Analysis of Orthotropic Bearing Non-linearity Using Non-linear FRFs

Dong-Ju Han*
Research Center, Sunaerosys Co.,
Songwon-ri, Nam-myeon, Yeongi-gun, 248-1, Chungnam 339-822, Korea

Among other critical conditions in rotor systems the large non-linear vibration excited by bearing non-linearity causes the rotor failure. For reducing this catastrophic failure and predictive detection of this phenomenon the analysis of orthotropic bearing non-linearity in rotor system using higher order frequency response functions (HFRFs) is conducted and is shown to be theoretically feasible as that of non-rotating structures. The complex HFRFs based on the Volterra series are newly developed for the process and investigated their features by using the simple forms of the FRFs associated with the forward and the backward modes.

Key Words : Orthotropic Bearing Non-Linearity, Non-Linear Frequency Response Functions (FRFs), Higher Order Volterra Kernel

Nomenclature

C_m, C_Δ : Dampings for linear mean, deviatoric properities
$C_{mn}, C_{\Delta n}$: Dampings for non-linear mean, deviatoric properities
c_y, c_z, c_{yz}, c_{zy} : Dampings for linear coefficients in y-z directions
c_{ny}, c_{nz} : Dampings for non-linear coefficients in y-z directions
D_m, D_Δ : Dynamic mean, deviatoric stiffnesses
$g(t)$: Complex input force
G : Magnitude of harmonic input
H : Frequency response
h : Kernel
J_P : Polar moment of inertia of rotor
j : Unit imaginary number $(=\sqrt{-1})$
K_m, K_Δ : Stiffnesses for linear mean, deviatoric properities
$K_{mn}, K_{\Delta n}$: Stiffnesses for non-linear mean, deviatoric properities
k_y, k_z, k_{yz}, k_{zy} : Stiffnesses for linear coefficients in y-z directions
k_{ny}, k_{nz} : Stiffnesses for non-linear coefficients in y-z directions
m : Rotor mass
N : Total number of coordinates
$p(t)$: Complex output response
t : Time
u : Integral parameter of kernel
$y(t), z(t)$: Real displacements in y-z coordinates
Ω : Rotational speed
ω : Rotational frequency

Superscripts

$(\bar{\ })$: Complex conjugate

Subscripts

m, Δ : Mean, deviatoric properties
m, n : Integers for index
y, z : y- and z-directional real-valued properties

* **E-mail** : djhan@sunaerosys.com
TEL : +82-41-864-2177; **FAX** : +82-41-864-2035
Research Center, Sunaerosys Co., Songwon-ri, Nam-myeon, Yeongi-gun, 248-1, Chungnam 339-822, Korea.
(Manuscript **Received** June 22, 2005; **Revised** January 9, 2006)

1. Introduction

Another special and local properties but generally accepted feature to deviate the rotor systems are the stator or bearing non-linearities, let alone manufacturing errors, clearances and joint surfaces. Especially the rolling element bearing, hydrodynamic bearings and squeeze film dampers are known to possess highly non-linear characteristics and their reliabilities are directly related to closer predictions and analysis of dynamic responses. Hence it is quite reasonable to investigate the non-linear analysis for effective diagnosis or the identification for bearing non-linearity. The mathematical non-linear analysis in rotor systems, by which the dynamic behaviors along with the parameter can be determined, still remains some far from the practical applications, however, they are useful and advantageous in explaining the physical phenomena. As a result, the main issues of the diagnosis for non-linear properties in stator have been analyzing the characteristics of its behavior from the signals in practice. In these respects, Lin (1993) presented a non-linear analysis of complex modes with hysteretic damping model in general structure and Ozguven (1993) also introduced the similar concept for non-linear frequency response. Tiwari (1995) suggested a non-linear parameter identification of rolling element stiffness by introducing probability density function with the model of a cubic non-linearity. Liangsheng (1993) introduced the concept of the pseudo-phase diagram and spectrum from the raw signals. Also other researches have been made using non-linear time series model as NARMAX for its relatively analytical easiness though its computational efforts and contamination by noise. Most of those researches are limited to simple general structures or simple rotor systems with isotropic bearing, which are in practice the same as the simple stationary structures, so that few attempts have been made in the analysis of the rotor systems with non-linear orthotropic bearing. As these reasons, in this study, in the sense that if the non-linearity can be expressed in polynomial form and the system is stable and time-invariant, the HFRFs based on the Volterra series (Tiwari and Vyas, 1995 ; Storer and Tomlinson, 1993 ; Vyas and Chatterjee, 2000 ; Zhang and Billings, 1993 ; Bedrosian and Stephen, 1971 ; Schetzen, 1990) are the practically valuable tools for analyzing the nature of non-linearity in wide class of structures, the complex HFRFs for rotor systems with orthotropic bearing are newly investigated and show the feasibility to further application. In particular, the non-linear complex FRFs by the non-linear complex modal analysis (Lin and Lim, 1993 ; Ozguven and Imregun, 1993) which utilizes between complex inputs and outputs for effective modal parameter identification and gives not only the directivity of the backward and forward modes but also separates those modes completely in the frequency domain so that effective modal parameter identification is possible (Lee et al., 2001), have been a new application to the case of non-linear rotating or stationary systems.

2. Representations of Bearing Non-linearity

The bearing non-linearity is mainly caused by the dynamic motion of the fluid film between the rolling element bearing and its journal of the hydrostatic or hydrodynamic bearing. The stiffness and damping characteristics of these bearings are non-linear functions of their displacements and clearances for relatively large amount of motions (Storer and Tomlinson, 1993). These two properties are coupled together by the motion parameters, i.e., the displacements, clearances and rotational speeds, so that changes of the damping result in those of the stiffness and vice-versa. For non-linear analysis of the bearing properties, however, the stiffness and damping characteristics are premised to be polynomial expressions to appropriate degrees for the closed form formulation, which is generally accepted to be reasonable in past studies (Storer and Tomlinson, 1993 ; Vyas and Chatterjee, 2000 ; Zhang and Billings, 1993 ; Worden and Manson, 1998 ; Han et al., 1998). In such a case, the trade-off studies are preceded for the polynomial degree for closer access to the damping and stiffness non-linearities. Normally in this case cubic form is available in effect. The other non-linear properties of the stator (bearing) beyond the assumed polynomial model are rubbing, radial clearance, hysteric or coulomb damping, bilinear stiffness etc, in which case the polynomial assumption is heavy and rough, however, the identification of these other ones are naturally deduced by resulting phenomenon of the polynomial model. Accordingly, in this study, the analysis of the bearing non-linearity is based on the assumption of that in polynomial form.

For the anisotropic rigid rotor, which is assumed to be simply supported at both ends and only one degree of freedom is used for the displacements in the y and z directions, with non-linearities (cubic) of dampings, c_{ny}, c_{nz} and stiffnesses, k_{ny}, k_{nz}, of the bearings (see Figs. 1 and 2) which is ubiquitous in Duffing oscillator (Worden and Manson, 1998), the equation can be derived in the cubic polynomial form of

$$\begin{aligned} &m\ddot{y}+c_y\dot{y}+J_p\Omega\dot{z}+c_{yz}\dot{z}+c_{ny}\dot{y}^3 \\ &+k_y y+k_{yz}z+k_{ny}y^3=g_y, \\ &m\ddot{z}+c_z\dot{z}+J_p\Omega\dot{y}+c_{zy}\dot{y}+c_{nz}\dot{z}^3 \\ &+k_z y+k_{zy}y+k_{nz}z^3=g_z, \end{aligned} \quad (1)$$

where m, c_y, c_{yz}, c_z, c_{zy}, k_y, k_{yz}, k_z, k_{zy}, J_p, Ω, are rotor mass, bearing dampings and their stiffnesses, rotor polar moment of inertia normalized by squared shaft length, and rotational speed, respectively. Applying the complex notations such as $p=y+jz$, $\bar{p}=y-jz$, $g=g_y+jg_z$ to Egs. (1), the equation leads to the following complex form

$$\begin{aligned} &M\ddot{p}+(C_m-jJ_p\Omega)\dot{p}+K_m p+C_\Delta\dot{\bar{p}}+K_\Delta\bar{p} \\ &+[C_{mn}(\dot{p}^3+3\dot{p}^2\dot{\bar{p}})+C_{\Delta n}(\dot{\bar{p}}^3+3\dot{p}\dot{\bar{p}}^2)] \\ &+[K_{mn}(p^3+3p^2\bar{p})+K_{\Delta n}(\bar{p}^3+3p\bar{p}^2)]=g, \end{aligned} \quad (2)$$

where the bracket term denotes the non-linear effects and their parameters are

$$\begin{aligned} &M=m,\ C_m=(c_y+c_z)/2+j(c_{yz}-c_{zy})/2, \\ &C_\Delta=(c_y-c_z)/2+j(c_{yz}+c_{zy})/2, \\ &K_m=(k_y+k_z)/2+j(k_{yz}-k_{zy})/2, \\ &K_\Delta=(k_y-k_z)/2+j(k_{yz}+k_{zy})/2, \\ &C_{mn}=(c_{ny}-c_{nz})/8,\ C_{\Delta n}=(c_{ny}+c_{nz})/8, \\ &K_{mn}=(k_{ny}-k_{nz})/8,\ K_{\Delta n}=(k_{ny}+k_{nz})/8. \end{aligned}$$

Here case the damping and stiffness parameters are assumed to be independent of the rotational speed for more or less higher speed range (Genta, 1988).

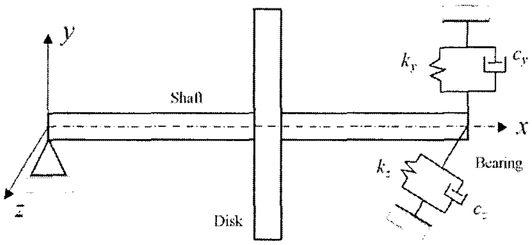


Fig. 1 Modeling a simple rotor system with orthotropic bearing

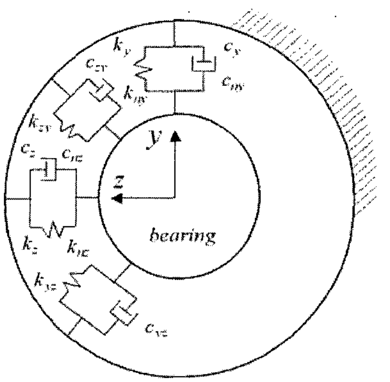


Fig. 2 Bearing stiffness and damping coefficient

3. Non-linear Response Analysis

The output $P(t)$ of a non-linear system in power of the input $g(t)$ is expressed by the well-known Volterra series (Bedrosian and Stephen, 1971), which is depicted in the form of

$$P(t)=\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\ \cdots,\ u_n)\prod_{r=1}^{n}g(t-u_r), \quad (3)$$

where $h_n(u_1, \cdots, u_n)$ is the n-th order kernel of the system, in this case the 1st order kernel $h_1(u_1)$ is the impulse response of a linear system.

The n-fold Fourier transformation of the kernel is the n-th order Volterra kernel that has an analogy to the n-th order transfer function, which is described as

$$\begin{aligned} &H_n(\omega_1,\ \cdots,\ \omega_n) \\ &=\frac{1}{n!}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\ \cdots,\ u_n)\,e^{-j\omega_r u_r}, \end{aligned} \quad (4)$$

where the 1st order Volterra kernel $H_1(\omega_1)$ is the transfer function of a linear system.

To derive the general form of an expression of the Volterra series by the series of harmonic inputs such as $g(t)=\sum_{m=1}^{p} g_m e^{j\omega_m t}$, the dummy coefficients α_s and the differential operator, which is the form as $D_\alpha^n \equiv \frac{\partial^n}{\partial\alpha_1\cdots\partial\alpha_n}\Big|_{\alpha_1=\cdots=\alpha_n=0}$, are used.

Applying this dummy variable and the sum of n exponential terms as the form of $A_n(\omega)=\sum_{s=1}^{n}\alpha_s e^{-j\omega u_s}$ Eq. (4), the n-th order Volterra kernel, or the n-th order transfer function of the system becomes

$$\mathrm{H}_n(\omega_1,\cdots,\omega_n)=\frac{1}{n!}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\cdots,u_n)D_\alpha^n\prod_{r=1}^{n}A_n(\omega_r), \quad (5)$$

and using the multinomial theorem, the term of power by the harmonic inputs becomes

$$\begin{aligned}\prod_{r=1}^{n} g(t-u_r) &= D_\alpha^n \exp\left[\sum_{r=1}^{n}\alpha_r g(t-u_r)\right]\\ &= D_\alpha^n\frac{1}{n!}\left[\sum_{r=1}^{n}\alpha_r g(t-u_r)\right]^n\\ &= D_\alpha^n\frac{1}{n!}\left[\sum_{r=1}^{n}\sum_{m=1}\alpha_r g_m e^{j\omega_m(t-u_r)}\right]^n\\ &= D_\alpha^n\frac{1}{n!}\left[\sum_{m=1}^{p} g_m e^{j\omega_m t}A_n(\omega_m)\right]^n\\ &= D_\alpha^n\left\langle\sum_{n_1=1}^{n}\sum_{n_2=1}\cdots\sum_{n_p=1}\frac{1}{n_1!\,n_2!\cdots n_p!}\left\{\prod_{m}^{p}[g_m e^{j\omega_m t}A_n(\omega_m)]^{n_m}\right\}\right\rangle,\end{aligned} \quad (6)$$

where n_m means the mth polynomial of the nth factorial number. Finally from Eqs. (5) and (6), the general complex form of Volterra series of the output by the input of pth multiple harmonics is derived by the following equation

$$\mathrm{P}(t)=\sum_{n=1}^{\infty}\left\langle\sum_{n_1=1}^{n}\sum_{n_2=1}\cdots\sum_{n_p=1}\frac{1}{n_1!\,n_2!\cdots n_p!}\left\{\prod_{m}^{p}[g_m e^{j\omega_m t}]^{n_m}\mathrm{H}_{n,n_m}(\omega_m)\right\}\right\rangle, \quad (7)$$

where $\sum_{i=1}^{p} n_i=n$, for all i; $n_i\geq 0(i=1\sim p,\ i$: integer) and $\mathrm{H}_{n,n_m}(\omega_m)$ denotes $\mathrm{H}_n(\omega_1,\cdots,\omega_m,\cdots,\omega_p)$ with the mth harmonic input of the ω_i equal to $+\omega_m$ and the remaining ω_{p-m} equal to $-\omega_m$.

4. Identification of Non-linear Frequency Responses

For the case of rotor system with orthotropic bearings by the input of sweeping single-tone excitation, $g(t)=Ge^{j\omega t}$, the solution form of the Volterra series in Eq. (7) can be deduced from Eq. (2) using the concept of a linear solution associated with the forward (H) and backward modes ($\bar{\mathrm{H}}$), which resaits in the form of

$$\begin{aligned}p(t)&=\mathrm{p}(t)+\tilde{\mathrm{p}}(t)\\ &=\mathrm{H}_1(\omega)Ge^{j\omega t}+\bar{\mathrm{H}}_1(\omega)\bar{G}e^{-j\omega t}\\ &\quad+\mathrm{H}_3(\omega,\omega,\omega)G^3e^{j3\omega t}+\bar{\mathrm{H}}_3(\omega,\omega,\omega)\bar{G}^3e^{-j3\omega t}\\ &\quad+\mathrm{H}_5(\omega,\omega,\omega,\omega,\omega)\dot{G}^2e^{j5\omega t}+\bar{\mathrm{H}}_5(\omega,\omega,\omega,\omega,\omega)\bar{G}^5e^{-j5\omega t}\\ &\quad+O(\mathrm{H}_7,\bar{\mathrm{H}}_7,\mathrm{H}_9,\bar{\mathrm{H}}_9,\cdots),\end{aligned} \quad (8)$$

where in this relation, the property of conjugate symmetry, i.e., $\mathrm{H}_n(-\omega,\cdots,-\omega)=\bar{\mathrm{H}}_n(\omega,\cdots,\omega)$ is used and only the principal diagonals of the three and five dimensions are employed for simplifying the physical interpretation,, i.e., for five dimensional functions, $\omega_1=\cdots=\omega_5=\omega$. Substituting Eq. (8) into Eq. (2) with considering the terms up to order three only and equating the coefficients of each exponential order and excitations, the successive formulations for the higher order (the 2nd and the 3rd) transfer functions with the 1st order transfer function can be obtained. Here for notational conveniences, simplifying the terms such that $\mathrm{H}_1(\omega)=p_{f1}$, where subscript f and b are forward and backward modes, respectively, $\bar{\mathrm{H}}_1(\omega)=p_{b1}$, $\mathrm{H}_3(\omega,\omega,\omega)=p_{f3}$, $\bar{\mathrm{H}}_3(\omega,\omega,\omega)=p_{b3}$, $\mathrm{H}_5(\omega,\omega,\omega,\omega,\omega)=p_{f5}$ and $\bar{\mathrm{H}}_5(\omega,\omega,\omega,\omega)=p_{b5}$, and using the harmonic probing method (Worden et al., 1997), for the terms of $Ge^{j\omega t}$, $\bar{G}e^{-j\omega t}$ $G^3e^{j3\omega t}$, $\bar{G}^3e^{-j3\omega t}$, $G^5e^{j5\omega t}$ and $\bar{G}^5e^{-j5\omega t}$, respectively, the linear equations for the 2nd and 3rd order transfer functions are then successively described as

$$\mathrm{D_m}(\omega)p_{f1}+\mathrm{D}_\Delta(\omega)\bar{p}_{b1}=1,$$

$$\tilde{\mathrm{D}}_\Delta(\omega)p_{f1}+\tilde{\mathrm{D}}_\mathrm{m}(\omega)\bar{p}_{b1}=0,$$

$$\begin{aligned}&\mathrm{D_m}(3\omega)p_{f3}+\mathrm{D}_\Delta(3\omega)\bar{p}_{b3}\\ &=-(c_{nf}j\omega^3+k_{nf})(p_{f1}^3+3p_{f1}^2\bar{p}_{b1})\\ &\quad-(c_{nb}j\omega^3+k_{nb})(\bar{p}_{b1}^3+3p_{f1}\bar{p}_{b1}^2),\end{aligned}$$

$$
\begin{aligned}
&\tilde{D}_\Delta(3\omega)\,p_{f3}+\tilde{D}_m(3\omega)\,\bar{p}_{b3}\\
&=-(\bar{c}_{nf}j\omega^3+\bar{k}_{nf})(\bar{p}_{b1}^3+3p_{f1}\bar{p}_{b1}^2)\\
&\quad-(\bar{c}_{nb}j\omega^3+\bar{k}_{nb})(p_{f1}^3+3p_{f1}^2\bar{p}_{b1}), \qquad (9)\\
&D_m(5\omega)\,p_{f5}+D_\Delta(5\omega)\,\bar{p}_{b5}\\
&=-3(3c_{nf}j\omega^3+k_{mf})(p_{f1}^2p_{f3}+p_{f1}^2\bar{p}_{b3}+2p_{f1}p_{f3}\bar{p}_{b1})\\
&\quad-3(3c_{nb}j\omega^3+k_{nb})(p_{f3}\bar{p}_{b1}^2+\bar{p}_{b1}^2\bar{p}_{b3}+2p_{f1}\bar{p}_{b1}\bar{b}_{b3}),\\
&\tilde{D}_\Delta(5\omega)\,p_{f5}+\tilde{D}_m(5\omega)\,\bar{p}_{b5}\\
&=-3(3\bar{c}_{nf}j\omega^3+\bar{k}_{nf})(p_{f3}\bar{p}_{b1}^2+\bar{p}_{b1}^2\bar{p}_{b3}+2p_{f1}\bar{p}_{b1}\bar{p}_{b3})\\
&\quad-3(3\bar{c}_{nb}j\omega^3+\bar{k}_{nb})(p_{f1}^2p_{f3}+p_{f1}^2\bar{p}_{b3}+2p_{f1}p_{f3}\bar{p}_{b1}),
\end{aligned}
$$

where

$$
\begin{aligned}
&D_m(\omega)=K_m-\omega^2M+j\omega(C_m-jJ_p\Omega),\\
&D_\Delta(\omega)=K_\Delta+j\omega C_\Delta,\\
&\tilde{D}_m(\omega)=\bar{K}_m-\omega^2\bar{M}+j\omega(\bar{C}_m-jJ_p\Omega),\\
&\tilde{D}_\Delta(\omega)=\bar{K}_\Delta+j\omega\bar{C}_\Delta,
\end{aligned}
$$

From Eqs. (9), the 1st, 2nd and 3rd order transfer functions, respectively, are obtained as

$$
\begin{aligned}
&p_{f1}=D(\omega)\tilde{D}_m(\omega),\\
&\bar{p}_{b1}=-D(\omega)\tilde{D}_\Delta(\omega),\\
&p_{f3}=D(3\omega)[f_3(p_{f1},\bar{p}_{b1})\tilde{D}_m(3\omega)\\
&\qquad-\bar{f}_3(p_{f1},\bar{p}_{b1})D_\Delta(3\omega)],\\
&\bar{p}_{b3}=D(3\omega)[\bar{f}_3(p_{f1},\bar{p}_{b1})D_m(3\omega)\\
&\qquad-f_3(p_{f1},\bar{p}_{b1})\tilde{D}_\Delta(3\omega)], \qquad (10)\\
&p_{f5}=D(5\omega)[f_5(p_{f1},\bar{p}_{b1},p_{f3},\bar{p}_{b3})\tilde{D}_m(5\omega)\\
&\qquad-\bar{f}_3(p_{f1},\bar{p}_{b1},p_{f3},\bar{p}_{b3})D_\Delta(5\omega)],\\
&\bar{p}_{b5}=D(5\omega)[\bar{f}_5(p_{f1},\bar{p}_{b1},p_{f3},\bar{p}_{b3})D_m(5\omega)\\
&\qquad-f_5(p_{f1},\bar{p}_{b1},p_{f3},\bar{p}_{b3})\tilde{D}_\Delta(5\omega)],
\end{aligned}
$$

where $D(\omega)=[D_m(\omega)\tilde{D}_m(\omega)-D_\Delta(\omega)]^{-1}$, $f_3(p_{f1}, \bar{p}_{b1})$, $\bar{f}_3(p_{f1}, \bar{p}_{b1})$, $f_5(p_{f1}, \bar{p}_{b1}, p_{f3}, \bar{p}_{b3})$ and $\bar{f}_5(p_{f1}, \bar{p}_{b1}, p_{f3}, \bar{p}_{b3})$, are right-hand sides of the 3rd to the 6th equations in Eqs. (9), respectively. For example of the linear system, i.e., in case of no medium bracket term in Eq. (3), the response associated with the 1st order transfer function becomes

$$p(t)=p_{f1}Ge^{j\omega t}+p_{b1}\bar{G}e^{-j\omega t}, \qquad (11)$$

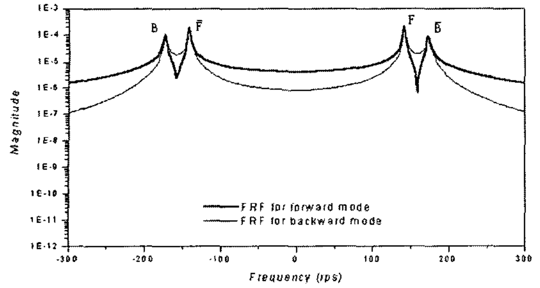
which is consistent with that shown in linear system (Genta, 1988).

Here as shown in the linear complex modal analysis for rotor systems, p_{f1} denotes the linear complex FRF corresponding to forward mode and p_{b1} denotes the complex FRF corresponding to backward one, whereas, in this non-linear analysis of the HFRFs, p_{f3} and p_{b3} (p_{f3} and p_{b5}) denote the 3rd (5th) order non-linear complex FRFs corresponding to forward and backward modes, respectively. From these non-linear complex FRFs, the property of the bearing non-linearity can be identified in practice.

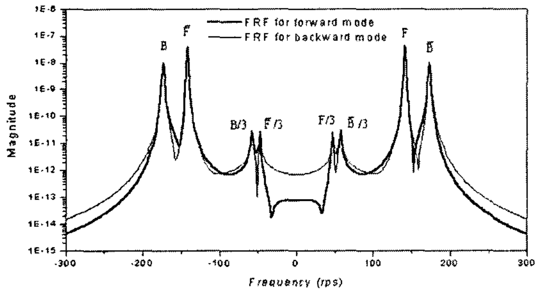
5. Numerical Example

In this simulation, the following numerical values have been used : m=10 kg, c_y=20 Ns/m, c_z=15 Ns/m, c_{nz}=3000 Ns/m, c_{nz}=2000 Ns/m, 20 Ns/m, $k_{yz}=3\times10^3$ N/m, $k_{zy}=-2\times10$ N/m, $k_y=3\times10^5$ N/m, $k_z=2\times10^5$ N/m, $k_{ny}=2\times10^{10}$ N/m, $k_{nz}=1.5\times10^{10}$ N/m, Ω=300 rps, J_p=0.05 kg/m^4. With these linear and non-linear parameters, Figures 3 show the simulated frequency response functions for the 1st (linear), the 3rd and the 5th order, respectively. In Eq. (2), due to the gyroscopic effect, which is caused by the cross-coupled damping coefficient herein, the FRFs are not symmetrical to negative frequencies so that complete FRFs according to the positive and the negative frequency ranges should be considered unlike the conventional ones.

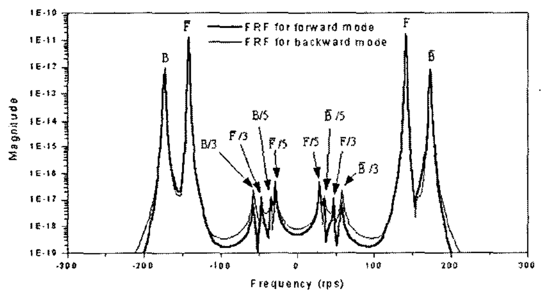
From Fig. 3(a), we can see the typical result of linear FRFs, in rotor system with orthotropic bearings, associated with one pair of the forward (F) and backward conjugate ($\bar{B}$) in the forward and backward modes, respectively, whereas in Fig. 3(b), the 3rd order FRFs show two pairs of the resonant peaks, one at $\omega_n s$ and the other at $\omega_n s/3$, which indicates the existence of cubic non-linearity. From Fig. 3(c), the 5th order FRFs show three pairs of the resonant peaks, $\omega_n s$, $\omega_n s/3$ and $\omega_n s/5$, which indicate the existence of cubic and quintic non-linearties, respectively. The peculiar feature of the results shown in Figs. 3(b) and (c) is that the (3rd and 5th) HFRFs do not show any distortion in frequency domain so that they look like the linear FRFs. This is due to the fact that for simplifying the physical interpretation, only the principal diagonals are employed so that they represent the exact Volterra kernel transforms, which is unique and independent of the excitation level. If the extra terms of HFRFs associated with additional sweeping multi-tone



(a) 1^{st} order (linear) FRFs



(b) 3^{rd} order FRFs



(c) 5^{th} order FRFs

Fig. 3 Non-linear FRFs for rotor system with orthotropic bearing non-linearity

excitation levels are introduced, these distortional phenomena are explicitly displayed. Another important feature in HFRF is that they cause energy transfer whereby an input at one frequency level influences the other frequency one, further more, the mode exchanges occur in forward and backward ones as shown in Figs. 3(b) and (c). This is also clearly depicted in Eqs. (10) in such a way that though in isotropic bearings the FRFs corresponding to forward modes are disappeared, whereas in linear FRFs, the backward ones are disappeared. We can see from these results that the HFRFs are fundamentally different from what one can expect and measure in linear ones in practice.

6. Conclusions

The analysis of orthotropic bearing non-linearity in rotor system using the complex HFRFs known to represent the non-linear degree of bearing orthotropy is newly investigated and shown to be valid as that of non-rotating structures or isotropic rotor systems.

The non-linear stiffness and damping forces in rolling or journal bearings are modeled by cubic polynomial form, in which HFRF can be deduced from Volterra series. Using principal diagonals of the dimensions of HFRFs for simplifying the physical interpretation at one excitation level, for which the computational efforts are lessen, the simple forms of dFRFs associated with the forward and the backward modes can be derived. The HFRFs show additional sub-harmonic resonant peaks, which indicate the existence of higher order non-linearties. Also another feature of HFRFs is that due to energy transfer the FRFs for forward and backward modes are exchanged so that they suggest the fundamental differences from what one can expect in linear ones.

We suggest, from this study, the feasibility to further application to detect non-linear properties in bearing, effectively.

Acknowledgments

This work is part of the results from the project "Tail Rotor Anti-Torque System for Helicopter" supported by Korea Aerospace Research Institute, Korea.

References

Bedrosian, E. and Stephen, O. R., 1971, "The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs," *Proceedings of IEEE*, Vol. 59, No. 12, pp. 1688~1971.

Genta, B., 1988, "Whirling of Unsymmetrical Rotors: A Finite Element Approach Based on

Complex Co-ordinates," *Journal of Sound and Vibration*, Vol. 124 (1), pp. 27~53.

Han, D. C., Choi, S. H., Lee, Y. H. and Kim, K. H., 1998, "The Nonlinear and Ball Pass Effects of a Ball Bearing on Rotor Vibration," *KSME International Journal*, Vol. 12, No. 3, pp. 396~404.

Kim, S. K. and Lee, C. W., 2001, "Complex Modal Testing of Asymmetric Rotors Using Magnetic Exciter Equipped with Hall Sensors," *KSME International Journal*, Vol. 15, No. 7, pp. 866~875.

Liangsheng, Q. and Xiao, L., 1993, "Study and Performance Evaluation of Some Nonlinear Diagonostic Methods for Large Rotating Machinery," *Mech. Mach. Theory*, Vol. 28, No. 5, pp. 699~713.

Lin, R. M. and Lim, M. K., 1993, "Identification of Nonlinearity from Analysis of Complex Modes," *International Journal of Analytical and Experimental Modal Analysis*, Vol. 8, pp. 285~299.

Ozguven, H. N. and Imregun, M., 1993, "Complex Modes Arising from Linear Identification of Non-linear Systems," *International Journal of Analytical and Experimental Modal Analysis*, Vol. 8, pp. 151~164.

Schetzen, M., 1990, *The Volterra and Wiener Theories of Nonlinear Systems*, John Wiley & Sons, Inc.

Storer, D. M. and Tomlinson, G. R., 1993, "Recent Developments in the Measurement and Interpretation of Higher Order Transfer Functions from Transfer Functions from Non-Linear Structures," *Mechanical Systems and Signal Processing*, Vol. 7, No. 2, pp. 173~189.

Tiwari, R. and Vyas, N. S., 1995, "Estimation of Non-Linear Stiffness Parameters of Rolling Element Bearings from Random Response of Rotor-Bearing Systems," *Journal of Sound and Vibration*, Vol. 187, pp. 229~239.

Vyas, N. S. and Chatterjee, A., 2000, "Non-linear System Identification and Volterra-Wiener Theories," *Proceeding of VETOMAC-I*, Oct. Bangalore, India.

Worden, K. and Manson, G., 1998, "Random Vibrations of a Duffing Oscillator Using the Volterra Series," *Journal of Sound and Vibration*, Vol. 217, pp. 781~789.

Worden, K., Manson, G. and Tomlinson. G. R., 1997, "A Harmonic Probing Algorithm for the Multi-Input Volterra Series," *Journal of Sound and Vibration*, Vol. 201, pp. 67~84.

Zhang, H. and Billings, S. A., 1993, "Analysis Non-Linear Systems in the Frequency Domain-I. The Transfer Function," *Mechanical Systems and Signal Processing*, Vol. 7, No. 6, pp. 5331~550.