Local and Global Isotropy Analysis of Caster Wheeled Omnidirectional Mobile Robot

Sung-bok Kim, Byoung-kwon Moon*

Abstract

The omnidirectional mobility of a mobile robot may lose significance in motion control, unless the isotropy characteristics of the mechanism is maintained well. This paper investigates the local and global isotropy of an omnidirectional mobile robot with three caster wheels. All possible actuations with different number and combination of rotating and steering joints are considered. First, the kinematic model based on velocity decomposition and the algebraic conditions for the local isotropy are obtained. Second, the geometric conditions for the local isotropy are derived and all isotropic configurations are fully identified. Third, the global isotropy index is examined to determine the optimal parameters in terms of actuation set, characteristic length, and steering link length.

Keywords : Omnidirectional mobility, Caster wheel, Isotropy analysis, Optimal design and control

I. Introduction

The omnidirectional mobility of a mobile robot is required to navigate in daily life environment which is restricted in space and cluttered with obstacles. Several omnidirectional wheel mechanisms have been proposed, including universal wheels, Swedish wheels, orthogonal wheels, ball wheels, and so on. Recently, caster wheels were employed to develop an omnidirectional mobile robot at Stanford University [1], which was commercialized by Nomadic Technologies. Since caster wheels do without small peripheral rollers or support structure, a caster wheeled omnidirectional mobile robot (COMR) can maintain good performance as payload or ground condition changes.

For a general form of wheeled mobile robots, a systematic procedure for kinematic modeling was described [2]. There have been previous work on a COMR. It was shown that at least four joints of two caster wheels should be actuated to avoid the singularity [3]. For some actuation sets, the global isotropy index was examined to optimize the design parameters [4]. For all possible actuation sets, the local isotropy was analyzed to identify the isotropic configurations. On the other hand, an isotropic omindirectional mobile robot with Swedish wheels was designed [6].

The purpose of this paper is to investigate the local and global isotropy of a COMR for the optimal design and control. This paper is organized as follows. In Section II, with the characteristic length introduced [6], the kinematic model and the isotropy conditions are obtained. For all possible actuation sets, Section III derives the geometric conditions for the local isotropy to identify all isotropic configurations. And, Section IV examines the global isotropy index to optimize the design and control parameters. Finally, the conclusion is made in Section V.

II. Kinematic Modeling and Isotropy Conditions

A. Kinematic Modeling

Consider a COMR with three caster wheels attached to a regular triangular platform moving on the xy plane, as shown in Fig. 1.

*School of Electronics and Information Eng. at Hankuk University of Foreign Studies
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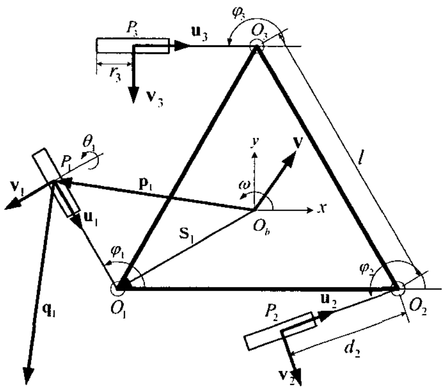


Fig. 91 A caster wheeled omnidirectional mobile robot.

Let l be the side length of the platform with the center O_b, and three vertices, O_i, $i=1,2,3$. For the i^{th} caster wheel with the center P_i, $i=1,2,3$, we define the following. Let d_i and r_i be the length of the steering link and the radius of the wheel, respectively. Let θ_i and ϕ_i be the angles of the rotating and the steering joints, respectively. Let $\mathbf{u}_i$ and $\mathbf{v}_i$ be two orthogonal unit vectors along the steering link and the wheel axis, respectively, such that

$$\mathbf{u}_i=\begin{bmatrix} -\cos\phi_i \\ -\sin\phi_i \end{bmatrix}, \quad \mathbf{v}_i=\begin{bmatrix} -\sin\phi_i \\ \cos\phi_i \end{bmatrix} \tag{1}$$

Note that

$$\mathbf{u}_i\ \mathbf{u}_i^t + \mathbf{v}_i\ \mathbf{v}_i^t = \mathbf{I}_2 \tag{2}$$

$$\sum \mathbf{u}_i = \mathbf{0} \Leftrightarrow \sum \mathbf{v}_i = \mathbf{0} \tag{3}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero vector. Let $\mathbf{s}_i$ be the vector from O_b to O_i, such that

$$\mathbf{s}_1=\frac{l}{\sqrt{3}}\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}, \quad \mathbf{s}_2=\frac{l}{\sqrt{3}}\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}, \quad \mathbf{s}_3=\frac{l}{\sqrt{3}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4}$$

Note that $\sum_1^3 \mathbf{s}_i = \mathbf{0}$. Let $\mathbf{p}_i$ be the vector from O_b to P_i, and $\mathbf{q}_i$ be the rotation of $\mathbf{p}_i$ by 90^o counterclockwise. Note that

$$\mathbf{p}_i = \mathbf{s}_i - d_i\mathbf{u}_i \tag{5}$$

$$\sum \mathbf{q}_i = \mathbf{0} \Leftrightarrow \sum \mathbf{p}_i = \mathbf{0} \tag{6}$$

$$\sum_1^3 \mathbf{p}_i = \mathbf{0} \Leftrightarrow \sum_1^3 \mathbf{u}_i = \mathbf{0} \tag{7}$$

Let $\mathbf{v}$ and ω be the linear and the angular velocities at O_b of the platform, respectively. For the i^{th} caster wheel, $i=1,2,3$, the linear velocity at the point of contact with the ground can be expressed by

$$\mathbf{v} + \omega\ \mathbf{q}_i = r_i\dot{\theta}_i\mathbf{u}_i + d_i\dot{\phi}_i\mathbf{v}_i \tag{8}$$

Premultiplied by $\mathbf{u}_i^t$ and $\mathbf{v}_i^t$, (8) becomes

$$\mathbf{u}_i^t\ \mathbf{v} + \mathbf{u}_i^t\ \mathbf{q}_i\ \omega = r_i\ \dot{\theta}_i \tag{9}$$

$$\mathbf{v}_i^t\ \mathbf{v} + \mathbf{v}_i^t\ \mathbf{q}_i\ \omega = d_i\ \dot{\phi}_i \tag{10}$$

Notice that the instantaneous motion of the wheel is decomposed into two orthogonal components of the rotating and the steering joints.

Assume that n $(3\le n\le 6)$ joints of a COMR are actuated. With the characteristic length L (>0) introduced [6], the kinematics of a COMR can be written as

$$\mathbf{A}\ \dot{\mathbf{x}} = \mathbf{B}\ \dot{\Theta} \tag{11}$$

where $\dot{\mathbf{x}} = [\ \mathbf{v}\ L\omega\ ]^t \in \mathbf{R}^{3\times 1}$ is the task velocity vector, and $\dot{\Theta} \in \mathbf{R}^{n\times 1}$ is the joint velocity vector, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{g}_1^t & \frac{1}{L}\ \mathbf{g}_1^t\ \mathbf{h}_1 \\ \vdots & \vdots \\ \mathbf{g}_n^t & \frac{1}{L}\ \mathbf{g}_n^t\ \mathbf{h}_n \end{bmatrix} \in \mathbf{R}^{n\times 3} \tag{12}$$

$$\mathbf{B} = \begin{bmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{bmatrix} \in \mathbf{R}^{n\times n} \tag{13}$$

are the Jacobian matrices. In (12), $\mathbf{g}_k$ corresponds to either $\mathbf{u}_i$ or $\mathbf{v}_i$, while $\mathbf{h}_k$ corresponds to $\mathbf{q}_i$. In (13), c_k corresponds to either r_i or d_i. It should be mentioned that the role of the characteristic length L is to make all three columns of $\mathbf{A}$ consistent in physical unit.

The expressions of $\mathbf{g}_k^t\ \mathbf{h}_k$ can be simplified as follows. In the case of the rotating joint for which $\mathbf{g}_k = \mathbf{u}_i$ and $\mathbf{h}_k = \mathbf{q}_i$,

$$\mathbf{g}_k^t\ \mathbf{h}_k = \mathbf{u}_i^t\ \mathbf{q}_i = \mathbf{v}_i^t\ \mathbf{p}_i = \mathbf{v}_i^t\ \mathbf{s}_i \tag{14}$$

And, in the case of the steering joint for which $\mathbf{g}_k = \mathbf{v}_i$ and $\mathbf{h}_k = \mathbf{q}_i$,

$$\mathbf{g}_k^t\ \mathbf{h}_k = \mathbf{v}_i^t\ \mathbf{q}_i = -\ \mathbf{u}_i^t\ \mathbf{p}_i = -\ \mathbf{u}_i^t\ \mathbf{s}_i + d \tag{15}$$

B. Isotropy Conditions

Based on (11), the isotropy conditions of a COMR can be stated as

$$A^t A \propto I_3 \quad (16)$$

$$B \propto I_6 \quad (17)$$

From (13) and (17), the isotropy condition on **B** is obtained by

$$c_k = d > 0, \quad k=1,\cdots,n \quad (18)$$

(18) indicates that three caster wheels should be identical in structure with the length of the steering link equal to the radius of the wheel.

From (12) and (16), the isotropy condition on **A** is obtained by

$$A^t A = \frac{n}{2} I_3 \quad (19)$$

which leads to the following three conditions:

$$\begin{aligned} C1 &: \sum_1^n g_k\, g_k^t = \frac{n}{2} I_2 \\ C2 &: \sum_1^n (g_k^t h_k)\, g_k = 0 \\ C3 &: \frac{1}{L^2}\sum_1^n (g_k^t h_k)^2 = \frac{n}{2} \end{aligned} \quad (20)$$

In general, **C1** and **C2** are functions of the steering joint angles, ϕ_k, $k=1,2,3$, from which the isotropic configurations are determined. With ϕ_k, $k=1,2,3$, known, **C3** determines the characteristic length required for the isotropy, denoted by L_{iso}.

C. Actuation Sets

Table 1 lists all possible actuation sets, Θ, of a COMR with different number and combination of rotating and steering joints. Prior to the isotropy analysis, let us examine the periodicity of $A^t A$. Let A_i, $i=1,2,3$, be the submatrix of **A**, corresponding to the i^{th} caster wheel, so that

$$A^t A = A_1^t A_1 + A_2^t A_2 + A_3^t A_3 \quad (21)$$

In general, A_i, $i=1,2,3$, is periodic with respect to the steering joint angle ϕ_i with period 2π:

$$A_i(\phi_i) = A_i(\phi_i \pm 2\pi) \quad (22)$$

However, when only the rotating joint is actuated,

$$A_i(\phi_i) = -A_i(\phi_i \pm \pi) \quad (23)$$

And, when both rotating and steering joints are not actuated, $A_i = 0$. These periodic properties allow us to consider the isotropy of a COMR over the reduced configuration space, denoted by Ω, instead of the entire configuration space, $0 \le \phi_1, \phi_2, \phi_3 \le 2\pi$. For instance, in the case of $\Theta = \{\theta_1, \phi_1, \theta_2\}$, $A^t A$ is a function of ϕ_1 with period 2π and a function of ϕ_2 with period π, so that Ω is given by $0 \le \phi_1 \le 2\pi$, $0 \le \phi_2 \le \pi$. Notice that the occurrence of the isotropy is independent of the steering joint of the unactuated cater wheel.

III. Local Isotropy Analysis

A. $\Theta = \{\theta_1, \theta_2, \theta_3\}$

First, under the first isotropy condition of **C1**, we have

$$\begin{aligned} c_1^2 + c_2^2 + c_3^2 &= 1.5 \\ c_1 s_1 + c_2 s_2 + c_3 s_3 &= 0.0 \end{aligned} \quad (24)$$

where $c_k = \cos(\phi_k)$ and $s_k = \sin(\phi_k)$, $k=1,2,3$. Satisfying (24), there are two different groups of possible distributions of { u_k, $k=1,2,3$ } on the unit circle. The first group of four distributions is given by

$$\phi_2 = \phi_1 + \frac{2\pi}{3},\ \phi_1 - \frac{\pi}{3}, \quad \text{and} \quad \phi_3 = \phi_1 + \frac{\pi}{3},\ \phi_1 - \frac{2\pi}{3} \quad (25)$$

Geometrically, u_1, u_2, and u_3 lie on three sides of a regular triangle in counterclockwise order. On the other hand, the second group of four distributions can be obtained by alternating u_2 and u_3, so that u_1, u_2, and u_3 lie on three sides of a regular triangle in clockwise order. Note that both groups of eight distributions can be characterized as

$$v_1 \pm v_2 \pm v_3 = 0 \quad (26)$$

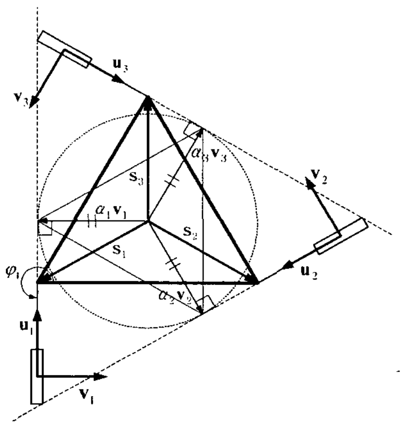
Second, under the second isotropy condition of **C2**, we have

$$\sum_1^3 (v_k^t s_k)\, v_k = \sum_1^3 \alpha_k v_k = 0 \quad (27)$$

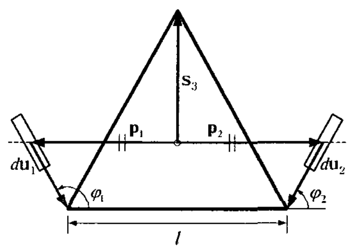
where $\alpha_1 = \frac{l}{\sqrt{3}}\cos(\phi_1 - \frac{2}{3}\pi)$, $\alpha_2 = \frac{l}{\sqrt{3}}\cos(\phi_2 + \frac{2}{3}\pi)$, and $\alpha_3 = \frac{l}{\sqrt{3}}\cos(\phi_3)$. It can be shown that the first group of four distributions given by (25) also satisfy (27). Fig. 2a) illustrates the isotropic configuration of a COMR, where three steering links are symmetric with respect to the center of the platform. On the other hand, the second group of four distributions cannot satisfy (27), and the isotropy of **A** cannot be achieved.

Finally, with (25) held, under **C3**, the characteristic length L_{iso} of an isotropic COMR is obtained by

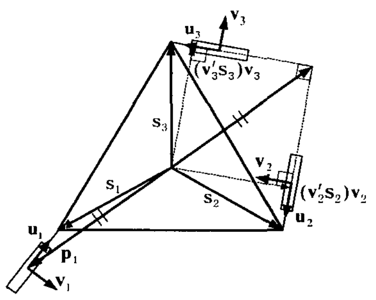
$$0 < L_{iso} = \sqrt{2}\,\alpha \le \sqrt{\frac{2}{3}}\, l \quad (28)$$



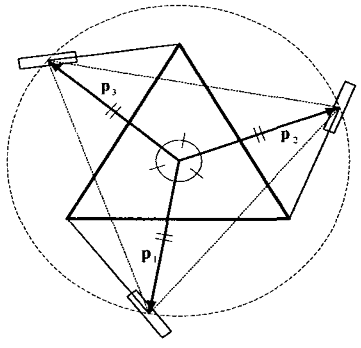
a) $\Theta = \{\theta_1, \theta_2, \theta_3\}$



b) $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2\}$



c) $\Theta = \{\theta_1, \phi_1, \theta_2, \theta_3\}$



d) $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$

Fig. 2 The isotropic configurations of a COMR.

where

$$|\alpha_1| = |\alpha_2| = |\alpha_3| = \alpha \tag{29}$$

If $L = L_{iso}$, the isotropy occurs as four points in the entire configuration space, which is the intersection of the lines given by (25) and the plane given by (28). Notice that the isotropic configurations change depending on the selection of L_{iso}. Otherwise, the isotropy does not occur. Similar analysis can be made for the case of $\Theta = \{\phi_1, \phi_2, \phi_3\}$.

B. $\Theta = \{\phi_1, \theta_2, \theta_3\}$

First, under **C1**, we have

$$\mathbf{u}_1 \pm \mathbf{v}_2 \pm \mathbf{v}_3 = \mathbf{0} \tag{30}$$

Next, under **C2**, we have

$$(\mathbf{u}_1{}' \mathbf{s}_1 - d)\,\mathbf{u}_1 + (\mathbf{v}_2{}' \mathbf{s}_2)\,\mathbf{v}_2 + (\mathbf{v}_3{}' \mathbf{s}_3)\,\mathbf{v}_3 = \mathbf{0} \tag{31}$$

which cannot be satisfied unless $d = 0$. This tells that the isotropy of **A** can be achieved only when caster wheels reduce to conventional wheels with no steering link. Similar analysis can be made for $\Theta = \{\phi_1, \phi_2, \theta_3\}$.

C. $\Theta = \{\theta_1, \phi_1, \theta_2\}$

First, under **C1**, we have

$$c_2^2 = 0.5, \quad c_2 s_2 = 0.0 \tag{32}$$

There does not exist ϕ_2 satisfying (32). This tells that the isotropy of **A** cannot be achieved at all. Similar analysis can be made for $\Theta = \{\theta_1, \phi_1, \phi_2\}$.

D. $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2\}$

First, **C1** holds always. Next, under **C2**, we have

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0} \tag{33}$$

which yields

$$\phi_1 = \arcsin\left(\frac{1}{2\sqrt{3}}\frac{l}{d}\right), \quad \phi_2 = \pi - \phi_1 \tag{34}$$

subject to $d \geq \frac{l}{2\sqrt{3}}$. Fig. 2b) illustrates the isotropic configuration, where the steering links of two caster wheels are symmetric with respect to y-axis, with the centers of two caster wheels and the center of the platform lying on the line of $y = \frac{l}{2\sqrt{3}}$. Note that the isotropic configuration does not exist if the length of the steering link is less than $\frac{l}{2\sqrt{3}}$.

Finally, with (34) held, under **C3**, the isotropic characteristic length L_{iso} is obtained by

$$L_{iso} = (\| \mathbf{p}_1 \| = \| \mathbf{p}_2 \|) \quad (35)$$

where

$$\| \mathbf{p}_1 \| = \frac{l}{2} - d\cos(\phi_1) \text{ and } \| \mathbf{p}_2 \| = \frac{l}{2} + d\cos(\phi_2) \quad (36)$$

Solving (35) yields

$$L_{iso} = \frac{l}{2} \pm \sqrt{d^2 - \frac{l^2}{12}} \quad (37)$$

which is a constant. If $L = L_{iso}$, the isotropy occurs at one point in the configuration space of (ϕ_1, ϕ_2), which satisfies (34) and (37). Otherwise, the isotropy does not occur.

E. $\Theta = \{\theta_1, \phi_1, \theta_2, \theta_3\}$

First, under **C1**, we have

$$\sin(2\phi_2) + \sin(2\phi_3) = 0.0 \quad (38)$$

which yields

$$\phi_3 = \phi_2 \pm \frac{\pi}{2} \quad (39)$$

Note that $\mathbf{v}_2$ and $\mathbf{v}_3$ should be perpendicular to each other. Next, under **C2**, we have

$$\mathbf{p}_1 + (\mathbf{v}_2^t \mathbf{s}_2)\mathbf{v}_2 + (\mathbf{v}_3^t \mathbf{s}_3)\mathbf{v}_3 = \mathbf{0} \quad (40)$$

Fig. 2c) illustrates the isotropic configuration, where the steering links of two caster wheels with active rotating joints are perpendicular to each other and the center of the other caster wheel with active rotating and steering joints is located in such a way as to satisfy (40).

Finally, with (39) and (40) held, under **C3**, the isotropic characteristic length L_{iso} is obtained by

$$L_{iso} = \left(\| \mathbf{p}_1 \| = \sqrt{(\mathbf{v}_2^t \mathbf{s}_2)^2 + (\mathbf{v}_3^t \mathbf{s}_3)^2}\right) \quad (41)$$

which is fixed. If $L = L_{iso}$, the isotropy occurs at eight or four points in the entire configuration space, which satisfy (39), (40), and (41). Otherwise, the isotropy does not occur. Similar analysis can be made for $\Theta = \{\theta_1, \phi_1, \phi_2, \phi_3\}$ and $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_3\}$.

F. $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3\}$

First, under **C1**, we have

$$c_3^2 = 0.5, \quad c_3 s_3 = 0.0 \quad (42)$$

for which there does not exist ϕ_3 and the isotropy of **A** cannot be achieved at all. Similar analysis can be made for $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \phi_3\}$.

G. $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$

First, **C1** holds always. Next, under **C2**, we have

$$\sum_1^3 \mathbf{p}_k = \mathbf{0} \quad (43)$$

which yields

$$\phi_2 = \phi_1 \pm \frac{2\pi}{3}, \quad \phi_3 = \phi_1 \mp \frac{2\pi}{3} \quad (44)$$

Fig. 2d) illustrates the isotropic configuration of a COMR. where the centers of three caster wheels are symmetric with respect to the center of the platform.

Finally, with (44) held, under **C3**, the isotropic characteristic length L_{iso} is obtained by

$$L_{iso} = \sqrt{\frac{1}{3}\sum_1^3 \| \mathbf{p}_k \|^2} \quad (45)$$

where

$$\| \mathbf{p}_1 \|^2 = d^2 + \left(\frac{l}{\sqrt{3}}\right)^2 - \frac{2dl}{\sqrt{3}}\cos\left(\phi_1 - \frac{\pi}{6}\right)$$
$$\| \mathbf{p}_2 \|^2 = d^2 + \left(\frac{l}{\sqrt{3}}\right)^2 - \frac{2dl}{\sqrt{3}}\cos\left(\phi_2 - \frac{5}{6}\pi\right)$$
$$\| \mathbf{p}_3 \|^2 = d^2 + \left(\frac{l}{\sqrt{3}}\right)^2 - \frac{2dl}{\sqrt{3}}\cos\left(\phi_3 + \frac{\pi}{2}\right) \quad (46)$$

If $L = L_{iso}$, the isotropy occurs at a single point in the entire configuration space, which are the intersection of the line given by (44) and the plane given by (45). Otherwise, the isotropy does not occur.

IV. Global Isotropy Analysis

A. Periodicity

Let us define the local isotropy index of a COMR, denoted by σ, as the ratio of the minimum to the maximum among the singular values of **A**. Note that σ is the inverse of the condition number of **A** and ranges between 0 and 1, that is, $0 \le \sigma \le 1$. In general, σ is a function of the actuation set Θ, the configuration (ϕ_1, ϕ_2, ϕ_3), the characteristic length L, and the steering link length d (equal to the wheel radius r):

$$\sigma = \sigma(\Theta, \phi_1, \phi_2, \phi_3, L, d) \quad (47)$$

For simulations, we set the side length of the platform as $l = 1.0$ [m], and the length of the steering link as $d = 0.1$ [m]. The size of d relative to l will be discussed at the end of this section.

For a given actuation set $\Theta = \{\theta_1, \theta_2, \theta_2\}$, Fig. 3 shows the plots of $\sigma(\phi_1 = 0, \phi_2, \phi_3)$ and $\sigma(\phi_1, \phi_2, \phi_3 = 0)$ for $0 \le \phi_2, \phi_3 \le 2\pi$ and $0 \le \phi_1, \phi_2 \le 2\pi$, respectively, with $L = \sqrt{\frac{2}{3}}\cos\left(\phi_1 - \frac{2}{3}\pi\right)\Big|_{\phi_1 = 0}$. From Fig. 3, the following can be observed. There are four isotropic configurations, $(\phi_1, \phi_2, \phi_3) = (0, \frac{2\pi}{3}, \frac{\pi}{3})$, $(0, \frac{2}{3}\pi, -\frac{2\pi}{3})$, $(0, -\frac{\pi}{3}, \frac{\pi}{3})$, and $(0, -\frac{\pi}{3}, -\frac{2\pi}{3})$, for which $L = L_{iso}$ and $\sigma = 1.0$. And, σ is periodic with respect to ϕ_1, ϕ_2, and ϕ_3 with period π, so that the reduced configuration space Ω is given by $0 \le \phi_1, \phi_2, \phi_3 \le \pi$.

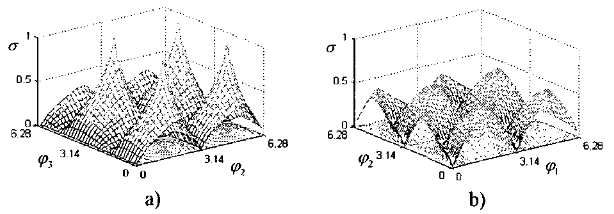


Fig. 3 The plots of σ in the case of $\Theta = \{\theta_1, \theta_2, \theta_2\}$: a) and b) $\sigma(\phi_1, \phi_2, \phi_3 = 0)$.

B. Optimal Characteristic Length

Let us define the global isotropy index of a COMR, denoted by $\bar{\sigma}$, as the average value of the local isotropy index σ over the reduced configuration space Ω. Now, $\bar{\sigma}$ is a function of the actuation set Θ, the characteristic length L, and the steering link length d:

$$\bar{\sigma} = \bar{\sigma}(\Theta, L, d) \quad (48)$$

By choosing $\bar{\sigma}$ as the optimization criterion, the optimal characteristic length, denoted by L_{opt}, can be determined according to Θ, which results in the maximum value of $\bar{\sigma}$, denoted by $\bar{\sigma}_{max}$. In the cases of $\Theta = \{\theta_1, \theta_2, \theta_3\}$, $\{\theta_1, \phi_1, \theta_2, \phi_2\}$, $\{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3\}$, and $\{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$, Fig. 4 shows the plots of $\bar{\sigma}$ for $0.0 < L \le 1.0$, all of which are convex with no exception. Table 1 lists L_{opt} and $\bar{\sigma}_{max}$ for all possible actuation sets, which can be divided into six groups with reference to the magnitude of $\bar{\sigma}_{max}$. From Table 1, the following can be observed. As the number of joints increases, $\bar{\sigma}_{max}$ becomes increasing strictly regardless of the existence of the isotropic configurations. With the same number of joints, for larger value of $\bar{\sigma}_{max}$, the rotating joint is preferred to the steering joint and the actuation of both rotating and steering joints per caster wheel is preferred. Notice that L_{opt} is about 0.6 [m], however, $\bar{\sigma}_{max}$ ranges between 0.26 and 0.89.

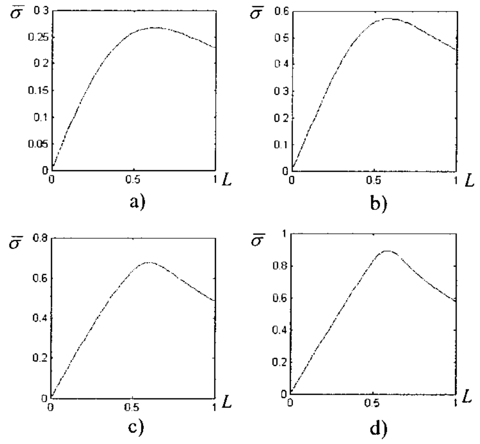


Fig. 4 The plots of $\bar{\sigma}$: a) $\Theta = \{\theta_1, \theta_2, \theta_2\}$, b) $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2\}$, c) $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3\}$, and d) $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$.

Table 1. The optimal characteristic length L_{opt} and the maximum global isotropy index $\bar{\sigma}_{max}$

Actuation set Θ	L_{opt} [m]	$\bar{\sigma}_{max}$	Group
$\{\theta_1, \theta_2, \theta_3\}$	0.62	0.2668	3A
$\{\phi_1, \theta_2, \theta_3\}$	0.63	0.2654	
$\{\phi_1, \phi_2, \theta_3\}$	0.63	0.2640	
$\{\phi_1, \phi_2, \phi_3\}$	0.63	0.2626	
$\{\theta_1, \phi_1, \theta_2\}$	0.59	0.3645	3B
$\{\theta_1, \phi_1, \phi_2\}$	0.59	0.3604	
$\{\theta_1, \phi_1, \theta_2, \phi_2\}$	0.58	0.5715	4A
$\{\theta_1, \phi_1, \theta_2, \theta_3\}$	0.60	0.5039	4B
$\{\theta_1, \phi_1, \phi_2, \theta_3\}$	0.61	0.5004	
$\{\theta_1, \phi_1, \phi_2, \phi_3\}$	0.61	0.4970	
$\{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3\}$	0.60	0.6765	5
$\{\theta_1, \phi_1, \theta_2, \phi_2, \phi_3\}$	0.60	0.6689	
$\{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi$	0.59	0.8918	6

C. Effect of Steering Link Length

Finally, let us discuss the effect of the length of the steering link, d, relative to the side length of the platform, l. In the case of $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$, Fig. 5 shows the plots of the maximum global isotropy index $\bar{\sigma}_{max}$ for $0.0 < L \le 1.0$ and $0.1 \le d \le 0.5$. From Fig. 5, it can be observed that L_{opt} becomes larger and $\bar{\sigma}_{max}$ becomes smaller, as d increases, This implies that the relative size of d should be kept small for high global isotropy index.

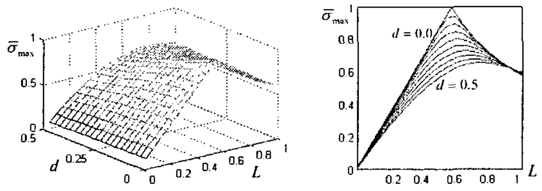


Fig. 5 The plots of $\bar{\sigma}_{max}$ in the case of $\Theta = \{\theta_1, \phi_1, \theta_2, \phi_2, \theta_3, \phi_3\}$.

V. Conclusion

This paper investigated the local and global isotropy of a caster wheeled omnidirectional mobile robot (COMR) under all possible actuations with different number and combination of rotating and steering joints. First, with the characteristic length introduced, the kinematic model and the isotropy conditions were obtained. Second, the geometric conditions for the local isotropy were derived and all isotropic configurations were fully identified. Third, the global isotropy index was examined to determine the optimal actuation set, characteristic length, and steering link length. It is hoped that the results of this paper can be useful for the optimal design and control of a COMR.

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Sung-bok Kim

received the B.S. degree in electronics eng. from Seoul National University in 1980, the M.S. degree in electrical and electronics eng. from KAIST in 1982, and the Ph.D. degree in electrical eng. from University of Southern Californica, U. S. A. in 1993. Since 1994, he has been with School of Electronics and Information Eng. at Hankuk University of Foreign Studies. His current research interests include the analysis, design and control of mobile robots and humanoids.

Byoung-kwon Moon

received the B.S. degrees in digital information engineering from Hankuk University of Foreign Studies in 2005. Currently, he is a graduate student at Hankuk University of Foreign Studies. His research interests include the design and control of omnidirectional mobile robots.