

A study on Indirect Adaptive Decentralized Learning Control of the Vertical Multiple Dynamic System

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The learning control develops controllers that learn to improve their performance at executing a given task, based on experience performing this specific task. In a previous work, the authors presented an iterative precision of linear decentralized learning control based on p-integrated learning method for the vertical dynamic multiple systems. This paper develops an indirect decentralized learning control based on adaptive control method. The original motivation of the learning control field was learning in robots doing repetitive tasks such as an assembly line works. This paper starts with decentralized discrete time systems, and progresses to the robot application, modeling the robot as a time varying linear system in the neighborhood of the nominal trajectory, and using the usual robot controllers that are decentralized, treating each link as if it is independent of any coupling with other links. Some techniques will show up in the numerical simulation for vertical dynamic robot. The methods of learning system are shown for the iterative precision of each link.

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1. Introduction

In general industrial applications, the automatic equipments are manufactured for the iterative simple periodic works following the pre-designed product schedule. However, the works have wrong trajectories out of the desired one because of the accumulative errors in the iterative periodic motion. This causes the problem at the industrial fields to maintain the good quality with precision. In order to overcome these problems, calibration can be used for adjustment of the initial and terminal points with measurement sensor installation in hardware. But, these methods decrease the productivity because of the investment for the non manufacture parts, and induce the shortage from product claim in which error can be found in the real time with specific range. To solve the problems, the learning control is recommended as one of the software methods instead of hardware methods to correct or exchange the mechanism¹⁻³.

Most of control algorithm needs the correct modeling for the system. But, the system identification methods can be used as a system modeling by data acquisition of the input and output signals because of the difficulties at the mathematical description⁴⁻⁷.

Therefore, indirect adaptive type decentralized learning control can be applied without system information in order to improve the iterative precision for the vertical multiple dynamic system. The vertical multiple dynamic system has nonlinear terms r

elated to gravitational force and interconnected terms in centrifugal force. The learning methods and convergence to zero tracking error will be studied to minimize the iterative error at the periodic works.

2. Systems

2.1 Vertical Multiple Dynamic Systems

In this paper, the control algorithms can be designed and applied to the systems for the rehabilitation including artificial legs and arms⁹⁻¹².

The system is included with four linkages moving on the vertical plane and two actuators as shown figure 1. And, the end point(p) of the fifth bar can be moved following the desired trajectory.

This type of mechanism is mostly used in the industrial vertical type robot because the payload can be reduced with motors installed at the base(o). This kind of structure can be designed and manufactured by decoupling the interconnected linkages in the coupled systems. So, we can solve the problem related to interconnected subsystems to control each system following its own output desired trajectory¹³.

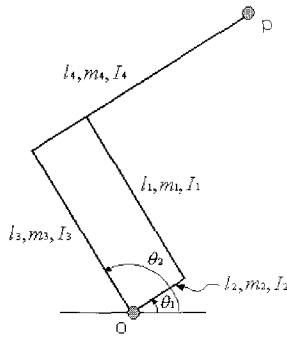


Fig. 1 Schematic diagram of the mechanism

2.2 Equation of Motion

The equations of motion are derived by Euler-Lagrange Method as follows.

$$\begin{aligned}
 d_{11}\ddot{\theta}_1 + d_{12}\ddot{\theta}_2 + h_{122}\dot{\theta}_2^2 + g_1 &= \tau_1 \\
 d_{21}\ddot{\theta}_1 + d_{22}\ddot{\theta}_2 + h_{211}\dot{\theta}_1^2 + g_2 &= \tau_2
 \end{aligned}
 \tag{1}$$

where, $d_{11} \sim d_{22}$ are mass and inertia terms, h_{122}, h_{211} are centrifugal force terms, g_1, g_2 are gravitational terms.

$$\begin{aligned}
 d_{11} &= I_2 + I_4 + (m_1 + m_2)l_2^2 + m_4l_4^2 \\
 d_{12} = d_{21} &= (m_1l_1l_2 + m_4l_3l_4)\cos(\theta_1 - \theta_2) \\
 d_{22} &= I_1 + I_3 + m_1l_1^2 + (m_3 + m_4)l_3^2
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 h_{122} &= (m_1l_1l_2 + m_4l_3l_4)\sin(\theta_1 - \theta_2) \\
 h_{211} &= -(m_1l_1l_2 + m_4l_3l_4)\sin(\theta_1 - \theta_2) \\
 g_1 &= g\{(m_1 + m_2)l_2 + m_4l_4\}\cos\theta_1 \\
 g_2 &= g\{m_1l_1 + (m_3 + m_4)l_3\}\cos\theta_2
 \end{aligned}
 \tag{3}$$

The physical properties of the above equations for the equations (2) and (3) are described in table 1. Where $g = 980[cm / s^2]$.

Table 1 Material properties used for calculation

Property	Unit	Symbol	Value
length	cm	l_1, l_3	25.2
		l_2	7.6
		l_4	30.7
		m_1, m_3	0.05
mass	g	m_2	0.5
		m_4	0.1
inertia moment	kg-cm ²	I_1, I_3	1.0600e-2
		I_2	2.6467e-3
		I_4	3.1500e-2

3. Indirect Decentralized Learning Control

A system with interconnected subsystems is described as follows.

$$\begin{aligned}
 x_i(k+1) &= \sum_{j=1}^s A_{ij}(k)x_j(k) + B_i(k)u_i(k) + w_i(k) \\
 y_i(k+1) &= C_i(k+1)x_i(k+1) \\
 k &= 0, 1, 2, \dots, p-1
 \end{aligned}
 \tag{4}$$

where, i and j are subsystem numbers, s is the total number of subsystems, p is the number of sampling number. $A_y(k)$ is subsystem matrix, $B_i(k)$ is input matrix, $C_i(k)$ is output matrix, $w_i(k)$'s are disturbances and repeated external forcing terms.

The difference operator of input and output is defined as follows.

$$\begin{aligned}
 \delta_r \underline{u}_i &= \underline{u}_i^r - \underline{u}_i^{r-1} \\
 \delta_r \underline{y}_i &= \underline{y}_i^r - \underline{y}_i^{r-1}
 \end{aligned}
 \tag{5}$$

Generally, in equations (4) and (5), the same initial value and the disturbance with external force happen in a periodical works, their terms can be cancelled in repetition domain as follows.

$$\delta_r \underline{y}_i = P_i \delta_r \underline{u}_i
 \tag{6}$$

where, δ_r is difference operator between r and $r-1$. P_i is

$$\begin{bmatrix}
 C(1)B(0) & 0 & \dots & 0 \\
 C(2)A(1)B(0) & C(2)B(1) & \dots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 C(p)(\prod_{k=1}^{p-1} A(k))B(0) & C(p)(\prod_{k=1}^{p-1} A(k))B(1) & \dots & C(p)B(p-1)
 \end{bmatrix}$$

The equation (6) can be rearranged as follows.

$$\delta_r \underline{y}_i = P_{ii} \delta_r \underline{u}_i + \sum_{\substack{j=1 \\ j \neq i}}^s P_{ij} \delta_r \underline{u}_j
 \tag{7}$$

where, P_{ii} and P_{ij} are the lower triangular matrices for subsystem i . The first term shows the response to the input for its own subsystem and the second term shows the response to the input for the other subsystem. This is the basic mathematical form in the decentralized learning control systems. The mathematical form for two subsystems in three time steps is shown in equation (7).

$$\begin{bmatrix}
 \delta_r y_1(1) \\
 \delta_r y_1(2) \\
 \delta_r y_1(3)
 \end{bmatrix} =
 \begin{bmatrix}
 C_1 B_1 & 0 & 0 \\
 C_1 A_1 B_1 & C_1 B_1 & 0 \\
 C_1 (A_1^2 + A_2 A_{21}) B_1 & C_1 A_1 B_1 & C_1 B_1
 \end{bmatrix}
 \begin{bmatrix}
 \delta_r u_1(0) \\
 \delta_r u_1(1) \\
 \delta_r u_1(2)
 \end{bmatrix}
 +
 \begin{bmatrix}
 0 & 0 & 0 \\
 C_1 A_{12} B_2 & 0 & 0 \\
 C_1 (A_1 A_{12} + A_2 A_{22}) B_2 & C_1 A_{12} B_2 & 0
 \end{bmatrix}
 \begin{bmatrix}
 \delta_r u_2(0) \\
 \delta_r u_2(1) \\
 \delta_r u_2(2)
 \end{bmatrix}
 \tag{8}$$

For causality, P_{ii} and P_{ij} are lower triangular matrices, P_{ij} has zero diagonal elements. We can consider several types of decentralized control and apply the control for one subsystem turned off the controller for the others. This is called the individual learning method. The equation (7) can be shown as follow, if the method is applied to the i th subsystem at the r th repetition.

$$\delta_r \underline{y}_i = P_{ii} \delta_r \underline{u}_i
 \tag{9}$$

The system matrix can be estimated from the input and output signals at one repetition for learning. If the estimation of system matrix is \hat{P}_{ii} , decentralized controller can be designed as follow.

$$\delta_r \underline{u}_i = \hat{P}_{i,r}^{-1} (\underline{y}_i^* - \underline{y}_{i,r}) \quad (10)$$

The estimation method is recursive least squares algorithm⁴. Let $\hat{P}_{l,r}$ represent the column vector which is the transpose of the l th row of $\hat{P}_{i,r}$, but with the zero elements to the right of the block diagonal deleted from the column vector. Let $\delta_r^i \underline{u}$ represent the quantity $\delta_r \underline{u}$ with the elements deleted that are multiplied by these zero elements in the product $P_{ii} \delta_r \underline{u}_i$. And let $\delta_r^i \underline{y}$ represent the l th row of $\delta_r \underline{y}$. Then the recursive least squares update is

$$\hat{P}_{l,r} = \hat{P}_{l,r-1} + M_{l,r-2} \delta_r^i \underline{u} \left[\frac{\delta_r^i \underline{y} - (\delta_r^i \underline{u})^T \hat{P}_{l,r-1}}{1 + (\delta_r^i \underline{u})^T M_{l,r-2} \delta_r^i \underline{u}} \right] \quad (11)$$

where,

$$M_{l,r-1} = M_{l,r-2} - \frac{M_{l,r-2} \delta_r^i \underline{u} (\delta_r^i \underline{u})^T M_{l,r-2}}{\alpha_r(k) + (\delta_r^i \underline{u})^T M_{l,r-2} \delta_r^i \underline{u}}; r \geq 2$$

$$\alpha_r(k+1) = \alpha_0 \alpha_r(k) + (1 - \alpha_0) \quad (12)$$

$$k = 0, 1, 2, \dots, p-1$$

The initial value $M_{l,0}$ is chosen as the identity matrix of the same dimension as the $\hat{P}_{l,r}$ corresponding \hat{P}_r . Note that this matrix need only be updated when the dimension of $\hat{P}_{l,r}$ increases when l is increased, and the same $M_{l,r}$ can be used for all rows corresponding to the same time step in the multiple output case. The weight factors α_r, α_0 are 0.95 and 0.99.

From equations (10) and (11), the design conditions can be estimated in the input and output signals without system information. And zero tracking error convergence can be guaranteed from equation (11).

4. Simulation

4.1 Desired trajectories

In figure 1, working boundary of θ_1 and θ_2 is needed to make a specific trajectories for an experimental test bed or a realistic mechanical realization. Home positions are $\theta_1 = 0^\circ, \theta_2 = 90^\circ$ and maximum ranges are $\theta_1 = -30^\circ \sim 30^\circ, \theta_2 = 60^\circ \sim 120^\circ$. In figure 2, maximum working range of end point(p) is shown as slash marks, a circle is desired trajectory with radius 50mm, and x and y axis are the relative distance from origin(o).

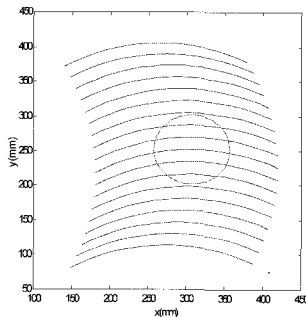


Fig. 2 Work-space and desired trajectory

In figure 3, (a) and (b) are desired trajectories for θ_1 and θ_2 in time domain.

4.2 PD controller design and state equations

The equation of motion is equation (1), and relative equation with torque and decentralization controller u_1, u_2 at each motor is

equation (13).

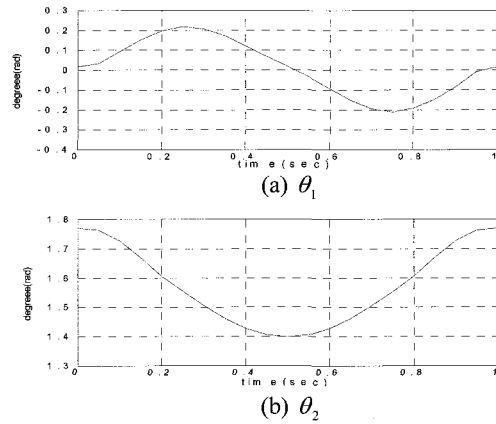


Fig. 3 Desired trajectory in Joint space

$$\tau_1 = K_{p1}(\theta_1^* - \theta_1) + K_{v1}(\dot{\theta}_1^* - \dot{\theta}_1) + u_1$$

$$\tau_2 = K_{p2}(\theta_2^* - \theta_2) + K_{v2}(\dot{\theta}_2^* - \dot{\theta}_2) + u_2 \quad (13)$$

where, $K_{p1}, K_{v1}, K_{p2}, K_{v2}$ are PD control gains with 3600, 100, 1800, 126.25, and poles are

$$p_i = \begin{bmatrix} -1.2037e+001 \pm 2.6260e+001i \\ -9.9960e+000 \pm 1.3767e+001i \end{bmatrix}$$

located in the left half plane, which estimated the system stable. The position error histories with PD control for each subsystem are shown in figure 4.

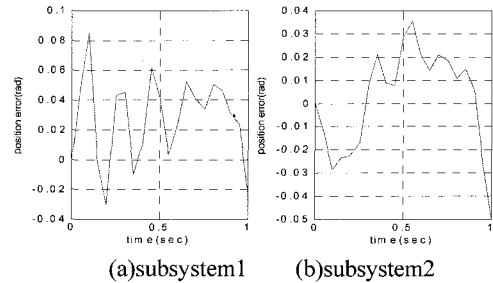


Fig. 4 Position error with PD control

For linearization of nonlinear system with each angle, let $\varepsilon\theta_1 = \theta_1 - \theta_1^*, \varepsilon\theta_2 = \theta_2 - \theta_2^*, \varepsilon\theta \cong 0$. And, plugging the equation (13) into the equation (1),

$$d_{11} \varepsilon \ddot{\theta}_1 + d_{12} \varepsilon \ddot{\theta}_2 = -K_{v1} \varepsilon \dot{\theta}_1 - K_{p1} \varepsilon \theta_1 + u_1 + w_1$$

$$d_{22} \varepsilon \ddot{\theta}_2 + d_{21} \varepsilon \ddot{\theta}_1 = -K_{v2} \varepsilon \dot{\theta}_2 - K_{p2} \varepsilon \theta_2 + u_2 + w_2 \quad (14)$$

where,

$$w_1 = -\{d_{11} \ddot{\theta}_1^* + d_{12} (\theta_1^*) \ddot{\theta}_2^* + h_{122} (\theta_1^*) \dot{\theta}_2^* \dot{\theta}_2^* + g_1 (\theta_1^*)\}$$

$$w_2 = -\{d_{21} (\theta_1^*) \ddot{\theta}_1^* + d_{22} \ddot{\theta}_2^* + h_{211} (\theta_1^*) \dot{\theta}_1^* \dot{\theta}_1^* + g_2 (\theta_2^*)\} \quad (15)$$

If the above equation is expressed by state matrix with $x = [\varepsilon\theta_1 \quad \varepsilon\dot{\theta}_1 \quad \varepsilon\theta_2 \quad \varepsilon\dot{\theta}_2]^T$, the state equation is⁸

$$\dot{x}(t) = A_c x(t) + B_c u(t) + w_c(t)$$

$$y(t) = Cx(t) \quad (16)$$

The discretized state matrices A, B are

$$\begin{aligned}
 A &= \begin{bmatrix} 3.7780e-001 & 2.0620e-002 & 2.7971e-002 & 2.5895e-003 \\ -1.6986e+001 & -9.4034e-002 & 4.3577e-001 & 5.8535e-002 \\ 5.5941e-002 & 2.8093e-003 & 7.4787e-001 & 2.7659e-002 \\ 8.7155e-001 & 8.0151e-002 & -8.0293e+000 & 1.8470e-001 \end{bmatrix} \\
 B &= \begin{bmatrix} 1.7283e-004 & -1.5539e-005 \\ 4.7183e-003 & -2.4210e-004 \\ -1.5539e-005 & 1.4007e-004 \\ -2.4210e-004 & 4.4607e-003 \end{bmatrix}
 \end{aligned}
 \tag{17}$$

4.3 Experimental results

In this section, individual learning method is applied to the experimental test. The weighting factor can be applied in a repetition variant or fixed type. The results are shown in position error in time domain and the response in repetition domain. The sampling rate is 0.05 second, and the number of repetition for each subsystem is 50. The error histories are shown in the summation of absolute position error in each repetition. Summation of Absolute Position Error(SAPE) is defined as the summation of position error for all time steps in each repetition.

$$SAPE = \sum_{k=1}^p |e_i^r(k)|$$

where,

i = subsystem number
 r = repetition number

The results for two conditions are as follows.

Condition 1: $\alpha(0) = 0.95$, $\alpha_0 = 0.8$ for the weighting factor.

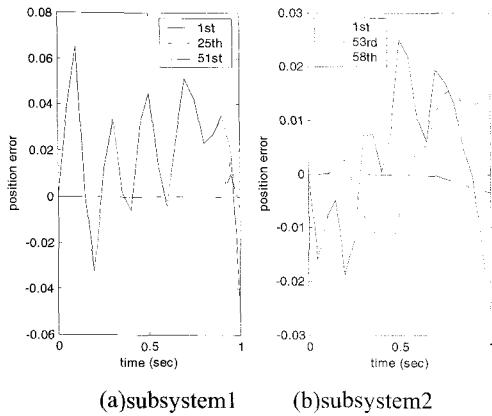


Fig. 5 Position error histories

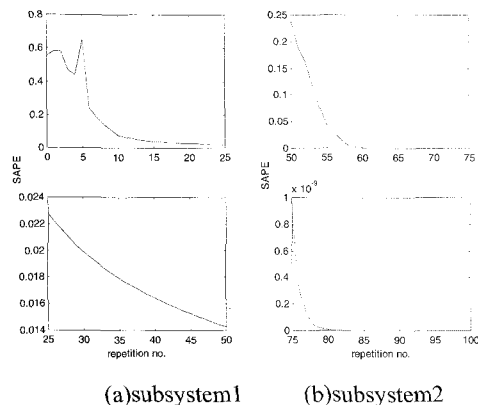


Fig. 6 Summation of absolute position error

Condition 2 : $\alpha_r(k) = 1.0$ for the fixed weighting factor.

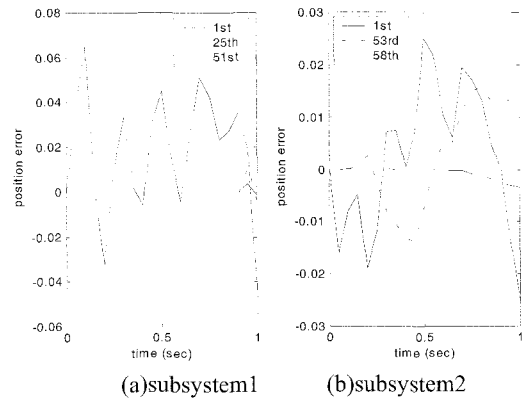


Fig. 7 Position error histories

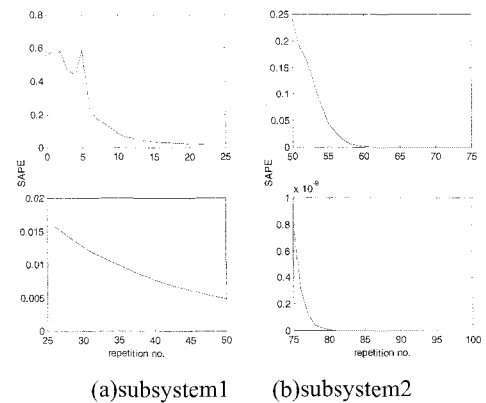


Fig. 8 Summation of absolute position error

Figure 5 and 7 show the position error histories for condition 1 and condition 2, and figure 6 and 8 show the magnitude of SAPE in repetition domain. The result of two conditions show similarly the convergence to zero for the position error histories and SAPE in the repetition domain. The convergence ratio of subsystem 1 is better than that of subsystem 2. This is caused from the mechanical properties of interconnected linkages in figure 3. And the effect of weighting factor change is so little in the controller for the system. So, the fixed value of weighting factor is more convenient and more efficient because of the computational simplicity in equation (12).

5. Conclusions

In this paper, the indirect adaptive type decentralized learning controller without system information is applied to the vertical multiple dynamic systems in numerical simulation. As the parameter estimation, recursive least square method is used and the variant value and fixed one are applied as a weighting factor. However, two methods are shown to improve the system iterative precision in the vertical multiple dynamic systems.

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