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Production-and-Delivery Scheduling with Transportation Mode Selection Allowed

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수송수단의 선택이 허용된 생산 및 배송 스케줄링에 관한 연구

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This paper considers a scheduling problem to minimize the sum of the associated scheduling (production/delivery times) cost and the delivery cost for an integrated system of a single production machine and various transportation vehicles with transportation mode selection allowed. Each transportation mode is provided with a fixed number of vehicles at the associated delivery time and cost. The proposed problem is characterized as being NP-hard. Some solution properties are also characterized. Therewith, three heuristic algorithms (called SPT-based, LWF-based and WSPT-based heuristic) and a branch-and-bound algorithm are derived. In order to evaluate the effectiveness and efficiency of the proposed algorithms, computational experiments are made with some numerical instances.

***Keywords*:** Production and Delivery, Branch-and-bound Algorithm, DP-Algorithm

1. Introduction

The coordination of logistics activities along various supply chain stages has received a lot research attention in production and operations management. One particular important issue in the research is the schedule coordination of production and delivery in supply chains. In many production systems, finished products are delivered from factory to multiple customer locations, warehouses, or distribution centers by vehicles.

Traditional scheduling models have mostly focused on determination of production schedules to minimize some performance measures, but without much considering any coordination between production and delivery schedules. Those scheduling models have implicitly assumed that there were infinitely many vehicles available for delivering finished jobs (products) to their destinations so as to deliver the finished jobs to customers without delay. Furthermore, such delivery schedules may not affect the associated production schedules. However, in the real world, the number of available vehicles is often limited and they may be available in a variety of different transportation modes. Moreover, a fixed number of vehicles may be available in each mode at its own delivery time and cost. The delivery cost is often composed of costs for driver hiring, gasoline and expressway toll fee. The delivery cost and time are also dependent on transportation modes but not on individual jobs.

This paper considers a coordinated production and delivery (two-stage) scheduling problem where selection of transportation mode is allowed. The focus is on the integration of production scheduling with delivery of finished products from a manufacturer to a warehouse. In the problem, jobs are processed at the manufacturer first and then delivered to the warehouse. The manufacturing system is represented by a single

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machine. And the finished jobs are delivered via various (different modes) vehicles.

The proposed two-stage problem considers the situation where all the jobs are processed sequentially without preemption so that no idle time is inserted between jobs. Moreover, as each job processing is finished, the finished job is delivered to a single warehouse by a vehicle in direct shipment. Each job completion time is the time by which the job is delivered to the warehouse. At the delivery stage, idle time is allowed in the situation where all the vehicles are on delivery duty or when it is more beneficial to insert idle time than immediately place any vehicle on delivery duty.

The objective of the proposed two-stage problem is to find a coordinated production and delivery schedule for minimizing the sum of the associated scheduling cost (represented by the sum of production and delivery times) and the delivery cost.

The proposed problem may be applied to military logistics. Reportedly, the concept of warfighting has changed from conventional arms to small and highly-mobile forces for military strength enforcement. Thus, the associated logistics system needs to support these forces at less total cost, and provide them with sufficient combat units at right time and place, which is, however, to be made subject to limited transportation assets including troops of cargo truck, ships, trains, planes and helicopters. Moreover, such combat units (ex : CBR-APC and Command-APC, Mortar-APC, Infantry-APC, etc) need to be produced at a manufacturer and delivered to the fighting forces as soon as possible via some limited transportation assets.

Few researches on the coordination of production and delivery scheduling have been reported. For example, Hall and Potts (2003) have studied a single machine problem with scheduling measure of minimizing flow time (cost) and delivery cost (in batch delivery), under the assumptions of infinite vehicle capacity, a sufficient number of vehicles and constant delivery cost. Potts (1980) has considered a single machine scheduling problem subject to release dates and subsequent delivery times under the assumptions of a sufficient number of vehicles. Cheng *et al*. (1996), Herrmann and Lee (1993), Yuan (1996), and Chen (1996) have considered a single machine scheduling problem with delivery being made in batch to minimize the sum of batch delivery cost and job-earliness penalty where they have not considered transportation times. Yang (2000) has studied the single machine scheduling problem subject to batch delivery dates.

Moreover, Chang (2004) has considered a single machine scheduling problem with batch delivery being made in an 1:2 network environment where one machine and two warehouses are involved. Li *et al*. (2005) and Park (2002) have considered distribution problems in the situation where the finished jobs (products) were delivered to multiple customer locations. Ahmadi (1992), Sung *et al*. (2006), Sung and Kim (2002) have considered a single machine problem where the finished jobs were delivered by a single vehicle.

Garcia *et al*. (2004) have studied a scheduling problem of ordering and vehicle assignment for production and distribution planning under the assumptions of no-wait immediate delivery to customers, homogeneous vehicles, and multiple suppliers. Yoo *et al*. (2001) have studied a single machine scheduling problem with earliness and tardiness measures subject to due-date window.

Hall *et al*. (2001) have analyzed a variety of problems under single and parallel machine environments where a set of available times for batch delivery were fixed before the schedule was determined.

Lee and Chen (2001) have studied a machine scheduling problem along with explicit transportation consideration. Two types of transportation situations were considered in the problem, and jobs were delivered in batch by transporters with zero return time.

As seen in the literature, no paper has considered machine scheduling problems with transportation mode selection allowed, which provides the authors with the motivation of considering the proposed coordinated single machine scheduling problem with transportation mode selection allowed.

Specifically it is a production-and-delivery scheduling problem with transportation mode selection allowed to minimize the aforementioned total cost. In the problem, the delivery is to be made via direct shipment at the delivery stage.

The organization of this paper is briefed as follows. Section 2 describes the proposed problem. Section 3 characterizes some solution properties. Section 4 derives three heuristic algorithms and a branch-and-bound algorithm, based on the characterized solution properties. In Section 5, the proposed heuristic algorithms and the branch-and-bound algorithm are tested for their performances through numerical experiments. Finally, Section 6 makes some concluding remarks.

2. Problem description

The proposed problem considers n weighted jobs to be processed on a single machine and m transportation modes for job delivery where each transportation mode has k_m vehicles. Each transportation mode requires its own delivery time and cost. After each job processing is finished at the production stage (the single machine), each finished job needs to be delivered to the warehouse by the vehicles. All the jobs and vehicles are available for processing at the production stage and for delivering at the delivery stage at time 0, respectively. All the jobs are processed without preemption so that no idle time need to be inserted between jobs, and all the finished jobs at the production stage are delivered by direct shipment via the vehicles at the delivery stage. The objective of the problem is to make an optimal decision on sequencing of the jobs to be processed at the production machine, and determine their associated delivery departure times and transportation modes at the delivery stage, so as to minimize the sum of job arrival times in sense of opportunity cost and delivery costs at the warehouse. In other words, the objective of solving the proposed problem is to find the optimal production and delivery schedule having minimum total cost.

For the proposed problem, some additional assumptions are considered as follows;

① The size of each job is equivalent to the capacity of each vehicle so that each vehicle is allowed to carry one job at a time.
② Each vehicle can make several round trips between the manufacturer and the warehouse.
③ The job completion time is the time at which the associated job is delivered to the warehouse.
④ The opportunity cost is only considered at the production stage.

In order to derive the mathematical expression of the problem, the following notation is introduced;

n : Number of the jobs.
m : Number of the transportation modes.
k_j : Number of the vehicles in transportation mode *j*, $j=1,2,\cdots,m$.
p_i : Processing time of job i, $i=1,2,\cdots,n$.
w_i : Weight of job i, $i=1,2,\cdots,n$.
t_j : Delivery time of transportation mode j, $j=1,2,\cdots,m$.
DC_j : Delivery cost of transportation mode j, $j=1,2,\cdots,m$.
Q_i : Completion time of job i at production stage, $i=1,2,\cdots,n$.
I_j : Idle time of job i at the delivery stage, $i=1,2,\cdots,n$.
R_i : Delivery departure time of job i, $(R_i=\sum_{v=1}^{i-1}p_v+p_i+I_i)$, $i=1,2,\cdots,n$.
x_{ijh} : a binary integer variable having the value 1 if the job i is delivered by the *h*-th vehicle of transportation mode j, and 0, otherwise.
C_i : Completion time of job i, $(C_i=R_i+\sum_{j=1}^{m}t_j\sum_{h=1}^{k_j}x_{ijh})$, $i=1,2,\cdots,n$.
$TC_w(\sigma)$: Total sum of weighted completion times and delivery cost of all the jobs in the sequence σ.

In the proposed problem, each weighted job completion time is represented by the sum of the associated weighted job processing times at the production stage and the weighted job delivery times at the delivery stage. Thus, the job completion time is expressed as $\sum_{i=1}^{n}w_i\left(R_i+\sum_{j=1}^{m}t_j\sum_{h=1}^{k_j}x_{ijh}\right)$ (which can be interpreted as the associated processing cost by considering the weight factor w_i as the cost(opportunity) of processing job i per unit processing time), and the delivery cost is expressed as $\sum_{i=1}^{n}\sum_{j=1}^{m}DC_j\sum_{h=1}^{k_j}x_{ijh}$.

Now, the total schedule cost is represented by the sum of all the weighted completion times and delivery costs, which is expressed as for a schedule S,

$$TC_w(S)=\sum_{i=1}^{n}w_i\left(R_i+\sum_{j=1}^{m}t_j\sum_{h=1}^{k_j}x_{ijh}\right)+\sum_{i=1}^{n}\sum_{j=1}^{m}DC_j\sum_{h=1}^{k_j}x_{ijh}. \tag{1}$$

3. Analysis

Hall and Potts (2003) have considered a production and delivery (two-stage) problem with only one transportation mode and only one vehicle employed, and then they have proved that their scheduling problem is NP-hard. Their scheduling problem is a special case of the proposed scheduling problem in this paper. This implies that the proposed problem in this paper is also NP-hard,

Proposition 1.

For the proposed problem, if two relations $p_u\le p_v$ and $w_u\ge w_v$ hold, then job u should precede job v in the optimal sequence.

Proof.

The proof can be made by using pairwise interchange argument.

Let *S* be a sequence where J_u precedes J_v and the relations $p_u\le p_v$ and $w_u\ge w_v$ hold, and *S'* be the sequence *S* but with the positions of J_u and J_v interchanged. Denote by $k(i)$ and $k(i)'$ the transportation mode to deliver job i in schedule *S* and *S'*, respectively. Without loss of generality, the following relations hold;

$k(u)=k(v)'$
$k(i)=k(i)'$, $i\in Z$,
$k(v)=k(u)'$,

where *Z* denotes the job set composed of jobs which are sequentially positioned between the jobs *u* and *v*. Denote by Q_i and Q_i' the completion time of job i at the production stage in the schedules *S* and *S'*, respectively. Then, due to the relation $p_u \le p_v$, the following relations hold ;

$$Q_v' - Q_u = p_v - p_u \ge 0,$$
$$Q_i' - Q_i = p_v - p_u \ge 0, \text{ for } i \in Z,$$
$$Q_u' - Q_v = 0,$$

The following relations also hold ;

$$R_u \le R_v',$$
$$R_i \le R_i', \text{ for } i \in Z,$$
$$R_v \le R_u',$$

where R_i and R_i' denote the delivery departure time of job i at the delivery stage in the schedules *S* and *S'*, respectively. Accordingly, it follows that

$$C_u \le C_v',$$
$$C_i \le C_i', \text{ for } i \in Z,$$
$$C_v \le C_u',$$

due to the relations $C_i = R_i + t_{k(i)}$ and $C_i' = R_i' + t_{k(i)'}$, where C_i and C_i' denote the completion time of job i in the schedules *S* and *S'*, respectively.
Moreover, denote by $TC_w(S)$ and $TC_w(S')$ the objective cost values of *S* and *S'*, respectively. Then, it follows that

$$TC_w(S) = \sum_{i=1}^{n} w_i C_i + \sum_{i=1}^{n} DC_{k(i)}, \text{ and}$$
$$TC_w(S') = \sum_{i=1}^{n} w_i C_i' + \sum_{i=1}^{n} DC_{k(i)'}$$

Now, it is going to be shown that $\triangle = TC_w(S') - TC_w(S) \ge 0$ as follows ;

$$\begin{aligned}\triangle &= TC_w(S') - TC_w(S) = \sum_{i=1}^{n} w_i C_i' - \sum_{i=1}^{n} w_i C_i \\ &= \sum_{i \in Z \cup \{u,v\}} w_i C_i' - \sum_{i \in Z \cup \{u,v\}} w_i C_i \\ &= \sum_{i \in Z} w_i (C_i' - C_i) + (w_v C_v' - w_u C_u) \\ &\quad + (w_u C_u' - w_v C_v) \\ &\ge (w_v C_u - w_u C_u) + (w_u C_v - w_v C_v) \\ &= (w_u - w_v)(C_v - C_u) \ge 0,\end{aligned}$$

since the relation $w_u \ge w_v$ holds and J_u precedes J_v in schedule *S*. This implies that $TC_w(S) \le TC_w(S')$. Thus, the proof is completed.

The results of Proposition 1 are useful for curtailing any associated part of the branches in the branch-and-bound scheme by reviewing the precedence relation between any pair of jobs. Moreover, the results of Proposition 1 can be applied for the associated heuristic algorithm derivation as in Section 4.

Proposition 2.

For the proposed problem, if two relations $t_y \le t_z$ and $DC_y \le DC_z$ hold, then it will be more beneficial to use vehicle y than vehicle z.

Proof.

The proof can be made by using pairwise interchange argument.

The total processing and delivery cost of job v with respect to vehicles y and z can be expressed as

$$TC_w(J_v)_y = w_v(Q_{v-1} + I_v + p_v + t_y) + DC_y \text{ and}$$
$$TC_w(J_v)_z = w_v(Q_{v-1} + I_v + p_v + t_z) + DC_z, \text{ respectively.}$$

It follows that

$$TC_w(J_v)_z - TC_w(J_v)_y = w_v(t_z - t_y) + (DC_z - DC_y),$$

which is positive.

This implies that it would be more beneficial to use vehicle y than vehicle z.

Thus, the proof is completed.

The results of Proposition 2 can be applied for the associated heuristic algorithm derivation as in Section 4.

Propositon 3.

If the relation $\min_{\{1 \le i \le n\}} p_i \ge \max_{\{1 \le j \le m\}} \{2t_j\}$ holds, then the optimal job sequence has each job i to be delivered via vehicle mode k which satisfies the relation $\min_{\{1 \le j \le m\}} \{w_i t_j + DC_j\}$.

Proof.

The relation $\min_{\{1 \le i \le n\}} p_i \ge \max_{\{1 \le j \le m\}} \{2t_j\}$ implies that the delivery departure time of k th job (R_k) in an arbitrary job sequence can be made at the time value $\sum_{i=1}^{k} p_i$. Therefore, no idle time need to be considered at the delivery stage.

In order to minimize the total processing and delivery cost, each job i should be delivered via a vehicle which satisfies the relation $\min_{\{1 \le j \le m\}} \{w_i t_j + DC_j\}$, since all the vehicles are available at the departure time, R_k.

Hence, the minimum total processing and delivery cost for the job i is expressed as follows ;

$$\begin{aligned}&\sum_{v=1}^{i} w_v \left(p_v + \sum_{j=1}^{m} t_j \sum_{h=1}^{k_j} x_{vjh} \right) + \sum_{v=1}^{i} \sum_{j=1}^{m} DC_j \sum_{h=1}^{k_j} x_{vjh} \\ &= \sum_{v=1}^{i} w_v p_v + \sum_{v=1}^{i} \min_{\{1 \le j \le m\}} \{w_v t_j + DC_j\}.\end{aligned}$$

Thus, the proof is completed.

The results of Proposition 3 can be applied for the associated heuristic algorithm derivation as in Section 4.

Remark 1.

With any given job processing sequence, the optimal delivery schedule (minimum delivery cost) can be derived by the following DP-algorithm :

If a production sequence is given, then the values of Q_i's can be found. Let $V_{j,h}$ be the completion time of the last job delivery which is made by the *h*-th vehicle of transportation mode *j*.

DP-Algorithm :

Indexing : Index all the jobs in non-decreasing order of Q_i's.

Value function :

$f_i(V_{1,1},\ldots,V_{1,k(1)},\ldots,V_{m,1},\ldots,V_{m,k(m)})=$ total minimum cost of a partial schedule with job i through n.

Optimal solution value : $f_1(0,0,\ldots,0)$.

Recursive relation :

$$f_i(V_{1,1},\ldots,V_{1,k(1)},\ldots,V_{m,1},\ldots,V_{m,k(m)})=$$
$$\min\begin{cases}\min\limits_{\{j,h|Q_i\le V_{j,h}\}}\begin{Bmatrix}f_{i+1}(V_{1,1},\ldots,V_{j,h}+2t_j,\ldots,V_{m,k(m)})\\ +w_i(V_{j,h}+t_j)+DC_j\end{Bmatrix}\\ \min\limits_{\{j,h|Q_i> V_{j,h}\}}\begin{Bmatrix}f_{i+1}(V_{1,1},\ldots,V_{j,h-1},Q_i+2t_j,\ldots,V_{m,k(m)})\\ +w_i(Q_i+t_j)+DC_j\end{Bmatrix}\end{cases}$$

In each recursive relation, the first term represents the situation where the associated *h*-th vehicle of transportation mode *j* is busy, so that any finished job should wait until reaching the time point $V_{j,h}$, as depicted in <Figure 1-(a)>. The second term represents the situation where the associated *h*-th vehicle of transportation mode *j* is available, so that any finished job is immediately delivered after the processing is finished, as depicted in <Figure 1-(b)>.

The complexity of the DP-Algorithm is of order $O(nX^T)$, where $T=\sum_{j=1}^{m} t_j$ and $X=\sum_{i=1}^{n} p_i+2n\times t_{\max}$.

4. Algorithms

4.1 Three Heuristic Algorithms

This section derives three heuristic algorithms, including SPT- based heuristic, LWF-based heuristic and WSPT-based heuristic, for the proposed problem.

4.1.1 SPT-based Heuristic

The SPT-based heuristic algorithm selects a job from among the current unscheduled job set σ' and arranges it to the lth position at each DP recursion stage $i=1,2,\cdots,n$. The job selection is made via the SPT rule which is applied to the current unscheduled job set σ'.

Step 0 : Index all the jobs in non-decreasing $p_{[i]}$ order such that $p_{[1]}\le p_{[2]}\le\cdots\le p_{[n]}$ (SPT order).

Step 1 : In case of ties, select a job which has the maximum weight among them (Proposition 1)

Step 2 : Find the objective cost value by DP-Algorithm implemented with the SPT-ordered sequence (Remark 1).

4.1.2 LWF-based Heuristic

The LWF-based heuristic algorithm selects a job from among the current unscheduled job set σ' and arranges it to the lth position at each DP recursion stage $i=1,2,\cdots,n$. The job selection is made via the LWF(largest weight first) rule which is applied to the current unscheduled job set σ'.

Step 0 : Index all the jobs in non-increasing $w_{[i]}$ order such that $w_{[1]}\le w_{[2]}\le\cdots\le w_{[n]}$ (LWF order).

Step 1 : In case of ties, select a job which has the minimum processing time among them (Proposition 1).

Step 2 : Find the objective cost value by DP-Algorithm implemented with the SPT-ordered sequence (Remark 1).

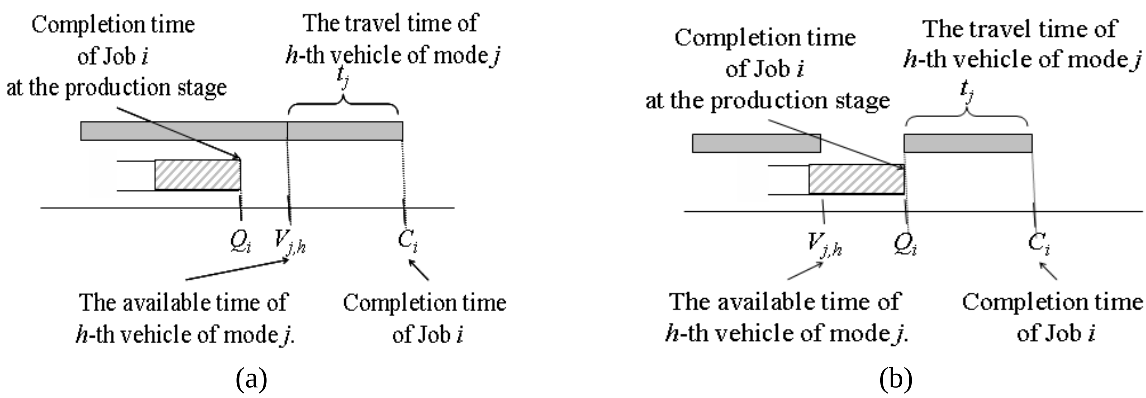

Figure 1. Two possible cases in DP algorithm.

4.1.3 WSPT-based Heuristic

The WSPT-based heuristic algorithm selects a job from among the current unscheduled job set σ' and arranges it to the lth position at each DP recursion stage $i=1,2,\cdots,n$. The job selection is made via the WSPT rule which is applied to the current unscheduled job set σ'.

Step 0 : Index all the jobs in non-decreasing $\frac{p_{[i]}}{w_{[i]}}$ order such that

$$\frac{p_{[1]}}{w_{[1]}} \le \frac{p_{[2]}}{w_{[2]}} \le \cdots \le \frac{p_{[n]}}{w_{[n]}} \text{ (WSPT order).}$$

Step 1 : In case of ties, select a job with a job which has the maximum weight among them (Proposition 1).

Step 2 : Find the objective cost value by DP-Algorithm implemented with the WSPT-ordered sequence (Remark 1).

4.2 Branch and Bound Algorithm

4.2.1 Branching Rule

A forward branching tree is to be considered, where each node corresponds to a subproblem. The subproblem is defined as a partial sequence representing a subset of jobs that are to be placed in front of the whole sequence. In each branching stage, a subproblem (i.e. node) is selected and partitioned into one or more subproblems that are defined by attaching one more unscheduled job to the end of the partial sequence associated with the subproblem being partitioned. To select a node in branching, a depth first rule is adapted in the algorithm. In the depth first rule, a node with most jobs in the corresponding partial sequence is selected for branching.

In case of ties, the branching rule selects a node with the minimum lower bound among them. In the branching, a node list will be kept, which is ranked by the depth levels and lower bounds for active nodes, and the first node in the list will be selected for branching.

4.2.2 Derivation of Lower Bound

This section is to derive a lower bound. For the convenience, some additional notations are introduced as follows :

σ denotes the partial sequence representing a subset of jobs, σ' denotes the partial sequence of jobs that are not contained in σ and n_σ denotes the number of jobs contained in σ, $TC_w(\sigma)$ denotes the total schedule cost of all the jobs which are contained in σ, and $TC_w(\sigma')$ denotes the total schedule cost of all the jobs which are contained in σ'.

For a full sequence S that starts with a given partial sequence σ and ends with σ', it's total schedule cost is expressed as $TC_w(S) = TC_w(\sigma) + TC_w(\sigma')$,

where $TC_w(\sigma)$ is computed by the DP-Algorithm which is suggested as in Remark 1, and the delivery departure time of job i (R_i) is easily obtained by the DP-Algorithm. Let all the jobs in the job set σ' be ordered by using the WSPT rule, and all the ordered jobs are delivered via the vehicles which satisfy the relation $\min_{\{1 \le m \le j\}}\{w_i t_j + DC_j\}$ for each job $i \in \sigma'$.

Then, the following relation is derived ;

$$\begin{aligned} TC_w(\sigma') &= \sum_{i \in \sigma'} w_i \left(C_i + \sum_{j=1}^{m} t_i \sum_{h=1}^{k_j} x_{ijh} \right) + \sum_{i \in \sigma'} \sum_{j=1}^{m} DC_j \sum_{h=1}^{k_j} x_{ijh}, \\ &\ge \sum_{i \in \sigma'} w_i C_i + \sum_{i \in \sigma'} \min_{\{1 \le j \le m\}}\{w_i t_j + DC_j\} \end{aligned} \quad (2)$$

where C_i is the completion time of the sequenced jobs at the production stage for each job $i \in \sigma'$.

Thus, the lower bound for the whole sequence that consists of the given partial sequence σ and the remaining job sequence σ' found by the WSPT order is derived as

$$\begin{aligned} LB &= TC_w(\sigma) + TC_w(\sigma') \\ &= TC_w(\sigma) + \sum_{i \in \sigma'} w_i C_i + \sum_{i \in \sigma'} \min_{1 \le j \le m}\{w_i t_j + DC_j\} \end{aligned} \quad (3)$$

4.2.3 Initial Upper Bound

In order to save the computational time in the suggested branch- and-bound algorithm, branching will start with an upper bound corresponding to a feasible solution value which is found prior to the implementation of the branch-and-bound algorithm. In this connection, a WSPT-based heuristic mechanism can be considered (Section 4.1.3). Moreover, the NP-hardness result provides the authors with the motivation of developing an effective heuristic algorithm. First of all, a job sequence is found by using the WSPT order, and then the upper bound value is found by DP-Algorithm implemented with the WSPT-ordered sequence (*Remark 1*). These algorithms may provide a good quality solution at little computational time.

4.2.4 Fathoming Rules

The branch-and-bound algorithm essentially solves the problem in exponential time in the worst case. In fact, up to $1+n+n(n-1)+\cdots+n!$ nodes may be generated in the entire tree, where up to $(n-k)$ nodes may be additionally generated from every node at level k, for k=0,1,...,n. However, the efficiency of the branch and bound algorithm may be improved by taking advantage of some dominance solution properties, the conditions of which, if satisfied, allow a reduction in the number of partial sequences that need to be examined in the tree.

This paper considers two fathoming rules. In the first fathoming rule, if the lower bound at any node is larger than the initial upper bound value, then the associated node will be fathomed, called Fathom 1. The second fathoming rule is due to the result of Proposition 1. In the second fathoming rule, if two jobs J_u and J_v are the candidates for any two consecutive job positions, and they satisfy the conditions $p_u \leq p_v$ and $w_u \geq w_v$, then the job u precedes the job v in an optimal sequence, called Fathom 2.

5. Computational Results

This section describes the numerical experiments that are made to evaluate the suggested branch-and-bound algorithm and to show the effectiveness and efficiency of each of the WSPT-based heuristic, the LWF-based heuristic and SPT-based heuristic. All the computational experiments are tested on Pentium Ⅳ processor with 1.7 GHz and 512 MB memory.

The test problems are generated with the number of jobs ranging from 8 to 20. In all the problems, the weight factors (w_i) are generated from the discrete uniform distribution over the range [10, 30] and the job processing times (p_i) are generated from the discrete uniform distribution over the range [10, 30]. And the delivery time factors of vehicles (t_j) are generated from the discrete uniform distribution over the range [10, 30] and the delivery cost factors of vehicles (DC_j) are generated from the discrete uniform distribution over the range [1000, 3000]. Two transportation modes are used, and for each transportation mode, seven identical vehicles are employed. For each problem instances (number of jobs), 20 problems are generated to get some associated test statistics.

The results of the experiments are presented in <Table 1>, which includes the average CPU times, the average number of generated nodes and the number of the problems solved optimally within 600 seconds(commonly considered in the literature). Solution gap is included as well, where the gap (%) is represented by $\left(\dfrac{TC_{heu} - TC_{opt}}{TC_{opt}}\right) \times 100$, and TC_{heu} denotes the feasible solution value generated by each of the SPT-based heuristic and the LWF-based heuristic and the WSPT- based heuristic, and TC_{opt} denotes the optimal solution value generated by the branch-and-bound algorithm within 600 seconds.

From <Table 1>, it is noted that the average gap of the WSPT-based heuristic shows that the WSPT-based heuristic becomes the most effective and efficiency heuristic as the number of jobs increases. However, the average gaps of the SPT-based heuristic and LWF-based heuristic increase as the number of jobs increases. Thereupon, the WSPT-based heuristic may be considered as the most effective heuristic among the three heuristic algorithms.

To review any effect of the fathoming rule in Section 4.2.4, <Table 2> is presented. The table shows the performance of the branch-and-bound algorithm (for <Table 1>) without Fathom 2 employed. By comparing <Table 1> and <Table 2>, it can be concluded that Fathom 2 fairly contributes to the efficiency of the branch-and-bound algorithm.

Table 2. Result of the branch-and-bound algorithm without Fathom 2 employed

n	Branch-and-bound-algorithm		
	Aver. Time(s)	Aver. Non	Npl
8	0.06	12.20	20
10	0.11	40.35	20
12	1.73	428.95	20
14	2.26	704.75	20
16	46.81	8011.11	18
18	17.05	7071.93	15
20	44.08	9974.86	15

Table 1. Summary of the computational results

n	SPT-based heuristic		LWF-based heuristic		WSPT-based heuristic		Branch-and-bound-algorithm		
	Max Gap	Aver. Gap	Max Gap	Aver. Gap	Max Gap	Aver. Gap	Aver. Time(s)	Aver. Non	Npl
8	29.069	16.833	7.595	3.155	0.299	0.0273	0.0396	12.2	20
10	32.636	23.856	5.189	3.069	0.271	0.0241	0.116	38.1	20
12	37.137	42.197	9.372	4.167	0.335	0.0607	0.991	385.45	20
14	47.412	25.773	8.301	3.935	0.279	0.0567	2.140	637	20
16	46.614	30.307	7.753	4.446	0.302	0.0593	30.520	7764.833	18
18	36.311	26.539	7.196	4.501	0.144	0.0343	72.451	31267.32	19
20	46.947	34.021	7.765	4.463	0.140	0.0405	67.076	23869.56	18

Note : Npl : number of the problems solved optimally by B&B within 600 seconds (out of 20 problems)
Non : number of the nodes in B&B, Gap : %

Table 3. Summary of the computational results for large-sized problems

n	SPT-based heuristic			LWF-based heuristic			WSPT-based heuristic		
	Max Gap	Aver. Gap	Aver. Time(s)	Max Gap	Aver. Gap	Aver. Time(s)	Max Gap	Aver. Gap	Aver. Time(s)
40	58.01	40.73	0.01	9.23	6.89	0.01	1.73	0.62	0.01
80	53.76	45.23	0.01	10.22	7.91	0.01	1.01	0.42	0.01
120	56.87	49.21	0.01	10.20	8.15	0.01	1.29	0.41	0.01
160	57.65	51.11	0.01	9.31	8.51	0.01	0.77	0.39	0.01
200	59.75	50.44	0.01	9.47	8.54	0.01	0.74	0.33	0.01

Therewith, the computational tests are carried out for large-sized problems of between 40 and 200 jobs. The test results are presented as in <Table 3>. All the cost parameters are randomly generated as in <Table 1> and 20 problems are generated for the performance tests. The gap (%) is represented by $\left(\frac{UB_{heu}-LB}{LB}\right)\times 100$, where UB_{heu} denotes the initial upper bound value generated by each of the WSPT-based heuristic and LWR based heuristic, and LB denotes the lower bound value generated by the scheme in Section 5.2.

It is observed from <Table 3> that the WSPT-based heuristic finds good solutions having the average gap value of about 0.44% with very small time elapsed. As the number of jobs increases, the total completion time is more sensitive to the total cost than the delivery cost. Therefore, the performance of the WSPT-based heuristic gets better. However, as the number of jobs increases, the performances of the SPT-based heuristic and the LWF-based heuristic get worse.

6. Conclusion

This paper considers a production-and-delivery scheduling problem with transportation mode selection allowed. The consideration of multiple transportation modes appears to be more-like realistic in the supply chain environment.

In the problem analysis, the proposed problem is proved as NP-hard in the strong sense. Thereupon, three heuristic algorithms and a branch-and-bound algorithm are derived, for which some solution properties are characterized. In order to evaluate the effectiveness and efficiency of the proposed algorithms, computational experiments are made with some numerical instances. The associated numerical experiments show that the WSPT-based heuristic algorithm is more effective and efficient than any of the SPT-based and the LWF-based heuristic algorithms. Moreover, the results indicate that the branch-and-bound algorithm is able to solve optimally the problem instances of up to 20 jobs in a reasonable time limit (within 600 seconds).

For further study, the immediate extension of the proposed model to two-machine flow/ parallel shops and also to different-capacity transportation modes would be interesting.

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