and so completes the proof. □

Assumption that the phase space M is compact in Theorem 3.2 cannot be removed as the following example shows.

EXAMPLE 3.3. Consider a homeomorphism f on $\mathbb{R}^3$ given by

$$f(x_1, x_2, x_3) = (x_1, 2x_2, \frac{1}{2}x_3).$$

Suppose f has the 2WISP_c. Let $\varepsilon = \frac{1}{2}$ and $\delta > 0$ be the corresponding constant in the definition of 2WISP_c. Define a map $\varphi : \mathbb{R}^3 \longrightarrow (\mathbb{R}^3)^{\mathbb{Z}}$ by

$$\varphi(x_1, x_2, x_3)_n = (x_1 + \frac{n\delta}{2},\ 2^n x_2,\ \frac{1}{2^n} x_3)$$

for any $n \in \mathbb{Z}$. It is clear that φ is a continuous δ-method for f. Let $x = (2,1,1) \in \mathbb{R}^3$. Then we can easily show that

$$O(x,f) \not\subset N_\varepsilon(\varphi(y))$$

for all $y \in \mathbb{R}^3$. The contradiction shows that f does not have the 2WISP_c.

References

[1] R. Corless and S. Y. Pilyugin, *Approximate and real trajectories for generic dynamical systems*, J. Math. Anal. Appl. **189** (1995), no. 2, 409–423.
[2] P. Diamond, K. Lee, and Y. Han, *Bishadowing and hyperbolicity*, Internat. J. Bifur. Chaos Appl. Sci. Engrg. **12** (2002), no. 8, 1779–1788.
[3] P. Kloeden and J. Ombach, *Hyperbolic homeomorphisms and bishadowing*, Ann. Polon. Math. **65** (1997), no. 2, 171–177.
[4] K. Lee, *Continuous inverse shadowing and hyperbolicity*, Bull. Austral. Math. Soc. **67** (2003), no. 1, 15–26.
[5] K. Lee and J. Park, *Inverse shadowing of circle maps*, Bull. Austral. Math. Soc. **69** (2004), no. 3, 353–359.
[6] S. Y. Pilyugin, *Shadowing in dynamical systems*, Lecture Notes in Mathematics 1706, Springer-Verlag, Berlin, 1999.
[7] ______, *Inverse shadowing by continuous methods*, Discrete Contin. Dyn. Syst. **8** (2002), no. 1, 29–38.
[8] S. Y. Pilyugin, A. A. Rodionova, and K. Sakai, *Orbital and weak shadowing properties*, Discrete Contin. Dyn. Syst. **9** (2003), no. 2, 287–308.
[9] K. Sakai, *Diffeomorphism with weak shadowing*, Fund. Math. **168** (2001), no. 1, 57–75.
[10] F. Takens, *On Zeeman's tolerance stability conjecture*, Lecture Notes in Mathematics 197, Springer-Verlag (1971), 209–219.

Taeyoung Choi, Department of Mathematics, Chungnam National University, Daejeon 305-764, Korea
E-mail: shadowcty@hanmail.net

Sungsook Kim, Department of Applied Mathematics, Paichai University, Daejeon 302-735, Korea
E-mail: sskim@mail.pcu.ac.kr

Keonhee Lee, Department of Mathematics, Chungnam National University, Daejeon 305-764, Korea
E-mail: khlee@math.cnu.ac.kr