Assessing the effect of stylus tip radius on surface roughness measurement by accumulation spectral analysis

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A spectral analysis and numerical simulation are employed to assess the effects of the stylus tip radius on measuring surface profiles. Original profiles with fractal spectral densities are generated and then are numerically traced with circular tipped stylus. Instead of their spectral densities, the accumulative power spectrums of traced profiles are analyzed. It is shown that the minimum wavelength of traced profile relates directly to the radius r of the stylus tip and the root-mean-square (rms) roughness σ_0 of original profile. From this accumulation spectral analysis, a formula is developed to estimate the minimum wavelength of traced profile. By using the concept of the minimum wavelength, an appropriate stylus tip radius can be chosen for the given rms roughness σ_0 of the profile.

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1. Introduction

The stylus profiler has been commonly used to measure surface roughness. However, owing to finite stylus tip size, a measured profile from the stylus profiler often differs from an original one. Thus, the effect of tip size of the stylus has been an important issue on measuring surface roughness. The effect of the stylus tip size on measuring surface roughness has been investigated by several researchers [1~6]. The smoothing effect of the stylus was investigated numerically by Radhakrishnan[1] who used digitized profile traces to compute the smoothing effect of the stylus. An investigation of the nonlinear filtering effect of the finite stylus tip radius was made by McCool[2] who developed a simulation model for assessing the distortion magnitude of the roughness parameters. Relation between the stylus tip radius and the root-mean-square (rms) roughness was investigated numerically by Mendeleyev[3] who developed a mathematical model of a random profile tracing.

Another approach was taken by Church and Takacs4 and Wu[5, 6]. Church and Takacs[4] investigated the spectral densities of measured surface profiles analytically. And, Wu[5, 6] used spectral analysis to investigate the effect of stylus tip curvature on measuring random profiles and fractal profiles. These approaches are based on the following idea: if the radius of the stylus tip is too large, the stylus will smooth the surface profile by filtering out high frequency components during measuring and there will exist a critical frequency above which the spectral density is filtered out. Thus, above the critical frequency, the spectral density is not correct. This critical frequency is the inverse of a critical sampling interval, so called, 'a minimum useful sampling interval' to be selected for surface profile analysis[6].

The object of the present study is to investigate the effect of the tip radius of the stylus on measuring surface roughness through spectral analysis. For this purpose, we analyze the accumulative power spectrums of measured surface profiles instead of their spectral density functions. The information contained in an accumulative power spectrum is basically identical to the information presented in a spectral density function, it is however, presented in a different form. The accumulative power spectrum plots the spatial frequency integral of power spectral density against spatial frequency, which has been employed as a useful guide to determine the critical sampling interval in surface measurement[7~10]. The critical sampling interval is determined by searching the smallest frequency which includes 95% of the power[10]. And this smallest frequency can be easily found from the accumulative spectrum graph. In this paper, the critical sampling interval calculated from this accumulative power spectrum is defined as 'minimum wavelength', which represents the shortest measurable wavelength limit with finite stylus tip radius.

In chapters 2 and 3, the generation of original surface profiles and the mathematical model of a profile tracing are explained. After that, numerical simulation is performed to analyze the trend of the accumulative power spectrums of traced profiles according to different stylus tip sizes. Finally, a formula for estimating the minimum wavelength to be measured with the known stylus tip radius is developed.

2. Surface profile generation

In this paper, real machined surface profiles are used for surface profile generation. Eight real machined surfaces with different

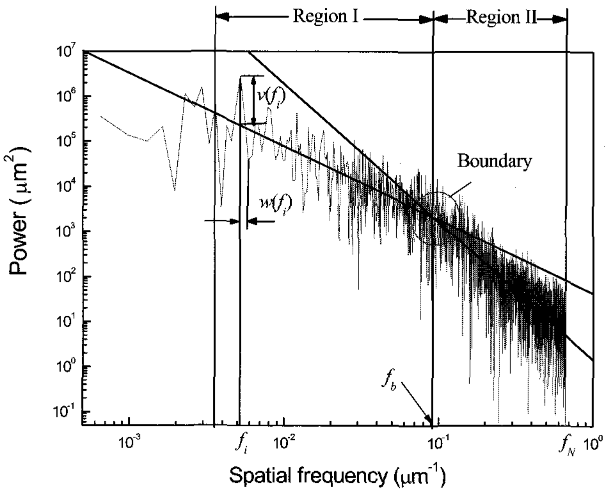

Fig. 1 Power spectrum of ground surface profile measured by the stylus with tip radius of 2 μm

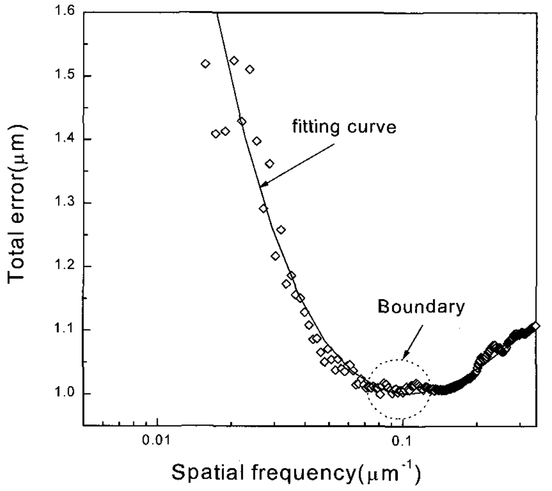

Fig. 2 The total error $e(f_n)$ calculated by Eq. (1) with log-log plot of Fig.1

machining processes such as grinding and lapping are measured by a stylus instrument, Form Talysurf II. And, the radius of stylus tip used is 2μm. All the measured profiles have a scan length of 3092μm and a sample interval of 0.75μm. To provide a true reproduction of original surface, the radius of the stylus tip should be theoretically zero. Unfortunately, measured profiles from the stylus with tip radius of 2 μm differ from original ones. In this paper, in order to reconstruct original profiles of sample surfaces, we assume that original surfaces have fractal geometry. This assumption implies that the power spectrums of original profiles follow power laws, namely the linear relation between the power spectrum $P(f)$ and the spatial frequency f in double logarithm coordination[11,12].

Fig. 1 shows the power spectrum of ground surface profile measured by the stylus with tip radius of 2μm. In Fig.1, f_b and f_N represent a boundary frequency and Nyquist frequency, respectively. And f_i represents an arbitrary frequency in regions I and II. It is found that the power spectrum of measured profile follows the power law up to a boundary frequency f_b, whereas above that it falls off. That is, the change of slope of the power spectrum occurs at the boundary frequency f_b in the log-log plot. This change of slope means that the stylus smoothes the original profile by filtering out wavelengths shorter than the radius of the stylus tip during measuring. In order to search the boundary frequency f_b, we use a least squares fitting to fit the data in the log-scale figure. The least squares fitting is simultaneously carried out in two frequency regions: below frequency f_n above frequency f_n. After least squares fitting, the objective function for searching the boundary frequency can be obtained from the following equation:

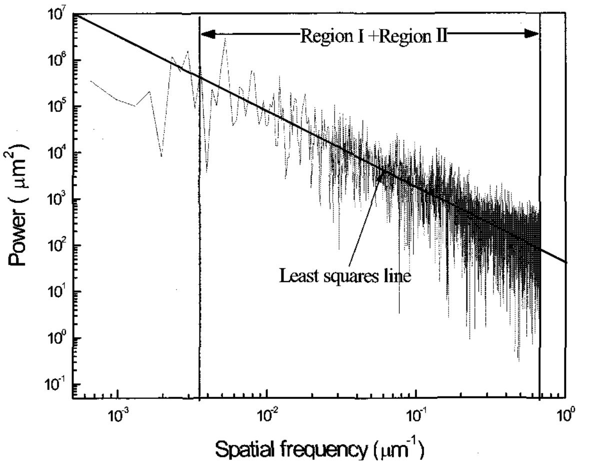

Fig. 3 An example of adjusted power spectrum

$$e(f_n)=\sum_{i=1}^{n} v(f_i)w(f_i)+\sum_{i=n+1}^{N} v(f_i)w(f_i) \quad (n=1,2,..,N-1) \tag{1}$$

where $v(f_i)$ indicates the absolute difference between a least squares line and a point of data at a frequency f_i ; $w(f_i)$ indicates a weighting value proportional to the interval between frequencies in the log-scale figure; N is the index of the Nyquist frequency f_N.

Fig. 2 shows the total error $e(f_n)$ calculated by Eq. (1) with log-log plot of Fig. 1. As shown in Fig. 2, the boundary frequency to minimize the calculated total error can be estimated from the fitting curve which is expressed as a quadratic equation. By using searching method described above, the boundary frequencies for the power spectrums of measured surface profiles can be found. As mentioned previously, we assumed that the original profiles are fractal surfaces and their power spectrums have the linear relation to the log-log plot. Based on this assumption, therefore, we can reconstruct the original profiles from the power spectrums of measured profiles by following procedures: Firstly, a boundary frequency f_b is calculated from the power spectrum of measured profile. Secondly, in the log-log plot of the power spectrum, the least-square fitting is carried out below f_b and then the slope and the intersection at f_b is calculated from the fitted line. Thirdly, the power spectrum of measured profile is adjusted so that the slope and intersection of the fitted line above f_b may be the same as those of the fitted line below f_b.

Fig.3 shows an example of power spectrum adjusted in this way. It is shown that the adjusted power spectrum has the linear relation to the log-log plot. From now on, the adjusted power spectrum can be written in its discrete form as follows:

$$P_k = P(f_k)\Delta f \qquad ,k=0,1,2,\ldots,N/2. \tag{2}$$

So, the amplitude at f_k is $\sqrt{P_k}$. Therefore, the Fourier transform of original profile is

$$\begin{aligned} &Z_0=\sqrt{P_0} \\ &Z_k=\sqrt{P_k}(\cos\phi_k + j\sin\phi_k) \qquad ,k=1,2,\ldots,N/2-1 \\ &Z_{N-k}=Z_k \qquad ,N/2<k<N \end{aligned} \tag{3}$$

where, ϕ_k is a set of random phase angles uniformly distributed between 0 and 2π.

Finally, by applying the inverse fast Fourier transform, the original profile z_l can be obtained by

$$z_l=\sum_{k=0}^{N-1} Z_k \exp(j\frac{2\pi kl}{N}) \qquad ,l=0,1,2,\ldots,N-1. \tag{4}$$

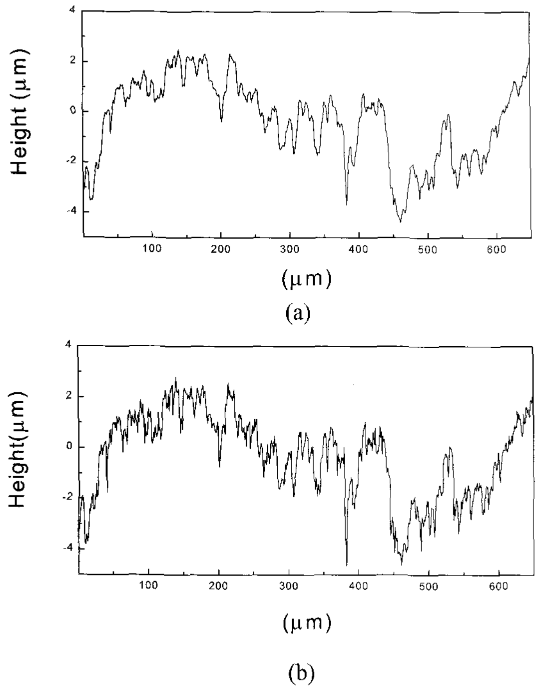

Fig. 4 An example of reconstruction of original profile: (a) a part of surface profile measured by the stylus with tip radius of 2 μm; (b) a part of reconstructed original profile

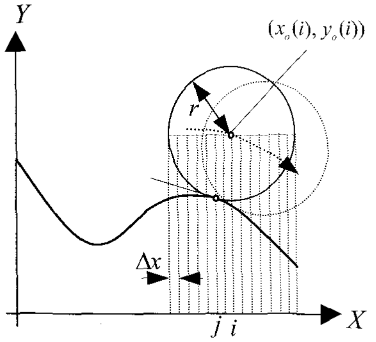

Fig. 5 A stylus of radius r in contact with a surface profile

Fig. 4 shows an example of reconstruction of original profile. It is shown that the short wavelengths filtered out by the stylus are restored in the generated profile.

3. A mathematical model of profile tracing

Our scheme for finding the locus of the stylus tip is the same as that of McCool[2] and Wu[5,6]. In order to derive the mathematical model of traced profiles, it is necessary to approximate a surface profile as a discrete series of points rather than a continuum. In order to guarantee the accuracy of the approximation, a small enough sampling interval must be chosen. In the mathematical model, the lower semicircle of stylus tip, which has a circular shape with a radius r, contacts the surface profile and the stylus is moved across the surface profile in discrete steps that may or may not equal the discrete interval. At each step, traced surface profile is drawn by the lowest point of the stylus tip. Fig. 5 shows a stylus of radius r in contact with a surface profile. The contact point has coordinates $x_c(j)$ and $y_c(j)$; the center of the stylus tip has coordinates $x_o(i)$ and $y_o(i)$; the lowest point of the stylus has coordinates $x_o(i)$ and $y_o(i)-r$. The index j of the contact point is determined by a search over the discrete points spanned by the stylus tip for the maximum of the function $y_c(k)+h(k)$, i.e.

$$y_c(j)+h(j)=\max_k[y_c(k)+h(k)]=y_o(i) \tag{5}$$

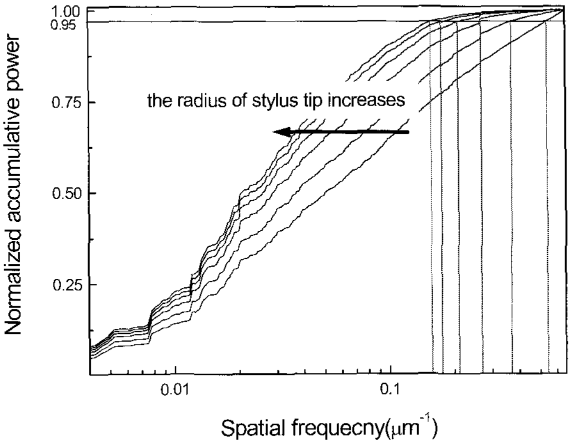

Fig. 6 An Accumulative power spectrum of profile vs. the stylus tip radius r

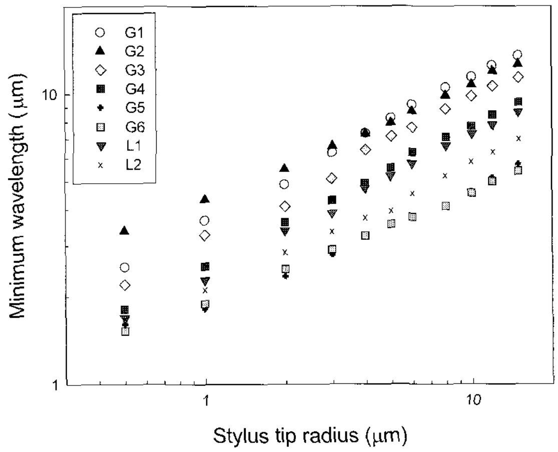

Fig. 7 Relationship between stylus tip radius and minimum wavelengths of traced profiles (G1, G2, G3, G4, G5 and G6 indicate original surface profiles generated from ground surface profiles; L1 and L2 indicate original surface profiles generated from lapped surface profiles)

where,

$$h(k)=\sqrt{r^2-(k-i)^2(\Delta x)^2} \tag{6}$$

The number of points spanned by the stylus tip is calculated as $2r/\Delta x$ and the range of indices k searched extends from $i-(r/\Delta x)$ to $i+(r/\Delta x)$. These equations are used for simulating the profile tracing.

4. Accumulation spectral analysis and results

The stylus of the surface measuring system acts as a low-pass filter. Generally, any frequency components with a wavelength shorter than the stylus tip radius cannot be observed. Therefore, the spectra of surface profiles measured by the stylus with finite tip radius start to decrease considerably (relative to peak spectra at low frequencies) at some high frequencies and then tend to zero. In this paper, to investigate this decline of spectra at high frequencies due to low-pass filtering of the stylus, the accumulation spectral analysis is employed.

4.1 Minimum wavelength of measured profile

The accumulative power spectrum plots the spatial frequency integral against spatial frequency. In a plot of accumulative power spectrum against frequency, the smallest frequency which includes 95% of the power is defined as the minimum wavelength of measured

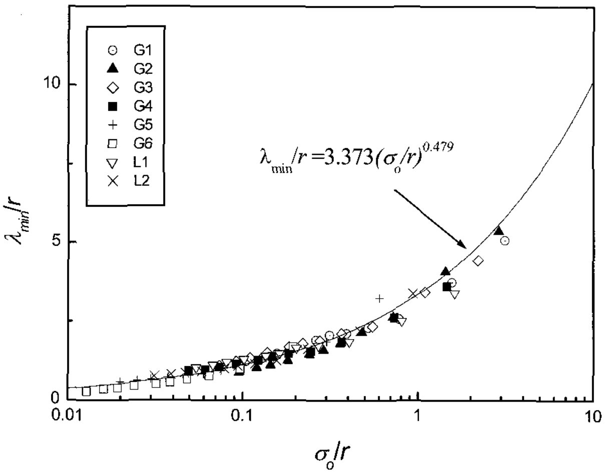

Fig.8 Minimum wavelength λ_{min} of traced profile as a function of the rms roughness σ_o of original profile and the stylus tip radius r

profile. Fig. 6 shows the influence of the stylus tip radius on the shape of accumulative power spectrum graph. For a specified percentage (e.g. 95%) in the accumulative power spectrum of traced surface profile, the value of its corresponding frequency decreases with increasing the stylus tip radius. Fig. 7 shows the minimum wavelengths of traced profiles according to the variation of stylus tip radius. The results of Fig. 7 are obtained from the accumulative power spectrums of the profiles numerically traced by the stylus with different radius. In this figure, the characters G1, G2, G3, G4, G5, and G6 indicate original surface profiles generated from ground surface profiles; L1 and L2 indicate original surface profiles generated from lapped surface profiles. And, they were obtained through the generation procedure described in chapter 2. As shown in Fig. 7, it can be found that minimum wavelength is directly proportional to the stylus tip radius in the log-log plot.

4.2 Minimum wavelength vs. rms roughness of original profile

Fig. 8 shows the relation between the minimum wavelength λ_{min} of traced profile and the rms roughness σ_0 of original profile. The rms roughness values can be obtained from the original surface profiles, which are given in Table 1. In Fig. 8, it is found that the minimum wavelength of traced profile directly relates to the stylus tip radius r and rms roughness σ_0 of original profile. Also, the minimum wavelength λ_{min} can be estimated by

$$\frac{\lambda_{min}}{r} \approx 3.373\left(\frac{\sigma_o}{r}\right)^{0.479} \tag{7}$$

In Eq. (7), if the rms roughness σ_0 of original profile and the stylus tip radius r are given, the minimum wavelength λ_{min} of measured profile can be estimated, namely, the shortest measurable wavelength limit with finite stylus tip radius can be estimated from Eq. (7). Based on this concept of minimum wavelength, a proper stylus tip radius can be determined when the rms roughness σ_0 of the profile is given.

Table 1 Rms roughness of original profiles

Profiles	Machining method	Rms roughness (μm)
G1	Grinding	1.574
G2	Grinding	1.437
G3	Grinding	1.096
G4	Grinding	0.729
G5	Grinding	0.301
G6	Grinding	0.223
L1	Lapping	0.746
L2	Lapping	0.467

5. Conclusions

This paper uses accumulation spectral analysis to assess the effects of the stylus tip radius on measuring surface profiles. Original surface profiles with fractal spectral densities are generated and numerically traced by the stylus with different radius. By using the accumulative power spectrums of traced profiles, the minimum wavelength λ_{min}, which represents the shortest measurable wavelength limit with finite stylus tip radius, is determined. It is found that the minimum wavelength of traced profile can be estimated from the stylus tip radius r and rms roughness σ_0 of original profile. By using this concept of minimum wavelength, a proper stylus tip radius can be determined for evaluating surface profiles.

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