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Transient Analysis and Leakage Detection Algorithm using GA and HS algorithm for a Pipeline System

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The impact of leakage was incorporated into the transfer functions of the complex head and discharge. The impedance transfer functions for the various leaking pipeline systems were also derived. Hydraulic transients could be efficiently analyzed by the developed method. The simulation of normalized pressure variation using the method of characteristics and the impulse response method shows good agreement to the condition of turbulent flow. The leak calibration could be performed by incorporation of the impulse response method with Genetic Algorithm (GA) and Harmony Search (HS). The objective functions for the leakage detection can be made using the pressure-head response at the valve, or the pressure-head or the flow response at a certain point of the pipeline located upstream from the valve. The proposed method is not constrained by the Courant number to control the numerical dissipation of the method of characteristics. The limitations associated with the discreteness of the pipeline system in the inverse transient analysis can be neglected in the proposed method.

Key Words : Pipeline Transients, Leakage Detection, Evolutionary Optimization

1. Introduction

Hydraulic transients in a pressurized pipeline system have been frequently analyzed by the method of characteristics (Roberson et al., 1995 ; Wylie and Streeter, 1993). The free vibration analysis assuming steady flow of the pipeline system (Wylie and Streeter, 1993) provides the natural frequencies and mode shapes. The propagation operator and characteristic impedance were derived in early 1960s (Brown, 1962). After then, Zielke and Rosl (1971) proposed a model for the frequency dependent friction of transient laminar flow on the platform of a method of character-

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istics. An experimental study of unsteady pipe flow friction modeling showed the validity of the Zielke's model for the laminar and the turbulent flow of low Reynolds number (Bergant et al., 2001). Integration of Fast Fourier Transform and hydraulic transient analysis had been used to control data noise and improve leak prediction (Kim et al., 2001). However, an alternative numerical method for frequency dependent pipeline transients is the impulse response method (Wylie and Streeter, 1993; Suo and Wylie, 1989).

In the present study, an alternative method for hydraulic transient analysis and leak detection algorithm is presented which requires the time series of pressure or flow-rates at any points of the pipeline about a simple operation of the valve. This paper addresses the free vibration analysis in the steady oscillatory flow and then the impulse response analysis of the pipeline system with leakages is presented. The frequency dependent friction is incorporated to consider the impact of unsteady friction in the laminar or turbulent flow of the low Reynolds number. The integrations of genetic algorithm (GA) or Harmony Search (HS) into the impulse response method provide the calibration methods of leak location. The results and discussions demonstrate the potential of the proposed method.

2. Transient Analysis in a Pipeline

The momentum and continuity equations for the transient flow in a pipeline are respectively given as follows:

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{f|Q|Q}{2DA}=0 \tag{1}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{2}$$

where x, t, a, g, A, Q, H, f and D are distance along a pipeline, time, the wave speed, the gravitational acceleration, the cross sectional area of a pipe, discharge of flow, the head, the Darcy-Weisbach friction factor, and the pipe diameter, respectively (Wylie and Streeter, 1993).

Derivations from equations (1) and (2), assuming a steady oscillatory flow and linearized friction, provide the complex head and discharge as a function of distance, x, as follows (Wylie and Streeter, 1993):

$$H(x)=H_v\cosh\gamma x-Z_C Q_U\sinh\gamma x \tag{3}$$

$$Q(x)=-\frac{H_U}{Z_C}\sinh\gamma x+Q_U\cosh\gamma x \tag{4}$$

where subscript U denotes the upstream section. Propagation constant γ, characteristic impedance Z_C, and complex frequency s' are respectively defined as:

$$\gamma=\sqrt{Cs'(Ls'+R)} \tag{5}$$

$$Z_C=\frac{\gamma}{Cs'} \tag{6}$$

$$s'=\sigma+i\omega \tag{7}$$

where the capacitance (C) is gA/a^2, the inertance L is $1/gA$, the resistance (R) is $f\bar{Q}/gAD^2$, σ is a decay factor, and ω is the frequency.

The hydraulic impedance $Z(x)$ is defined as the ratio of the complex head to the complex discharge.

$$Z(x)=\frac{H(x)}{Q(x)} \tag{8}$$

The impedance transfer function at the upstream end as a function of impedance at the downstream end can be expressed as:

$$Z_U=\frac{Z_D+Z_C\tanh\gamma l}{1+(Z_D/Z_C)\tanh\gamma l} \tag{9}$$

where l is the length of a pipeline. If the upstream impedance is determined by the constant head reservoir, then equation (9) can be used to configure the frequency characteristics of the system. The relationship of a Fourier transform can be made between the response function and the transfer function in a reservoir-pipe-valve (RPV) system. When a discharge impulse is imposed at the downstream valve of the RPV system, the pressure head response at the valve with the reservoir boundary condition of $H_u=0$, is expressed as (Suo and Wylie, 1989)

$$r_{Dh}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty(-Z_C\tanh\gamma l)\,e^{i\omega t}d\omega\right] \tag{10}$$

where Re represents the "real part of".

The pressure head response at x upstream distance from valve is

$$r_{xh}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty Z_c(\sinh\gamma x - \cosh\gamma x\tanh\gamma l)\,e^{i\omega t}d\omega\right] \tag{11}$$

The pressure flow response at x upstream distance from valve is

$$r_{xq}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^\infty (\cosh\gamma x - \sinh\gamma x\tanh\gamma l)\,e^{i\omega t}d\omega\right] \tag{12}$$

Assuming the discharge pulse at the valve is $\Delta q_D(t)$ and convolving the pressure head response, or the pressure flow response provides the pressure head change at the valve or the pressure head and discharge changes at x upstream section from valve as (Suo and Wylie, 1989)

$$\Delta h_D(t)=\int_0^t r_{Dh}(t-T)\Delta q_D(T)\,dT \tag{13}$$

$$\Delta h_x(t)=\int_0^t r_{xh}(t-T)\Delta q_D(T)\,dT \tag{14}$$

$$\Delta q_x(t)=\int_0^t r_{xq}(t-T)\Delta q_D(T)\,dT \tag{15}$$

The valve boundary condition of the RPV system is

$$q_D(t)=(C_dA)_t\sqrt{2gh_D(t)} \tag{16}$$

where $q_D(t)=q_{D0}+\Delta q_D(t)$ and $h_D(t)=h_{D0}+\Delta h_D(t)$, and q_{D0} are h_{D0} the initial discharge and pressure head, respectively.

The valve boundary condition of the reference state such as the initial state of closing a valve or the final state of opening a valve is

$$q_{Dr}=(C_dA)_r\sqrt{2gh_{Dr}} \tag{17}$$

where the subscript r refers to the reference state. In conjunction with the valve boundary conditions of equations (16) and (17), the application of discrete convolution to equation (13) provide the explicit expression of the plus sign of discharge variation at the valve as,

$$\Delta q_D(t)=-b+\sqrt{b^2-c} \tag{18}$$

where $c=q_{D0}^2-\tau^2q_{Dr}^2(h_{D0}+\Delta h'_D)/h_{Dr}$, $b=q_{D0}-\tau^2q_{Dr}^2r_{Dh}(0)\Delta t/2h_{Dr}$, $r_{Dh}(0)$ is the pressure response function depending upon the discharge pulse at the valve, and τ is the relative opening of the valve.

The frequency dependent friction can be accounted by replacing the propagation operator, γx, and the characteristic impedance, Z_c, by the following equations as (Wylie and Streeter, 1993 ; Suo and Wylie, 1989)

$$\Gamma(s')=\frac{s'x}{a}\left(1-\frac{2J_1\left(i\frac{D}{2}\sqrt{\frac{s'}{\nu}}\right)}{i\frac{D}{2}\sqrt{s'\nu}\,J_0\left(i\frac{D}{2}\sqrt{\frac{s'}{\nu}}\right)}\right)^{-1/2} \tag{19}$$

$$Z(s)=\frac{a}{gA}\left(1-\frac{2J_1\left(i\frac{D}{2}\sqrt{\frac{s'}{\nu}}\right)}{i\frac{D}{2}\sqrt{\frac{s'}{\nu}}\,J_0\left(i\frac{D}{2}\sqrt{\frac{s'}{\nu}}\right)}\right)^{-1/2} \tag{20}$$

where J_0 and J_1 are the first-type Bessel function of zero and first order, respectively.

The impedance transfer function with a leak is derived in the RPV system. The schematic view of this system is shown in Fig. 1. The impedance at the upstream of leak position is expressed as

$$Z_{upleak}=-Z_c\tanh\gamma x_{up} \tag{21}$$

where x_{up} is the distance from upstream reservoir to the leak point.

The impedance at the downstream of leak position can be derived using the point matrix for leak (Mpesha et al., 2001) as

$$Z_{downleak}=\frac{Z_{upleak}}{1-\frac{Q_{olk}}{2\cdot H_o}Z_{upleak}} \tag{22}$$

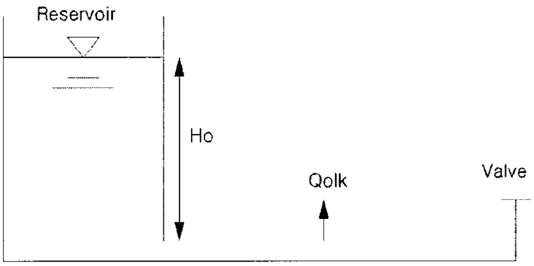

Fig. 1 A single pipeline system with one leak

where Q_{olk} is the mean discharge of leak and H_o is the mean head pressure. Therefore, when a discharge impulse is imposed at the downstream valve of the RPV system with a leak, the pressure head response at the valve can be expressed as

$$r_{Dh}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^{\infty}\left(\frac{-Z_c\tanh\gamma x_{down}+Z_{downleak}}{1-\frac{Z_{downleak}}{Z_c}\tanh\gamma x_{down}}\right)\cdot e^{i\omega t}d\omega\right] \quad (23)$$

where x_{down} is the distance from the leak point to the downstream valve.

The pressure head response between the valve and leak position is

$$\gamma_{x1h}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^{\infty}(Z_{valve}\cosh\gamma x_1+Z_c\sinh\gamma x_1)\cdot e^{i\omega t}d\omega\right] \quad (24)$$

where x_1 is the distance from the valve to the response point and Z_{valve} is the hydraulic impedance at the valve position.

The flow response between the valve and leak position is

$$\gamma_{x1q}(t)=\frac{1}{\pi}\mathrm{Re}\left[\int_0^{\infty}\left(\frac{Z_{valve}}{Z_c}\sinh\gamma x_1+\sinh\gamma x_1\right)\cdot e^{i\omega t}d\omega\right] \quad (25)$$

3. Optimization

3.1 Harmony search

The Harmony Search (HS) Algorithm is used to minimize the impact of multiple local optima (Geem, 2000). The HS based on a simple idea that musical performance seeks a consonant harmony. A musical performance is made from the multiple instruments. Practices are repeated for the best play. The best aesthetic play can be determined by the objective function. Objective function is evaluated by the set of parameters generated by the component variables. The procedure of HS is as follows;

Step 1. The environments of performance are determined. The algorithm parameters such as the number of variables, the range of variables, the size of harmony memory, the harmony memory considering rate, the pitch adjusting rate and the stopping criteria are specified. The harmonic memory is produced and sorted by the objective function.

Step 2. A new harmony is produced from the harmonic memory. The hybrid between variables of existing harmony and a randomized variable is made to explore a better combination.

Step 3. The harmonic memory is renewed by the evaluation of objective function.

Step 4. The computation is terminated if the stopping criterion is satisfied. Otherwise repeat the procedure from the Step 2.

3.2 Leak detection with genetic algorithm

Detection of leak position and area is an important issue in field of hydraulic engineering. Many algorithms of leak detection are used the measured time series of pressure or discharge to calibrate the possible leak locations (Liggett and Chen, 1994; Nash and Karney, 1999; Vitkovsky et al., 2000). In this study, the detection scheme of the leak parameters was designed using the genetic algorithm (GA). The impulse response method is integrated into the GA. The GA is a powerful search tool that utilized evolutionary based principles to find optimal solutions (Goldberg, 1989). As in the case of evolution, the GA starts with a population of potential solution. A string of chromosome represents a possible solution of the problem. The fitness of each solution is estimated by employing an objective function. The fitness of each solution is estimated by employing an objective function. The objective function of the time series of the pressure head can be expressed as

$$\mathit{Minimize}\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t))^2\right\} \quad (26)$$

where $h_m(t)$ is the time series of the measured or specified pressure head and $h_o(t)$ is the time series of the optimized pressure head.

The objective function of the time series of flow rate is

$$\mathit{Minimize}\left\{\sum_{t=1}^{end}(Q_m(t)-Q_o(t))^2\right\} \quad (27)$$

where $Q_m(t)$ is the time series of the measured or specified discharge and $Q_o(t)$ is the time series of the optimized discharge.

The hybrid objective function can be expressed as

$$\text{Minimize}\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t))^2+(Q_m(t)-Q_o(t))^2\right\} \quad (28)$$

Constraints of the system can be feasibly considered in the objective function by integrating the penalty function. Fig. 2 shows the flowchart of the integration of the impulse response method with GA for the leak detection. Furthermore, HS has been incorporated with the impulse response method as the similar way of the GA integration.

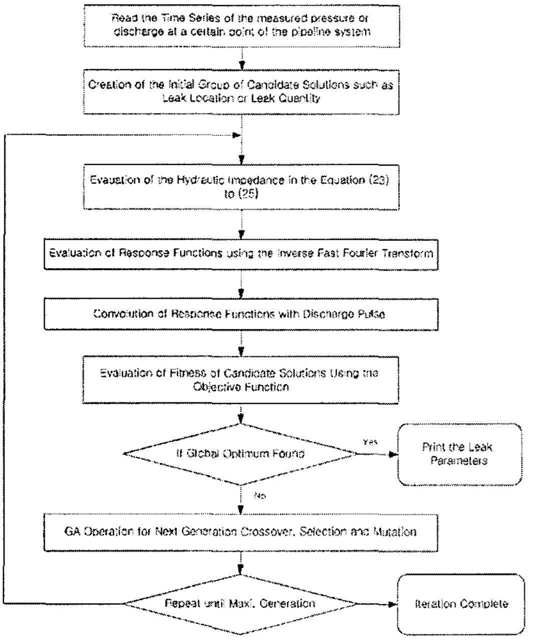

Fig. 2 The flowchart of leak detection by genetic algorithm

4. Application Examples

Consider a simple horizontal pipeline system with a control valve at the downstream end of the pipeline. Assume that the pipeline passes water from a constant head supply reservoir to another reservoir at various flow rates depending upon the differences of pressure heads between two reservoirs. As Fig. 1 illustrates, the pipeline is 100 m in length and 0.015 m in diameter. The Darcy-Weisbach friction factor of the pipeline is assumed to 0.02. The control valve executes a closure from a full gate opening. The wave propagation speed is 1,300 m/sec. Water hammers are introduced from the valve closures in 0.01 and 0.3 seconds.

4.1 Transient analysis

The impulse response method is applied to calculate the time series of pressure head and discharge at the 50 m point from the valve. Considering the length of pipeline and wave speed, the frequency range is truncated at 82.68 rad/sec. The number of sample for FFT is determined to 2048. Both turbulent and laminar flow conditions are used for the simulation of water hammer. Fig. 3

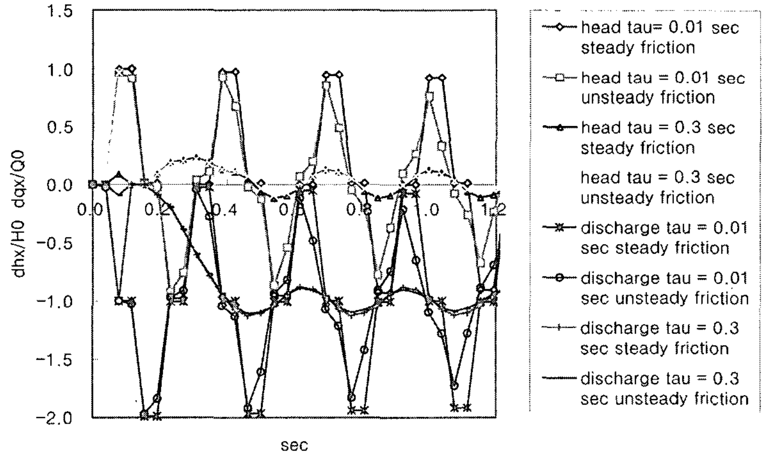

Fig. 3 The normalized pressure heads and discharges due to sudden and slow valve closures on the laminar flow ; Steady friction : Constant friction, Unsteady friction : Frequency dependent friction

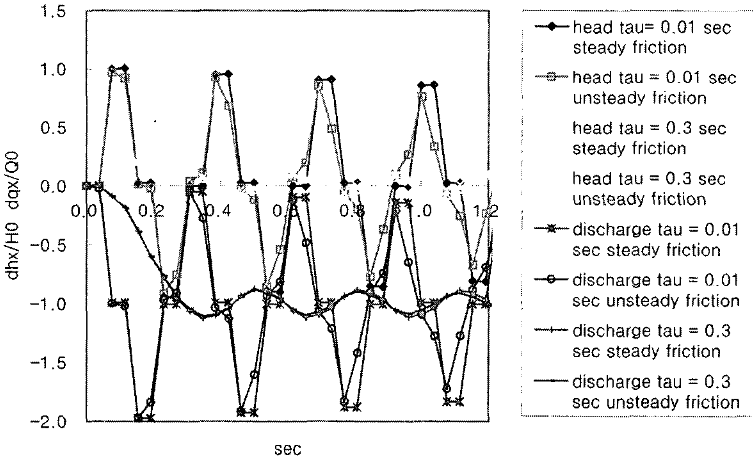

Fig. 4 The normalized pressure heads and discharges due to sudden and slow valve closures on the turbulent flow ; Steady friction : Constant friction, Unsteady friction : Frequency dependent friction

shows the normalized pressure and discharge variations on a laminar flow (Reynolds Number=1282).

Depending upon the valve operation and friction consideration, the time series of pressure and discharge illustrate distinct oscillatory behaviors with damping. The pressure and discharge of unsteady friction by the frequency-dependent friction factor provide additional mitigation to the water hammer (Fig. 3). The impact of unsteady friction appear to even more significant in the sudden valve closure. The time series of normalized pressure and discharge on a turbulent flow are shown in Fig. 4 (Reynolds Number=6410.26). The trend of variations is similar to the laminar flow. The phases of the slow valve closure (0.3 second) appear different to the laminar flow. This difference is primarily associated with the impact of the resistance per unit length, R, to the propagation of wave. The resistance of the laminar flow is related to the diameter of a pipeline and the viscosity of fluid, the resistance of the turbulent flow is additionally influenced by the variation of velocity, which can be sensitive to the valve operation. The method of characteristics (Wylie and Streeter, 1993) is applied to validate the impulse response method. Fig. 5 compares the simulation of normalized pressure variation using the method of characteristics and the impulse response method on the turbulent flow. The results of two methods illustrate good agreement. Suo and Wylie (1989) reported similar result on the laminar flow (Reynolds Number=81.89). This study shows the validity of the impulse response method on the turbulent flow (Reynolds Number=6410.26).

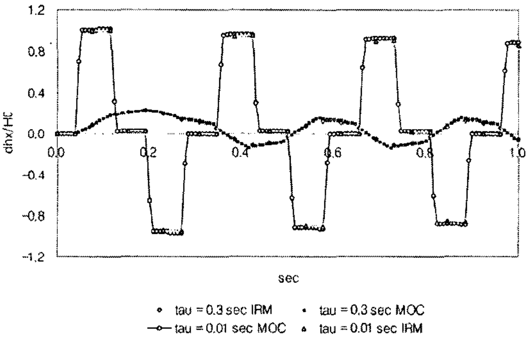

Fig. 5 The transients due to sudden and slow valve closures on the turbulent flow with constant friction ; MOC : Method of characteristics, IRM : Impulse response method

4.2 Leak detection by GA and HS

Proposed leak detection algorithm is applied to a pipeline with a leakage at 11.11 m from the control valve of the downstream reservoir (Fig. 1). The quantities of leak are assumed to be 5% and 0.2% to the mean discharge. The pressure and discharge data were assumed to record at 50 m from the upstream reservoir over 2 seconds from the valve operation.

Assuming the leak quantity can be easily evaluated from pre-calibration or it is known before the calibration through the measurement of discharges, the leak location remains a primary variable for calibration. Leak calibration of fre-

Table 1 The calibrations of leakage for a pipeline system by the Impulse response method with genetic algorithm ; the numbers of underline are the exact location of leak in meter, F-D friction ; frequency dependent friction, X_{leak} ; the location of Leak in meter

	Flow	Laminar (V_0=0.1 m/sec, Re=1282.05)		Turbulent (V_0=0.5 m/sec, Re=6410.26)	
Objective Function	Valve Closure	Sudden τ=0.01 sec	Slow τ=0.3 sec	Sudden τ=0.01 sec	Slow τ=0.3 sec
	Parameters	X_{leak}	X_{leak}	X_{leak}	X_{leak}
$Mini\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t)_x^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$
	F-D Friction Q_{leak}/Q_0 : 0.2%	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.12 $\underline{11.11}$
$Mini\left\{\sum_{t=1}^{end}(Q_m(t)-Q_o(t)_x^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$
	F-D Friction Q_{leak}/Q_0 : 0.2%	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$
$Mini\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t)_{valve}^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$	11.11 $\underline{11.11}$
	F-D Friction Qleak/Q0 : 0.2%	11.11 $\underline{11.11}$	11.12 $\underline{11.11}$	11.11 $\underline{11.11}$	11.12 $\underline{11.11}$

Table 2 The calibrations of leakage for a pipeline system by the impulse Response method with harmony search ; the numbers of underline are the exact location of Leak in meter, F-D friction ; Frequency dependent friction, X_{leak} ; The location of leak in meter

	Flow	Laminar (V_0=0.1 m/sec, Re=1282.05)		Turbulent (V_0=0.5 m/sec, Re=6410.26)	
Objective Function	Valve Closure	Sudden τ=0.01 sec	Slow τ=0.3 sec	Sudden τ=0.01 sec	Slow τ=0.3 sec
	Parameters	X_{leak}	X_{leak}	X_{leak}	X_{leak}
$Mini\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t)_x^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$
	F-D Friction Q_{leak}/Q_0 : 0.2%	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.12 $\underline{11.11}$
$Mini\left\{\sum_{t=1}^{end}(Q_m(t)-Q_o(t)_x^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$
	F-D Friction Q_{leak}/Q_0 : 0.2%	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$
$Mini\left\{\sum_{t=1}^{end}(h_m(t)-h_o(t)_{valve}^2)\right\}$	F-D Friction Q_{leak}/Q_0 : 5%	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$	11.18 $\underline{11.11}$
	F-D Friction Qleak/Q0 : 0.2%	11.18 $\underline{11.11}$	11.12 $\underline{11.11}$	11.18 $\underline{11.11}$	11.12 $\underline{11.11}$

quency dependent friction is performed to consider the impact of unsteady friction to multiple local solutions of the GA. Considering feasibility of data acquisition, three types of objective functions are used for the detection of leak location. These are the objective functions of the pressure or discharge at a certain point of pipeline and the pressure head at the valve. 2,500 iterations with identical input parameters of GA are used.

Table 1 shows GA calibration results of leak point. Most cases provide excellent predictions of leak location. Negligible differences were found between sudden and slow valve closure condition. Table 2 shows HS calibration results of leak point. No differences of leak calibration were found between laminar and turbulent flow condition. The application examples show that impulse response method has the leak detection capability up to 0.2% of the mean discharge. Smaller leakage than 0.2% of the mean discharge can be detected if the proposed algorithm employs more elaborated options of the evolutionary optimization. Theoretically, if the valid digit of number in computation can be extended, the calibration of leak can be performed even finer scale.

5. Conclusions

An alternative leak detection method was developed and applied for the pipeline systems. The pressure head or discharge responses are derived in any point of various pipeline systems including the impact of leakage. The genetic algorithm and the harmony search are incorporated into the impulse response method to calibrate the location and quantity of leakage. The leak detection capability of the proposed method is evaluated in various conditions such as the valve closing operation, laminar or turbulent flow, and steady or frequency-dependant friction assumptions. The accuracy associated with the prediction of leak location is apparently improved than the other approaches. The interpolation issue of the platform of the method of characteristic can be neglected. The propose method can be applied in the calibration of the friction of pipeline segments or other energy dissipater such as internal valve. The experimental verification of the impulse response method could be a future research objective.

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