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A Study on Uncertainty Analyses of Monte Carlo Techniques Using Sets of Double Uniform Random Numbers

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Abstract

Structural uncertainties are generally modeled using probabilistic approaches in order to quantify uncertainties in behaviors of structures. This uncertainty results from the uncertainties of structural parameters. Monte Carlo methods have been usually carried out for analyses of uncertainty problems where no analytical expression is available for the forward relationship between data and model parameters. In such cases any direct mathematical treatment is impossible, however the forward relation materializes itself as an algorithm allowing data to be calculated for any given model. This study addresses a new method which is utilized as a basis for the uncertainty estimates of structural responses. It applies double uniform random numbers (i.e. DURN technique) to conventional Monte Carlo algorithm. In DURN method, the scenarios of uncertainties are sequentially selected and executed in its simulation. Numerical examples demonstrate the beneficial effect that the technique can increase uncertainty degree of structural properties with maintaining structural stability and safety up to the limit point of a breakdown of structural systems.

Keywords: Monte Carlo Simulation, Double Uniform Random Number, Uncertainty, Structural Parameter, Structural Response, Stability and Safety

1. INTRODUCTION

Numerical analysis of structural responses involves the consideration of some impacts, for example the impact of a standard on numerical properties of applied structural parameters. In order to perform the calculation, analysts must first: 1) specify the equation or model that will be used; 2) define the quantities in the equation; and 3) provide numerical values for each quantity. In the simplest, the equation is unambiguous (i.e. this contains all relevant quantities and on others), each quantity has a single numerical value, and the calculation results in a single value. However, non-ambiguity and precision of structural properties are rarely the case. In almost all cases the model and/or the numerical values for each quantity in the model are not completely known (i.e. there is uncertainty) and depend upon other conditions (i.e. there is variability).

The numerical analysis involves accounting for the uncertainty and the variability. While the simplest analysis involves a single numerical value for each quantity in a calculation, arguments can arise about what the appropriate value is for each quantity. Explicit analysis of the uncertainty and the variability is intended to provide more complete information to the decision process.

This flexibility in structural modeling has led to two areas, which can and must be considered;

1) Computational accuracy considering numerical round-off and ill conditioning.

2) Quantification of structural response uncertainties due to uncertain properties of the structural model.

In the first area, the structural input is assumed to be precise and the error to lie only in the numerical computations. The objective of such studies is to contain the errors within set bounds. The second area assumes that the machine errors are contained but opens for consideration of the measurement of the structural parameters as probabilistic and non-probabilistic methods.

In non-probabilistic methods (i.e. interval analysis introduced by Moore (1966)), it is possible to apply models of the uncertainty, which are independent of such detailed knowledge. However it is difficult to apply these results to practical engineering problems due to the complexity of the algorithm.

Since the structural parameters are random variables in probabilistic methods, then the structural response quantities (i.e. internal force, displacement, stress, strain and so on) are also random variables and related to the structural parameters (i.e. loading condition, Young's modulus, cross-sectional area, length of member and so on). However, given that only a small amount of the statistical information about the initial data such as structural parameters and loading conditions can be known in a few specialized cases, indeed, the probabilistic approaches cannot deliver reliable solutions at the required accuracy without sufficient experience and experimental data to prove the valid assumptions made regarding the probability densities of random variables or functions involved.

A Monte Carlo method of the probabilistic approaches has a long history, but its application to the solution of scientific problems begins with Ulam et al. (1947) who used the Monte Carlo method in nuclear reaction studies. The name 'the Monte Carlo method' was first used by them. The method is executed by appropriate stochastic models of statistic extractions using random variables. One advantage of the approach is simplicity of the calculation algorithm contrary to interval analysis. Another one is to be able to obtain the error of calculation. Therefore the method can be suitable for engineering problem admitting computational error of 5~10%.

In this paper we show how one can quantify response uncertainties and also make probabilistic statements about prescribed levels of structural responses as a modified version of conventional Monte Carlo simulation. Contrary to classical Monte Carlo approach, this method does not require many statistic data in order to measure the uncertainty model. The numerical algorithm for structural analysis is not complex and complicate in the comparisons with interval method.

For the purpose of making efficiency of this modified method, sets of *Double Uniform Random Number* (i.e. DURN) are devised for analytical and numerical processes. In DURN technique, conventional table of uniform random numbers is modified as the table with 100 groups, which include 5 random numbers in each group. In the modified table, statistical properties of the combination of each group are calculated through sequential generation processes. It is assumed that this is similar to uncertainty characteristics of structural parameters which are random variables in Monte Carlo analysis algorithm. Since the generation of random numbers occurs repetitively in Monte Carlo analysis, this processes are regarded as the DURN technique for quantities of response uncertainties.

In this study, a plane truss modeled as 10 degrees of freedom is considered as a numerical example. Sequential scenarios for the loading conditions, Young's modulus, the length of member and the cross-sectional area are assumed through the application of sequential generation of uniform random number in order to describe uncertainties of structural response. Finally, this study presents the numerical dependence of structural parameters and applied loads by the uncertainty of structural responses.

2. MATHEMATICAL BACKGROUND

In analytical probabilistic methods, we can consider Taylor's series expansion to formulate a linear relationship between the response random variables and the random structural parameters. In order to understand such a mathematical model, we consider the notion of Taylor's series expansion.

Consider a single function which is dependent on $\mathbf{n}$ parameters, $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_\mathbf{n}$, that is, random variables. Therefore, the $\mathbf{m}$ functions of these parameters $\mathbf{f}(\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_\mathbf{n})$ are also sets of random variables. Now let $\bar{\mathbf{a}}_\mathbf{j}$ be the mean of the random variables $\mathbf{a}_\mathbf{j}$, and then Taylor's series expansion about the mean of random variables, say $\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \cdots, \bar{\mathbf{a}}_\mathbf{n}$ is

$$\mathbf{f}(\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_\mathbf{n}) = \mathbf{f}(\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \cdots, \bar{\mathbf{a}}_\mathbf{n}) + \sum_{\mathbf{j}=1}^{\mathbf{n}} \frac{\partial \mathbf{f}(\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \cdots, \bar{\mathbf{a}}_\mathbf{n})}{\partial \mathbf{a}_\mathbf{j}} (\mathbf{a}_\mathbf{j} - \bar{\mathbf{a}}_\mathbf{j}) \quad (1)$$

In a linear statistical model the higher terms than linear term are neglected. The corresponding matrix equation for m functions of $\mathbf{n}$ parameters is

$$\underset{\mathbf{m}\times 1}{\{\mathbf{f}(\mathbf{a})\}} = \underset{\mathbf{m}\times 1}{\{\mathbf{f}(\bar{\mathbf{a}})\}} + \underset{\mathbf{m}\times\mathbf{n}}{\left[\frac{\partial \mathbf{f}(\bar{\mathbf{a}})}{\partial \mathbf{a}}\right]} \underset{\mathbf{n}\times 1}{\{\mathbf{a} - \bar{\mathbf{a}}\}} \quad (2)$$

The mean value of each random function is obtained by taking the expected value of Eq. (2) and is

$$\mathbf{E}\langle\{\mathbf{f}(\mathbf{a})\}\rangle = \{\mathbf{f}(\bar{\mathbf{a}})\} + \left[\frac{\partial \mathbf{f}(\bar{\mathbf{a}})}{\partial \mathbf{a}}\right] \mathbf{E}\langle\{\mathbf{a} - \bar{\mathbf{a}}\}\rangle \quad (3)$$

Since each term $\mathbf{E}\langle\{\mathbf{a} - \bar{\mathbf{a}}\}\rangle$ is zero, it follows that

$$\mathbf{E}\langle\{\mathbf{f}(\mathbf{a})\}\rangle = \{\mathbf{f}(\bar{\mathbf{a}})\} \quad (4)$$

In the numerical probabilistic method, computationally these random numbers of structural parameters are then substituted into the response equation to obtain a set of random numbers that reflect the uncertainty in the structural response. Most problems in structural analysis are solved using the finite element method, wherein the force equilibrium equations governing the displacement of a structure are written as follows:

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\} \quad (5)$$

where $\mathbf{K}$ is the symmetric stiffness matrix, $\mathbf{u}$ is the vector of displacements for each degree of freedom, and $\mathbf{F}$ is the vector of applied forces. Using the same general notation for these random variables $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_\mathbf{n}$ and noting that the random functions under consideration are generalized displacements, one may be rewrite Eq. (2) as

$$\underset{\mathbf{m}\times 1}{\{\mathbf{u}(\mathbf{a})\}} = \underset{\mathbf{m}\times 1}{\{\mathbf{u}(\bar{\mathbf{a}})\}} + \underset{\mathbf{m}\times\mathbf{n}}{\left[\frac{\partial \mathbf{u}(\bar{\mathbf{a}})}{\partial \mathbf{a}}\right]} \underset{\mathbf{n}\times 1}{\{\mathbf{a} - \bar{\mathbf{a}}\}} \quad (6)$$

The mean response is the solution to matrix equation and is

$$\mathbf{E}\langle\{\mathbf{u}(\mathbf{a})\}\rangle = \{\mathbf{u}(\bar{\mathbf{a}})\} = [\mathbf{K}]^{-1}\{\mathbf{F}\} \quad (7)$$

with $\bar{\mathbf{a}}_\mathbf{j}$ substituted everywhere for $\mathbf{a}_\mathbf{j}$ before solving. The values of $\{\mathbf{u}(\mathbf{a})\}$ depend on structural parameter vectors $\mathbf{\Phi}$ with bound error or uncertainties. But it is assumed that we have no data about the value of $\mathbf{\Phi}$ in this paper, and uncertainty values of structural parameters is replaced by generation values of random number through the Monte Carlo simulation. The probability density function is a powerful mathematical tool which enables one to perform problems of the uncertainty and variability in the structural analysis. Figure 1 shows a general shape of the cumulative distribution function (i.e. CDF).

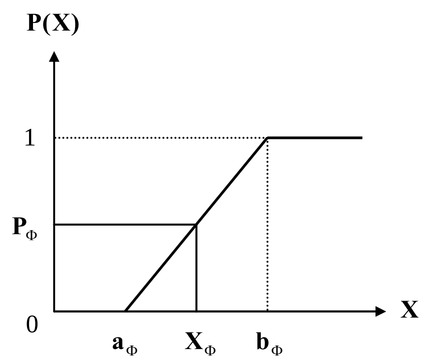


Figure 1. Cumulative distribution function

We define the mean of $\mathbf{\Phi}$ by

$$\mathbf{M}_\mathbf{\Phi} = (\mathbf{a}_\mathbf{\Phi} + \mathbf{b}_\mathbf{\Phi}) / 2 \quad (8)$$

and its variance is

$$\mathbf{V}_{\Phi} = (\mathbf{b}_{\Phi} - \mathbf{a}_{\Phi})^2 / 12 \qquad (9)$$

In a similar form, they are expressed as

$$\mathbf{a}_{\Phi} = \mathbf{M}_{\Phi} - \sqrt{3\mathbf{V}_{\Phi}} \qquad (10)$$

$$\mathbf{b}_{\Phi} = \mathbf{M}_{\Phi} + \sqrt{3\mathbf{V}_{\Phi}} \qquad (11)$$

where $\mathbf{a}_{\Phi} < \mathbf{b}_{\Phi}$ is assumed.

In order to generate a new uniform random number $\mathbf{X}_{\Phi}$ between $\mathbf{a}_{\Phi}$ and $\mathbf{b}_{\Phi}$, the $\mathbf{i}$ th generator of the Monte Carlo simulation is executed by Eq. (12) as introduced by Hart (1982).

$$\overset{(i)}{\mathbf{X}}_{\Phi} = \mathbf{a}_{\Phi} + \left[\frac{\mathbf{b}_{\Phi} - \mathbf{a}_{\Phi}}{1.00 - 0.}\right]\left(\overset{(i)}{\mathbf{P}} - 0.00000\right)$$

$$= \mathbf{M}_{\Phi} - \sqrt{3\mathbf{V}_{\Phi}}\left(1 + 2\overset{(i)}{\mathbf{P}}\right) \qquad (12)$$

where, $\overset{(i)}{\mathbf{P}}$ is the probability values of uniform random number values between 0 and 1. $\overset{(i)}{\mathbf{X}}_{\Phi}$ denote the value of the random number for a uniform PDF over the range $\mathbf{a}_{\Phi}$ to $\mathbf{b}_{\Phi}$, which are listed by the Table 1.

The equations of the mean and variance of $\mathbf{X}_{\Phi}$, obtained by calculation of the $\mathbf{n}$ th uniform random number is respectively written as follows.

$$\mathbf{m}_{\Phi} = \mathbf{M}_{\Phi} - \sqrt{3\mathbf{V}_{\Phi}}\left(1 - \frac{1}{\mathbf{n}}\sum_{i=1}^{\mathbf{n}}\overset{(i)}{\mathbf{P}}\right) \qquad (13)$$

$$\boldsymbol{\sigma}_{\Phi} = \frac{1}{\mathbf{n}}\sum_{i=1}^{\mathbf{n}} 12\mathbf{V}_{\Phi}\left(\overset{(i)}{\mathbf{P}} + \frac{1}{\mathbf{n}}\sum_{j=1}^{\mathbf{n}}\overset{(j)}{\mathbf{P}}\right)^2 \qquad (14)$$

In case that Young's modulus $\mathbf{E}$, cross-sectional area $\mathbf{A}$, and element length $\mathbf{L}$ are deterministic and the applied load $\mathbf{P}$ is the random parameter in linear truss system, the values of random displacements is expressed as Eq. (15) according to the mean of arithmetic calculation and harmonic combination.

$$\mathbf{U}(\mathbf{I}) = (1 + \mathbf{X}(\mathbf{I}))\left[\boldsymbol{\alpha}\,\frac{\mathbf{P(I)L}}{\mathbf{EA}}\right] \geq 2\sqrt{\mathbf{X}(\mathbf{I})}\left[\boldsymbol{\alpha}\,\frac{\mathbf{P(I)L}}{\mathbf{EA}}\right] \qquad (15)$$

where, $\mathbf{X(I)}$ denote the random numbers obtained by Monte Carlo simulation. $\boldsymbol{\alpha} \geq \mathbf{0}$ is the weighting coefficient in order to indicate actual structural response values which are reversely proportional to axial rigidity. From Eq. (15), the quadratic operation of the left and right term as follows is considered for the purpose of similarity of solutions.

$$(1 + \mathbf{X}(\mathbf{I}))^2 \geq \left(2\sqrt{\mathbf{X}(\mathbf{I})}\right)^2 = 4 \times \mathbf{X}(\mathbf{I}) \qquad (16)$$

Therefore final random displacements are formulated as Eq. (17) for Monte Carlo simulation.

$$\mathbf{U}(\mathbf{I}) = 4 \times \mathbf{X}(\mathbf{I})\left[\boldsymbol{\alpha}\,\frac{\mathbf{P(I)L}}{\mathbf{EA}}\right] \qquad (17)$$

These values of the response formulation which are devised in this study are analyzed by using the algorithm described in Section 3. If any of the other parameters on the response system were random, then the flow chart of Figure 4 would be modified to reflect the generation of a set of random numbers for each random variable.

3. THE GENERATION OF DOUBLE UNIFORM RANDOM NUMBERS AND MONTE CARLO ALGORITHMS

The random structural response set is then analyzed by using techniques related with a mathematical function, that is, a probability distribution function (i.e. PDF). This type of analysis is called a Monte Carlo analysis. Monte Carlo analysis is a powerful engineering tool that enables one to perform a statistical analysis of the uncertainty in structural engineering problems, being particularly useful for complex problems where numerous random variables are related through nonlinear equation. It is often helpful to visualize a Monte Carlo analysis as an experiment that is performed by a computer rather than in a structural engineering laboratory.

The fundamental step in a Monte Carlo analysis is the generation of a set of random numbers. Figure 2 illustrates the generation of double uniform random numbers between 0 and 1. Contrary to conventional Monte Carlo simulation using only grouping 1, both grouping 1 and 2 are executed through the sequential method in this present approach.

The unit of sequential five random numbers of grouping 1 is used for the construction of grouping 2, for example a grouping 2-1 has ten random numbers and a grouping 2-n (n=constant) consists of 5×(n+1) uniform random numbers. The groupings have repetitive numbers with other groupings. The numbers is named as dual uniform random numbers.

Table 1 lists the means and standard deviations of uniform random numbers which are obtained by the generation of Figure 2. This table is reproduced with permission from the RAND Corporation, A Million Random Digits with 100,000 Normal Deviates, The Free Press, Glencoe, IL. These random numbers can be used for any range of the uniform probability density function if one scales them properly. In this paper they are scaled by 10^{-5}.

Figure 3 shows convergence of the mean and standard deviation, while the number of the groupings increases in case of 2^{nd} groupings.

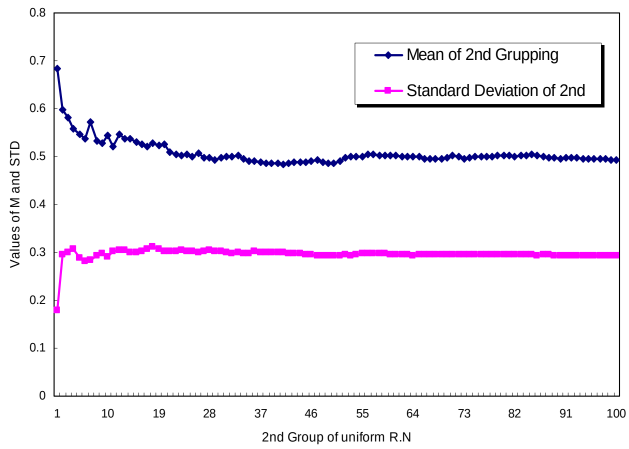


Figure 3. Convergence, Mean and Standard Deviation for generation values of one set of n *Dual Uniform Random Number* for each parameter in the response by processes of 2^{nd} grouping.

It can be seen that the means and standard deviations become converged after some generation of the groupings.

These random numbers can be mechanically or electronically generated, however today in practice most random number generation is accomplished by using digital computer algorithm for the random number generation. Uniform random numbers have characteristics that for a selected range of values (i.e. 0.0 to 1.0) the generated random numbers are equally likely to occur anywhere in the range.

Figure 4 shows relationships between mean and standard deviation in DURN of grouping 2 (i.e. 2^{nd} group) and typical random numbers of grouping 1 (i.e. 1^{st} group). It can be found that the relationship is more stable in grouping 2 than in grouping 1.

The grouping method of the random number used in this process is applied to numerical examples of the structural analysis to consider uncertainty properties of structural parameters. Basically it is assumed that characteristics of this uncertainty are expressed as dual uniform random numbers.

According to this method, modified parameters have an influence on the uncertain behaviors of structures using numerical approach. The Monte Carlo analysis involves the generation of one set of **n** random numbers for each random parameter in the equation system. The response equation is then solved by using each random number in the set. Therefore, the response equation is solved n times; i.e. i = 1, 2, … , n and the algorithm is shown to Figure 5.

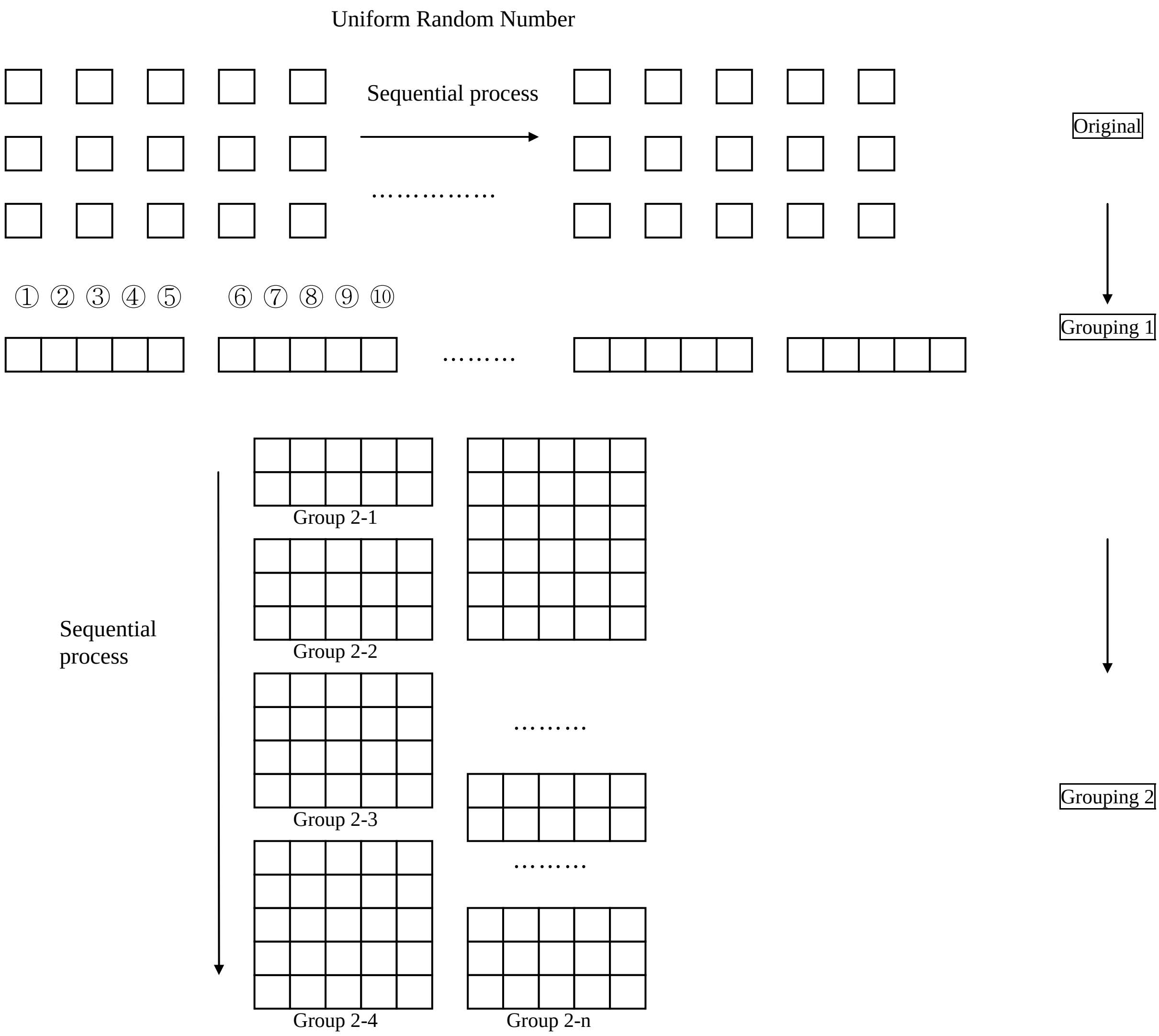


Figure 2. The generation of double uniform random numbers

Table 1. Uniform Random Numbers between 0 and 1.0 by the 1st and 2nd grouping

Uniform Random Number					1st	1st group		2nd	2nd group	
					Group	M of RN	STD of RN	Group	M of RN	STD of RN
0.52478	0.80249	0.94132	0.17938	0.58815	1	0.68456	0.17938	1	0.68456	0.17938
0.69379	0.75228	0.14327	0.38270	0.06070	2	0.51126	0.38270	1-2	0.59791	0.2962
0.60929	0.41999	0.75908	0.33985	0.03428	3	0.54717	0.33985	1-3	0.58100	0.30002
0.02333	0.55696	0.74838	0.34966	0.87260	4	0.49192	0.34966	1-4	0.55873	0.30599
0.73595	0.66224	0.48078	0.22980	0.47813	5	0.49976	0.22980	1-5	0.54693	0.28898
0.39404	0.84131	0.65097	0.27269	0.11997	6	0.49036	0.27269	1-6	0.53750	0.28253
0.89716	0.85258	0.45790	0.18956	0.80321	7	0.78694	0.18956	1-7	0.57314	0.28312
0.16964	0.03060	0.46517	0.19071	0.11580	8	0.23920	0.19071	1-8	0.53140	0.29346
0.92862	0.27419	0.01450	0.35661	0.66047	9	0.49796	0.35661	1-9	0.52768	0.29666
0.71899	0.36567	0.86135	0.19715	0.75099	10	0.70371	0.19715	1-10	0.54528	0.29163
0.22835	0.16089	0.15190	0.33379	0.01919	11	0.28546	0.33379	1-11	0.52166	0.30184
0.89366	0.79546	0.93484	0.21588	0.44497	12	0.81065	0.21588	1-12	0.54575	0.30501
0.72559	0.12790	0.62356	0.31782	0.01736	13	0.41259	0.31782	1-13	0.53693	0.30554
0.25192	0.67106	0.46105	0.25724	0.93170	14	0.55555	0.25724	1-14	0.53693	0.30076
0.29104	0.21746	0.26348	0.31040	0.96995	15	0.42615	0.31040	1-15	0.52955	0.30054
0.88593	0.64236	0.63684	0.35699	0.06462	16	0.47215	0.35699	1-16	0.52596	0.30209
0.28633	0.16576	0.04380	0.40796	0.92435	17	0.45610	0.40796	1-17	0.52185	0.30663
0.90116	0.35414	0.93209	0.41137	0.04688	18	0.63485	0.41137	1-18	0.52813	0.31149
0.25355	0.80915	0.54579	0.23222	0.26430	19	0.45118	0.23222	1-19	0.52408	0.30734
0.68860	0.46306	0.49654	0.15830	0.37473	20	0.55458	0.15830	1-20	0.52560	0.30124
0.33307	0.01964	0.08425	0.15429	0.22225	21	0.20726	0.15429	1-21	0.51045	0.30321
0.50240	0.65528	0.49875	0.15523	0.21962	22	0.38249	0.15523	1-22	0.50463	0.30119
0.19902	0.87990	0.27342	0.39601	0.91578	23	0.47056	0.39601	1-23	0.50315	0.30379
0.93932	0.73369	0.29798	0.27507	0.35494	24	0.55007	0.27507	1-24	0.50510	0.30173
0.59346	0.81973	0.33935	0.31474	0.19524	25	0.39549	0.31474	1-25	0.50072	0.30172
0.71327	0.26803	0.50298	0.27037	0.80215	26	0.64981	0.27037	1-26	0.50645	0.30100
0.77208	0.27023	0.06893	0.29891	0.23677	27	0.27308	0.29891	1-27	0.49781	0.30306
0.77618	0.80332	0.83758	0.38293	0.11925	28	0.52656	0.38293	1-28	0.49884	0.30461
0.38189	0.50829	0.23503	0.11794	0.22415	29	0.32974	0.11794	1-29	0.49301	0.30151
0.08150	0.69777	0.63467	0.33019	0.98759	30	0.61811	0.33019	1-30	0.49718	0.30215
0.73842	0.21414	0.70298	0.28047	0.38562	31	0.58847	0.28047	1-31	0.50012	0.30106
0.49343	0.48794	0.12103	0.22955	0.48270	32	0.47033	0.22955	1-32	0.49919	0.29856
0.40024	0.77646	0.93069	0.33381	0.09165	33	0.57891	0.33381	1-33	0.50160	0.29888
0.65485	0.20689	0.05504	0.26259	0.54207	34	0.32034	0.26259	1-34	0.49627	0.29876
0.21213	0.43832	0.08981	0.25397	0.17227	35	0.32594	0.25397	1-35	0.49141	0.29831
0.08978	0.09167	0.98391	0.45123	0.16900	36	0.44460	0.45123	1-36	0.49011	0.30185
0.34231	0.25722	0.83032	0.24308	0.33356	37	0.40231	0.24308	1-37	0.48773	0.30021
0.38200	0.87759	0.25428	0.31748	0.57414	38	0.42641	0.31748	1-38	0.48612	0.29995
0.68819	0.23146	0.31250	0.36503	0.98215	39	0.46156	0.36503	1-39	0.48549	0.30069
0.39941	0.56251	0.35906	0.29199	0.91653	40	0.47343	0.29199	1-40	0.48519	0.29976
0.67277	0.72117	0.02202	0.30570	0.45731	41	0.40968	0.30570	1-41	0.48335	0.29937
0.31867	0.73980	0.77984	0.24623	0.68632	42	0.55770	0.24623	1-42	0.48512	0.29794
0.74215	0.33177	0.60284	0.21187	0.67708	43	0.65249	0.21187	1-43	0.48901	0.29694
0.73266	0.27707	0.62588	0.31771	0.82683	44	0.50820	0.31771	1-44	0.48945	0.29667
0.30923	0.55932	0.44947	0.23886	0.73490	45	0.43221	0.23886	1-45	0.48817	0.29519
0.41241	0.39695	0.93703	0.26606	0.75972	46	0.56718	0.26606	1-46	0.48989	0.29429
0.79158	0.44809	0.91230	0.21168	0.76405	47	0.67352	0.21168	1-47	0.49380	0.29364
0.45273	0.13961	0.07686	0.15830	0.56554	48	0.27428	0.15830	1-48	0.48923	0.29363
0.61797	0.11641	0.56057	0.25308	0.10413	49	0.31440	0.25308	1-49	0.48566	0.29345
0.60556	0.20007	0.50560	0.21419	0.76447	50	0.54525	0.21419	1-50	0.48685	0.29187
0.32880	0.91712	0.80519	0.31986	0.91743	51	0.64911	0.31986	1-51	0.49003	0.29263
0.81661	0.87645	0.97966	0.12685	0.68338	52	0.86941	0.12685	1-52	0.49733	0.29488
0.93857	0.62684	0.69329	0.20635	0.36704	53	0.64104	0.20635	1-53	0.50004	0.29383
0.95972	0.10432	0.12700	0.43994	0.22976	54	0.47309	0.43994	1-54	0.49954	0.29601
0.15747	0.81707	0.78208	0.40522	0.09448	55	0.56290	0.40522	1-55	0.50069	0.29748
0.88350	0.98995	0.55438	0.30440	0.76712	56	0.68337	0.30440	1-56	0.50395	0.29803
0.12531	0.61284	0.36690	0.30201	0.93136	57	0.48877	0.30201	1-57	0.50369	0.29757
0.20442	0.07849	0.24628	0.38926	0.94938	58	0.45411	0.38926	1-58	0.50283	0.29859
0.70112	0.29047	0.44450	0.15860	0.54380	59	0.51882	0.15860	1-59	0.50310	0.29662
0.23386	0.74448	0.24921	0.23121	0.34010	60	0.43746	0.23121	1-60	0.50201	0.29546
0.76457	0.11487	0.23516	0.37138	0.99315	61	0.49308	0.37138	1-61	0.50186	0.29611
0.41037	0.22604	0.08644	0.16838	0.39325	62	0.32551	0.16838	1-62	0.49902	0.29517
0.04988	0.34119	0.83998	0.31374	0.59481	63	0.50591	0.31374	1-63	0.49913	0.29494
0.72606	0.53118	0.46093	0.23271	0.12257	64	0.49686	0.23271	1-64	0.49909	0.29378
0.67104	0.89193	0.94370	0.40902	0.13156	65	0.54652	0.40902	1-65	0.49982	0.29508
0.34760	0.22498	0.22718	0.08052	0.14861	66	0.25390	0.08052	1-66	0.49610	0.29451
0.31612	0.07636	0.39612	0.33109	0.60668	67	0.47016	0.33109	1-67	0.49571	0.29455
0.80655	0.08970	0.95824	0.38049	0.18151	68	0.52585	0.38049	1-68	0.49615	0.2953
0.84563	0.45806	0.55982	0.26850	0.10492	69	0.47244	0.26850	1-69	0.49581	0.29458
0.92449	0.75234	0.21109	0.39001	0.86452	70	0.56774	0.39001	1-70	0.49684	0.29555

0.94489	0.67479	0.86294	0.11807	0.70350	71	0.81366	0.11807	1−71	0.50130	0.2961
0.59120	0.49290	0.07089	0.21871	0.35105	72	0.41989	0.21871	1−72	0.50017	0.29509
0.24389	0.09212	0.15037	0.07712	0.28243	73	0.19999	0.07712	1−73	0.49606	0.29524
0.89242	0.23692	0.58754	0.29373	0.94522	74	0.70131	0.29373	1−74	0.49883	0.29578
0.90389	0.01511	0.82235	0.35445	0.41802	75	0.53960	0.35445	1−75	0.49937	0.29611
0.19507	0.49489	0.78013	0.30814	0.97496	76	0.57110	0.30814	1−76	0.50032	0.29596
0.23543	0.67054	0.65695	0.22364	0.43458	77	0.55940	0.22364	1−77	0.50108	0.29499
0.13676	0.67354	0.11554	0.35462	0.93058	78	0.48951	0.35462	1−78	0.50094	0.29529
0.54657	0.98176	0.73082	0.35412	0.59665	79	0.57470	0.35412	1−79	0.50187	0.29568
0.31012	0.58915	0.18652	0.30655	0.82362	80	0.55827	0.30655	1−80	0.50257	0.29549
0.82597	0.13649	0.32871	0.30578	0.78240	81	0.55572	0.30578	1−81	0.50323	0.29529
0.44282	0.08068	0.18809	0.15652	0.42348	82	0.27199	0.15652	1−82	0.50041	0.29498
0.22094	0.89973	0.96165	0.29096	0.71395	83	0.69362	0.29096	1−83	0.50274	0.29534
0.91968	0.21450	0.15780	0.35516	0.61364	84	0.55358	0.35516	1−84	0.50334	0.29567
0.75901	0.83257	0.34382	0.24427	0.28217	85	0.54742	0.24427	1−85	0.50386	0.29491
0.39102	0.16808	0.64409	0.20447	0.18986	86	0.38015	0.20447	1−86	0.50242	0.29415
0.57480	0.26665	0.94966	0.37672	0.08562	87	0.38626	0.37672	1−87	0.50109	0.29494
0.41471	0.16026	0.01428	0.17177	0.26924	88	0.25457	0.17177	1−88	0.49829	0.29488
0.21490	0.75455	0.23666	0.17177	0.42368	89	0.42223	0.17177	1−89	0.49743	0.29406
0.41277	0.64903	0.39797	0.15939	0.25185	90	0.39581	0.15939	1−90	0.49630	0.293
0.04836	0.94539	0.89573	0.41079	0.22015	91	0.57253	0.41079	1−91	0.49714	0.29403
0.66605	0.54935	0.33738	0.22834	0.14412	92	0.47343	0.22834	1−92	0.49688	0.29321
0.89237	0.39638	0.62149	0.31357	0.38607	93	0.46853	0.31357	1−93	0.49658	0.29309
0.25721	0.67362	0.00361	0.25942	0.34228	94	0.36369	0.25942	1−94	0.49516	0.29282
0.45606	0.89931	0.18908	0.30053	0.32658	95	0.40559	0.30053	1−95	0.49422	0.29272
0.17168	0.10807	0.97879	0.39810	0.66671	96	0.55776	0.39810	1−96	0.49488	0.29353
0.75667	0.73238	0.21734	0.25986	0.70311	97	0.54279	0.25986	1−97	0.49538	0.293
0.59063	0.23225	0.80580	0.26811	0.16181	98	0.42447	0.26811	1−98	0.49465	0.29259
0.52086	0.09782	0.66578	0.26118	0.15138	99	0.40437	0.26118	1−99	0.49374	0.29219
0.18925	0.24311	0.19964	0.30952	0.90842	100	0.41376	0.30952	1−100	0.49294	0.29215

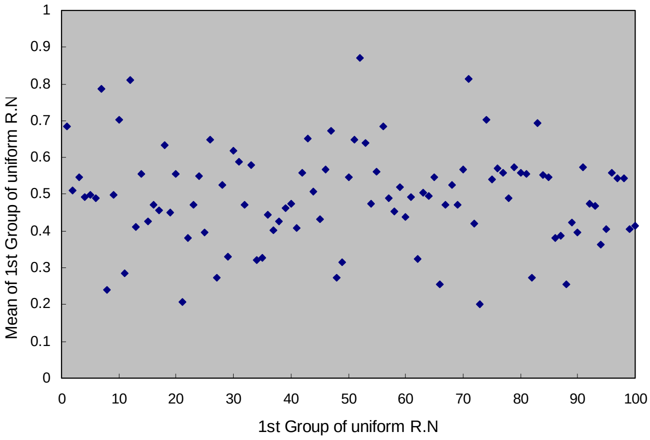
Table 2. Random displacement u_1 using the 2^{nd} grouping

(a) Iteration=50~250

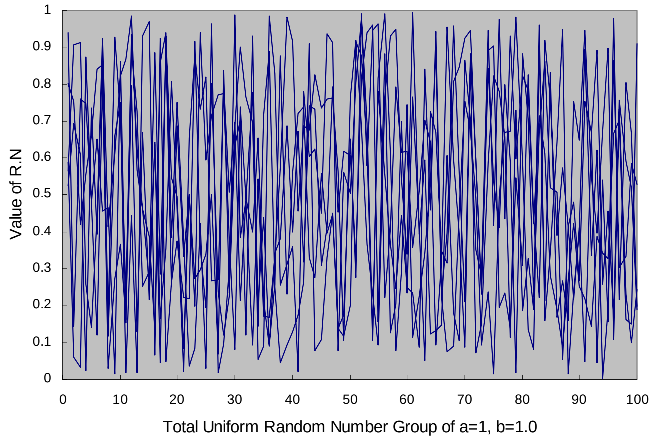
2^{nd} Group	50		100		150		200		250	
	M50	STD50	M100	STD100	M150	STD150	M200	STD200	M250	STD250
1	−0.10963	0.02833	−0.11025	0.02918	−0.11084	0.02871	−0.11001	0.02869	−0.11069	0.02851
1~10	−0.0822	0.04365	−0.08316	0.04495	−0.08407	0.04422	−0.08279	0.04419	−0.08383	0.04391
1~20	−0.07939	0.04521	−0.08038	0.04656	−0.08133	0.04581	−0.08	0.04578	−0.08108	0.04549
1~30	−0.07701	0.04654	−0.07803	0.04793	−0.079	0.04715	−0.07764	0.04713	−0.07875	0.04683
1~40	−0.07634	0.04692	−0.07737	0.04832	−0.07835	0.04753	−0.07697	0.04751	−0.0781	0.04721
1~50	−0.07757	0.04623	−0.07857	0.04761	−0.07954	0.04684	−0.07819	0.04682	−0.07929	0.04652
1~60	−0.07831	0.04582	−0.07931	0.04718	−0.08027	0.04642	−0.07893	0.04639	−0.08003	0.0461
1~70	−0.07788	0.04606	−0.07889	0.04743	−0.07985	0.04666	−0.0785	0.04664	−0.0796	0.04634
1~80	−0.07836	0.04579	−0.07936	0.04716	−0.08031	0.04639	−0.07897	0.04637	−0.08007	0.04608
1~90	−0.07819	0.04589	−0.07919	0.04726	−0.08015	0.04649	−0.07881	0.04646	−0.0799	0.04617
1~100	−0.07803	0.04597	−0.07904	0.04735	−0.08	0.04658	−0.07865	0.04655	−0.07975	0.04626

(b) Iteration=300~500

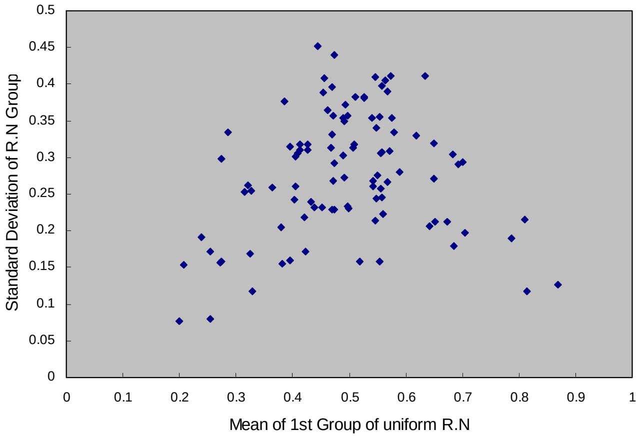
2^{nd} Group	300		350		400		450		500	
	M300	STD300	M350	STD350	M400	STD400	M450	STD450	M500	STD500
1	−0.11035	0.02906	−0.11085	0.0289	−0.1098	0.02879	−0.11061	0.02855	−0.10979	0.02859
1~10	−0.08332	0.04477	−0.08408	0.04452	−0.08247	0.04435	−0.08371	0.04398	−0.08246	0.04404
1~20	−0.08055	0.04638	−0.08134	0.04612	−0.07967	0.04595	−0.08096	0.04556	−0.07966	0.04562
1~30	−0.0782	0.04774	−0.07901	0.04748	−0.0773	0.0473	−0.07862	0.0469	−0.07728	0.04696
1~40	−0.07754	0.04812	−0.07836	0.04786	−0.07663	0.04768	−0.07797	0.04728	−0.07662	0.04734
1~50	−0.07875	0.04742	−0.07955	0.04717	−0.07785	0.04698	−0.07916	0.04659	−0.07784	0.04665
1~60	−0.07949	0.04699	−0.08028	0.04674	−0.0786	0.04656	−0.0799	0.04617	−0.07858	0.04623
1~70	−0.07906	0.04724	−0.07986	0.04699	−0.07817	0.0468	−0.07947	0.04641	−0.07815	0.04647
1~80	−0.07953	0.04697	−0.08032	0.04672	−0.07864	0.04653	−0.07994	0.04614	−0.07862	0.0462
1~90	−0.07936	0.04707	−0.08016	0.04681	−0.07847	0.04663	−0.07978	0.04624	−0.07846	0.0463
1~100	−0.07921	0.04716	−0.08001	0.0469	−0.07832	0.04672	−0.07962	0.04633	−0.0783	0.04639



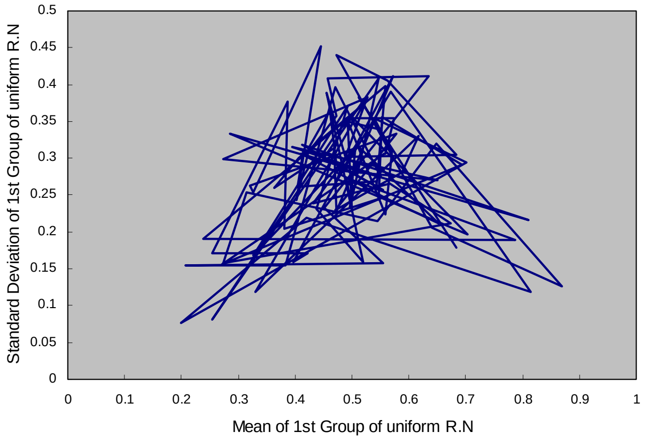
(a) The point distribution of total uniform random number



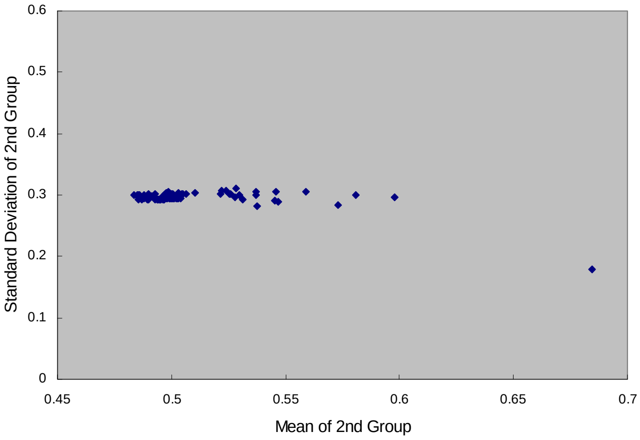
(b) The line distribution of time dependency of total uniform random number



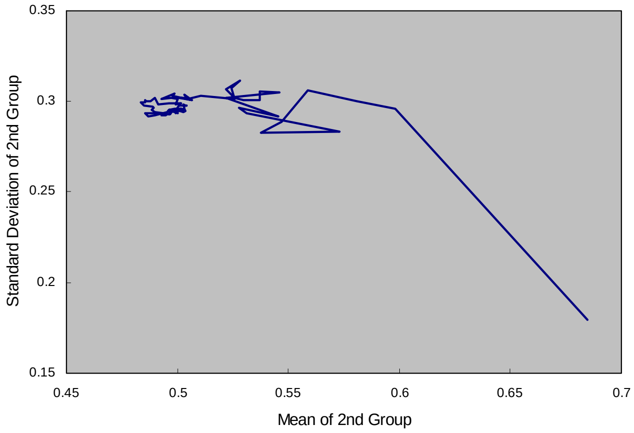
(c) The point distribution of the mean & standard deviation of 1st uniform R.N. Group



(d) The line distribution of time dependency of the mean & standard deviation of 1st uniform R.N. Group



(e) The point distribution of the mean & standard deviation of 2nd uniform R.N group



(f) The line distribution of time dependency of the mean & standard deviation of 2nd uniform R.N Group

Figure 4. Relationships between mean and standard deviation in sequential generation process of 1st and 2nd grouping using uniform random numbers with permission from the RAND Corporation, A Million Random Digits with 100,000 Normal Deviates, The Free Press.

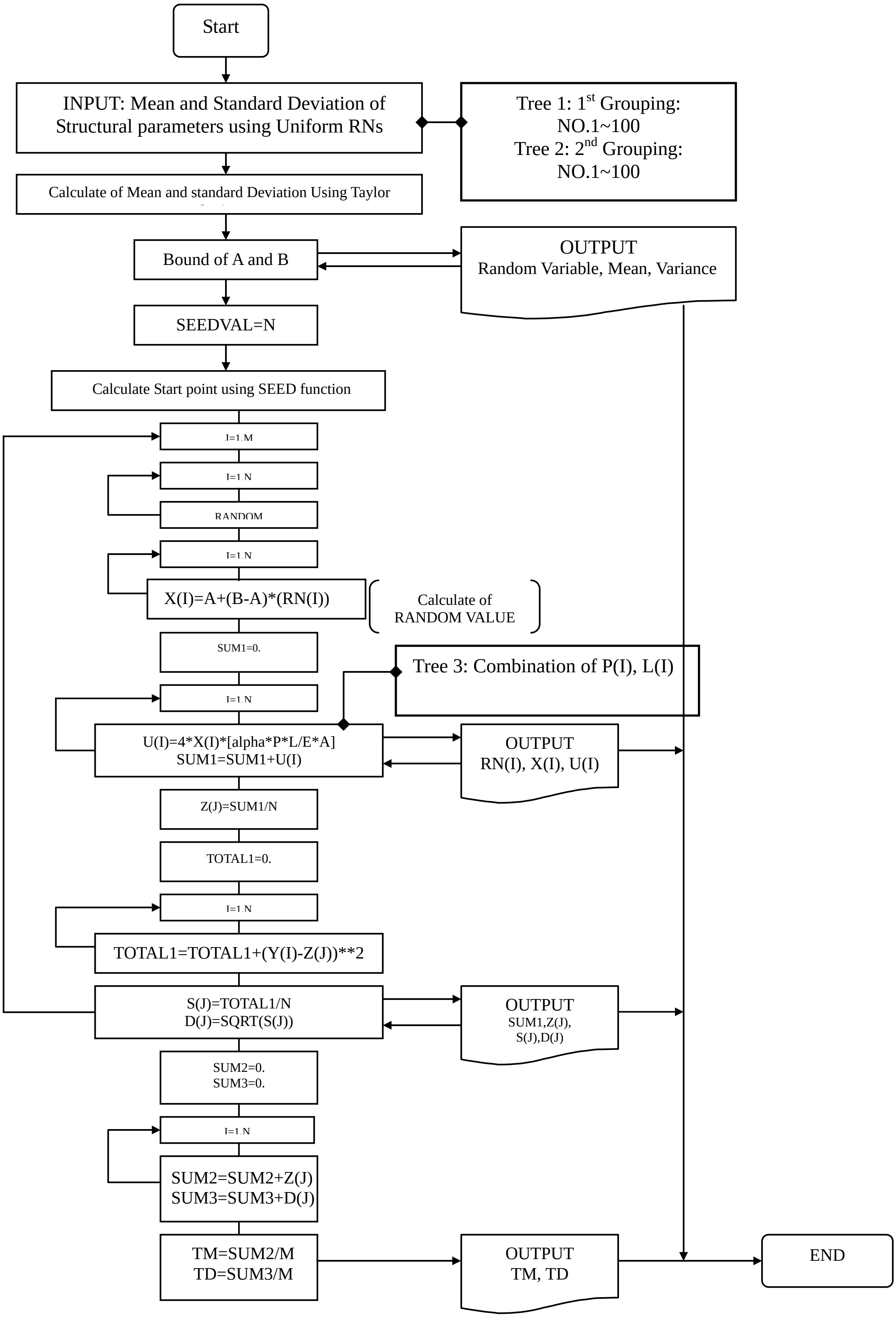


Figure 5. Algorithm of Monte carlo Analysis with generation of one set of *Dual Uniform Random Numbers* for each parameter in the response by processes of 1^{st} and 2^{nd} grouping.

4. NUMERICAL EXAMPLE

A symmetrical truss under a concentrated load **P** at joint 3 is shown in Figure 6. All bars have the same axial rigidity **EA** and length **L** and the seven-element truss has five nodal points and 10 degree of freedom is shown in Figure 7.

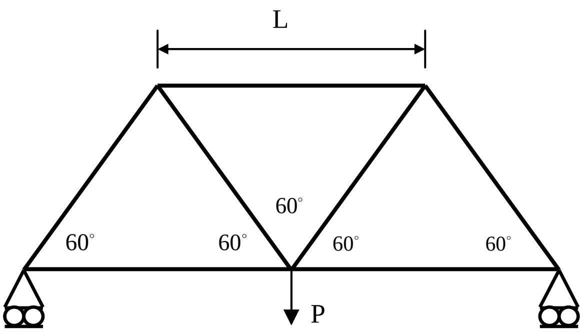


Figure 6. Truss structure

The global stiffness matrix and the applied force vector are

$$\mathbf{K} = \sum_{i=1}^{7} \frac{\mathbf{E}_i \mathbf{A}_i}{\mathbf{L}_i} \mathbf{k}_i \tag{18}$$

$$\mathbf{F} = (\mathbf{P})^{\mathrm{T}} \tag{19}$$

where **F** defines the applied external force vector and concentrated load **P** take a value of -1000 kN , $\mathbf{E}_i$ is

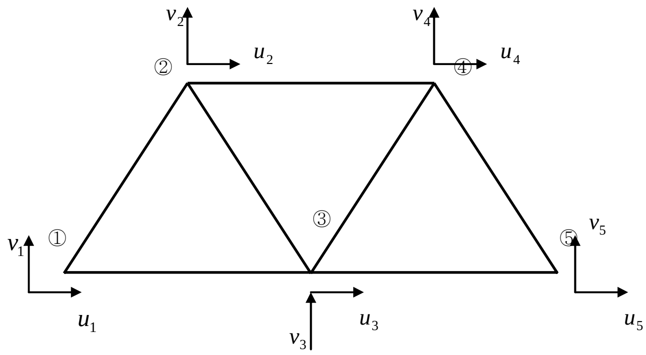


Figure 7. Nodal displacements (DOF=10)

Young's modulus of the **i** th member, $\mathbf{A}_i$ is the cross-sectional area of the **i** th member, $\mathbf{L}_i$ is the length of the **i** th member. The local element matrices is $\mathbf{k}_i$. Young's modulus for each member is assigned a value of 2.0×10^5 N/m^2, the cross-sectional area of members 1-6 are assigned values of 9.0×10^{-2} m^2, and the length L (see figure) take a value of 10 m .

Each structural parameter, **P**, **E**, **A**, and **L** are assigned by uniform random number sets, and the 1st and 2nd groups of uniform random numbers as shown in Table 1 are composed of the scenario of 100 cases of uncertainty characteristics for the structural response respectively. Also in the tree graph of combinations of structural parameters 2^4 cases of structural responses occurs.

Table 2 illustrates the means and standard deviations of random displacements u_1 according to 2nd grouping of Table 1 and it is shown in Figure 8 and 9.

Figure 8 and 9 illustrate that the results of uncertainty of structural responses are changed through each scenario. Here, M and STD denote mean and standard deviation respectively. The number 200 is total useful number of random number in M200. As the number of the uniform random number is increased, the uncertainty of structural responses makes toward the stabile location of the structural behavior. The range of the random displacement is changed by the upper bound (i.e. the 4th case with the number of an iteration=50) and the lower bound (i.e. the 1st case with the number of an iteration=350). The errors of the upper and lower bound compared with the numerical exact solution (i.e. –0.160375) are 52.399 % and 47.572 %, respectively.

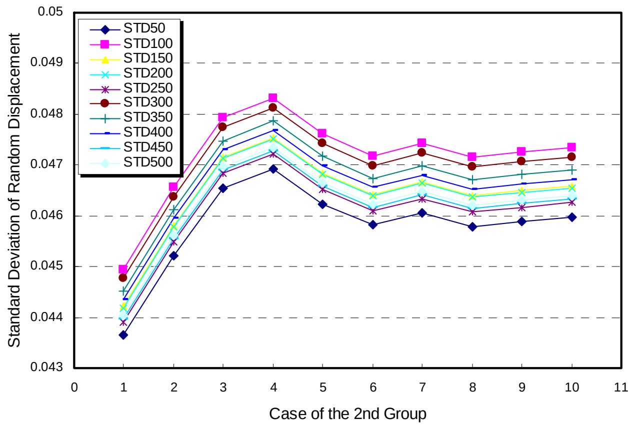


Figure 8. Convergence of Standard Deviation of random displacement u_1 for generation values of one set of n *Dual Uniform Random Number* for each parameter in the response by processes of 2nd grouping

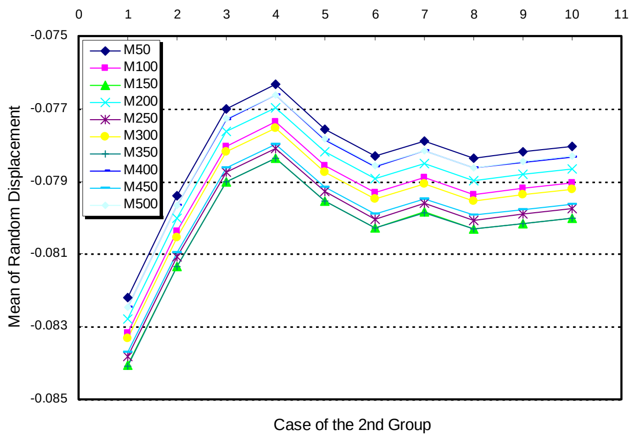


Figure 9. Convergence of Mean of random displacement u_1 for generation values of one set of n *Dual Uniform Random Number* for each parameter in the response by processes of 2nd grouping

Figure 10 shows the degree of the uncertainty with respect to each scenario and number of the iteration. Here uncert denotes uncertainty and the number 100 in uncert100 is numbers of useful RN. It is investigated that the 4^{th} case with the number of an iteration=100 and 1^{st} case with the number of an iteration=250 have the maximum value (i.e. the upper bound value) 62.453 % and the minimum value (i.e. the lower bound value) 52.379 % of scenarios of random displacements, respectively.

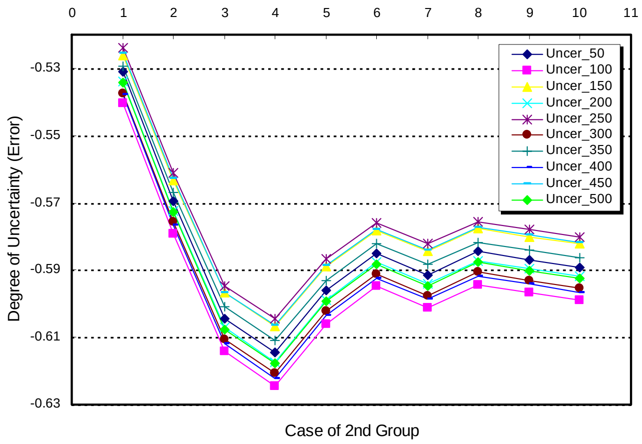


Figure 10. Convergence of uncertainty degree (error) in Mean and Standard Deviation for generation values of one set of n *Dual Uniform Random Number* for each parameter in the response by processes of 2^{nd} grouping

5. CONCLUSIONS

Monte Carlo methods provide a systematic way of dealing with complex problems for which we have incomplete knowledge of the relationship between data and model parameters. This is the case, e.g., for many highly complicated problems of the structural responses, where the forward relation is insusceptible to mathematical analysis, and is only given by complex algorithm. Monte Carlo methods can be devoted to sampling from a probability density. Although the Monte Carlo algorithm of this technique is simple and well mapped-out, many theoretical and practical problems concerning their speed of processes remain to be solved, and this random solution appears great error in comparison with the numerical exact solution. Therefore many random numbers and sequential processes need in order to be close to exact solution. This results in computational burdens.

However in this paper the sampling DURN technique by 1^{st} and 2^{nd} grouping of the random number can make selective grouping cases without additional iteration of the simulation and the groupings are utilized in cases where a resolution and uncertainty analysis is called for. Comparisons of structural behaviors are investigated by each grouping and it saves computational time in order to obtain approximate and appropriate solutions.

This study addresses the modified Monte Carlo algorithms through dual uniform random numbers currently in use in applications. Several variants of these algorithms exist many of which are adaptations of the basic methods, or exploit special properties of the problem considered.

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