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Assessment of Acoustic Iterative Inverse Method for Bubble Sizing to Experimental Data

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Abstract – Comparative study was carried out for an acoustic iterative inverse method to estimate bubble size distributions in water. Conventional bubble sizing methods consider only sound attenuation for sizing. Choi and Yoon [IEEE, 26(1), 125-130 (2001)] reported an acoustic iterative inverse method, which extracts the sound speed component from the measured sound attenuation. It can more accurately estimate the bubble size distributions in water than do the conventional methods. The estimation results of acoustic iterative inverse method were compared with other experimental data. The experimental data show good agreement with the estimation from the acoustic iterative inverse method. This iterative technique can be utilized for bubble sizing in the ocean.

Keywords – bubble size distribution, inverse method, iterative bubble sizing, sound attenuation, sound speed

1. Introduction

A number of experiments have shown that the upper layers of the ocean contain a large number of air bubbles (Clay and Medwin 1977; Thorpe *et al.* 1992). Small-sized bubbles are formed in the upper layer of ocean by breaking waves, ship-induced agitation, and biological activity, forming clouds at depths down to tens of meters by Langmuir circulation (Thorpe 1984). Bubble clouds are well known to work as strong sound scatterers and ambient noise sources (Yoon and Choi 1994). Therefore, researchers have investigated an acoustic remote technique which estimates the bubble size distribution in order to know the source mechanism of the ambient noise and the effect of sound propagation in the ocean (Medwin 1970; Vagle and Farmer 1992; Thorpe *et al.* 1992; Su *et al.* 1994; Commander and Moritz 1989; Commander and McDonald 1991). Recently the sound speed variation is recognized as an important parameter for bubble sizing as well as for sound attenuation (Choi *et al.* 1994; Choi and Yoon 1995; Choi 1996; Duraiswami *et al.* 1998). However, the measurement of the sound speed at each frequency in the sea is very difficult. To overcome this difficulty, a new iterative inverse method was proposed by Choi and Yoon (Choi and Yoon 2001).

In this paper, using experimental data, we propose a comparison between the iterative inverse method and conventional bubble sizing methods. This iterative technique can be utilized for practical bubble sizing in the ocean.

2. Bubble Sizing Theory using Sound Attenuation

The extinction cross section of a single bubble on sound wave in water (Clay and Medwin 1977) can be given by

$$\sigma \equiv \frac{\Pi}{I} = \frac{\Pi}{P^2/\rho c} = \frac{4\pi a\delta}{(\omega_o^2/\omega^2 - 1)^2 + \delta^2} \cdot \frac{c}{\omega} \qquad (1)$$

where Π is the scattered and absorbed power, I the incident intensity, P the root-mean-square pressure of incident wave, ρ the water density, c the sound speed in bubble-free water, a the bubble radius, ω_o the angular resonance frequency of the bubble, ω the angular

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frequency of incident sound, and δ the total damping constant of the bubble. The damping constant is made up of the thermal, viscous and re-radiation damping constants.

If the incident plane wave intensity is I_o, the intensity after traversing a distance x is

$$I(x) = I_o e^{-\sigma x} \tag{2}$$

The sound pressure becomes

$$P(x) = P_o e^{-\frac{1}{2}\sigma x} = P_o e^{-\alpha x} \tag{3}$$

The excess attenuation due to a bubble is

$$\alpha = \frac{1}{2}\sigma \tag{4}$$

For the bubbles of several sizes, the non-dispersive attenuation α can be written as follows

$$\alpha = \frac{1}{2}\int \sigma n da \tag{5}$$

where nda is the bubble number density.

From sound attenuation obtained by sound scattering on bubbles, we can obtain the bubble size distribution by inversing of Eq. (5) using the singular value decomposition (SVD) method (Strang 1980). However, the inverse results only using the sound attenuation can be not exactly correct without the sound speed correction (Choi and Yoon 2001). The sound speed cannot be easily measured in bubbly water. Therefore, the iterative method to estimate the sound speed from the sound attenuation has been introduced by Choi and Yoon (2001). By using the iterative method, we test the bubble sizing with the experimental results of Silberman (1957).

3. Utility of Iterative Inverse Method

From the iterative inverse method, we can obtain approximately the sound speed. Using this technique we can get more exactly the bubble size distribution, because the effect of sound speed variation is included in the sound attenuation (Choi and Yoon 2001).

Let us consider the non-dispersive and dispersive extinction cross section of a bubble as follows,

$$\sigma_0 \equiv \frac{\Pi}{I} = \frac{\Pi}{P^2/\rho c_0} = \frac{4\pi a\delta}{(\omega_o^2/\omega^2 - 1)^2 + \delta^2}\cdot\frac{c_0}{\omega}$$

(non-dispersive cross section), (6)

$$\sigma \equiv \frac{\Pi}{I} = \frac{\Pi}{P^2/\rho c} = \frac{4\pi a\delta}{(\omega_o^2/\omega^2 - 1)^2 + \delta^2}\cdot\frac{c}{\omega}$$

(dispersive cross section), (7)

where c_0 is the sound speed in bubble-free water as a constant value. $c(\omega)$ is the sound speed in bubbly water as a non-constant value.

Using the extinction cross section of a bubble, we can describe the sound attenuation as follows,

$$\alpha = \frac{1}{2}\int \sigma n da = \left\{\frac{1}{2}\int \sigma_o n da\right\}\frac{c}{c_o} = \alpha_o\frac{c}{c_o} \tag{8}$$

From the dispersive sound attenuation of Eq. (8), we can see that it contains the sound speed variation in bubbly water. By using the iterative method with the dispersive sound attenuation, the sound speed can be successfully estimated (Choi and Yoon 2001) as shown in Fig. 1.

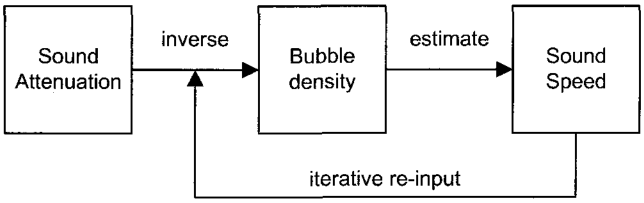

Fig. 1. Procedure of iterative inverse method.

4. Results of Bubble Size Distribution inversed from Silberman's Sound Attenuation Data

To understand the utility of the iterative inverse method, we can estimate the bubble size distribution from the sound attenuation data measured by Silberman (1957). The void fractions are selected for four cases (0.0377%, 0.22%, 0.53%, 1%). Silberman measured the sound attenuation and did not measure the sound speed in bubbly water. To get the bubble size distribution he assumed the averaged bubble radii as 1.03 mm, 2.1 mm, 2.18 mm and 2.64 mm corresponding to void fractions of 0.0377%, 0.22%, 0.53% and 1%, respectively. However, his data and the estimated bubble size distribution were not coincident. To overcome this lacuna in his analysis, we

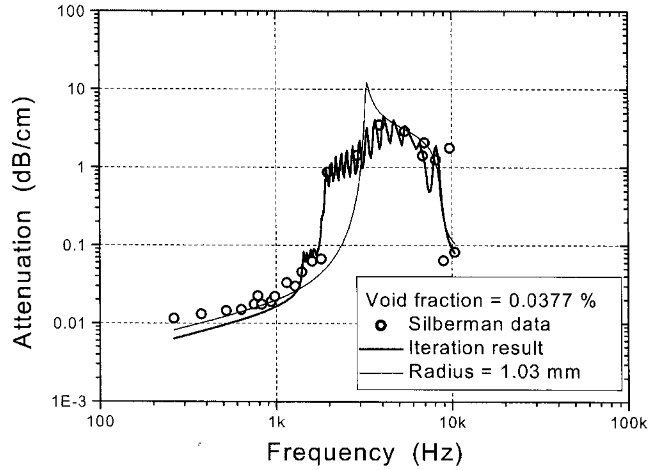

Fig. 2. Sound attenuation coefficient against incident sound frequency in bubbly water (void fraction: 0.0377%).: data of Silberman, heavy solid line: results of iterative inverse method, solid line: bubble radius of 1.03 mm.

introduced the iterative inverse method considering the sound speed variation and compared his experimental data as shown in Figs. 2 to 9.

The sound attenuations estimated by the iterative inverse methods are shown in Figs. 2 in the case of void fraction 0.0377%. The symbol (○) and heavy solid line represent the measured data and the iterative result, respectively, in Fig. 2. The solid line represents the theoretical calculation in the case of bubble radius, 1.03 mm by Silberman.

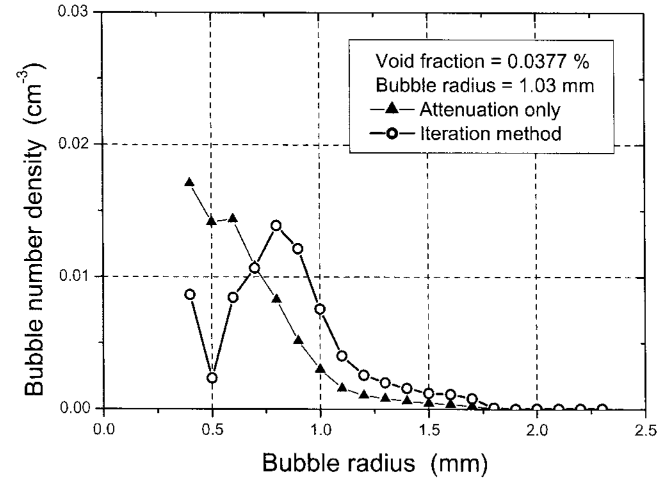

Fig. 3. Bubble size distribution estimated from measured sound attenuation (void fraction: 0.0377%). -▲- : the case considering only sound attenuation, -○- : the case of iterative inverse method considering sound attenuation with sound speed variation.

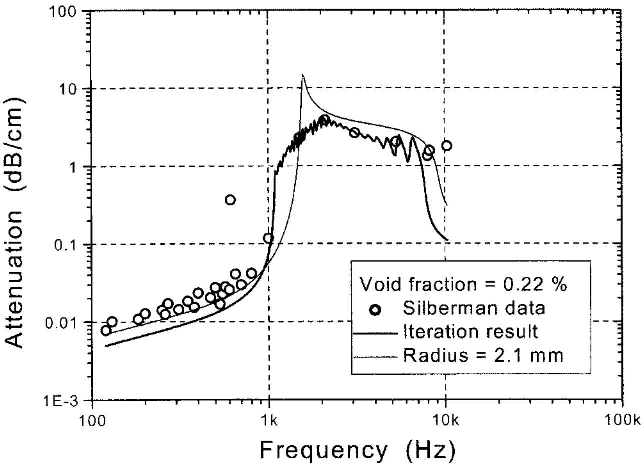

Fig. 4. Sound attenuation coefficient against incident sound frequency in bubbly water (void fraction: 0.22%).: data of Silberman, heavy solid line: results of iterative inverse method, solid line: bubble radius of 2.1 mm.

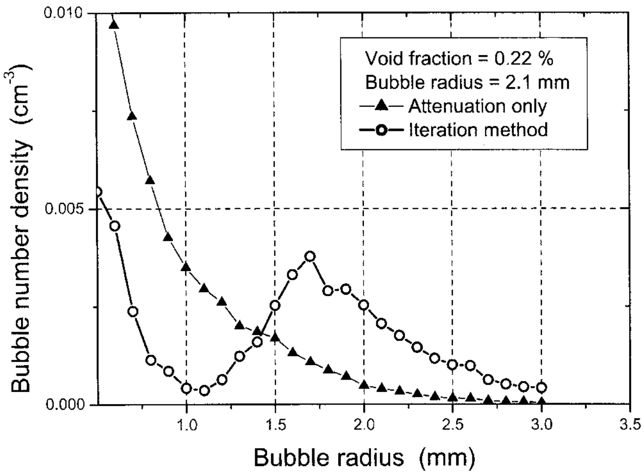

Fig. 5. Bubble size distribution estimated from measured sound attenuation (void fraction: 0.22%). -▲- : the case considering only sound attenuation, -○- : the case of iterative inverse method considering sound attenuation with sound speed variation.

The bubble size distribution estimated by the iterative inverse method from the measure attenuation data of Fig. 2 is shown in Fig. 3. The Symbols, -▲- and -○- are the traditional inverse result using only sound attenuation and the new iterative inverse result, respectively. The iterative inverse estimate consideration of the sound speed variation is better than the traditional inverse estimate that uses sound attenuation only. The bubble radius of the center of bubble distribution by the iterative inverse method is estimated at about 0.75 mm rather than 1.03 mm estimated by Silberman.

Next is in the case of void fraction of 0.22% shown in Figs. 4 and 5.

In Fig. 4, the iterative result (heavy solid line) shows good fit with the measured data from Silberman, but the Silberman's theoretical result does not agree as neatly. Here, because the vertical axis has logarithmic scale, the solid line has large discrepancy with the measured data.

From Fig. 5 the center of bubble size distribution is estimated about 1.7 mm rather than 2.1 mm by Silberman.

The case of the void fraction of 0.53% is shown in Figs. 6 and 7. The theoretical result by Silberman and the iterative inverse result have some differences with the measured data of Silberman. However, the iterative estimate has small difference with Silberman's data at high frequency. We have showed that the iterative inverse method has less correctness in the case of void fraction over 0.1%. In Fig. 7, The bubble sizing results by the iterative method represent better approximations than that by the method considering only sound attenuation and the result of bubble radius, 2.18 mm. The estimated center radius of bubble distribution is about 3.1 mm.

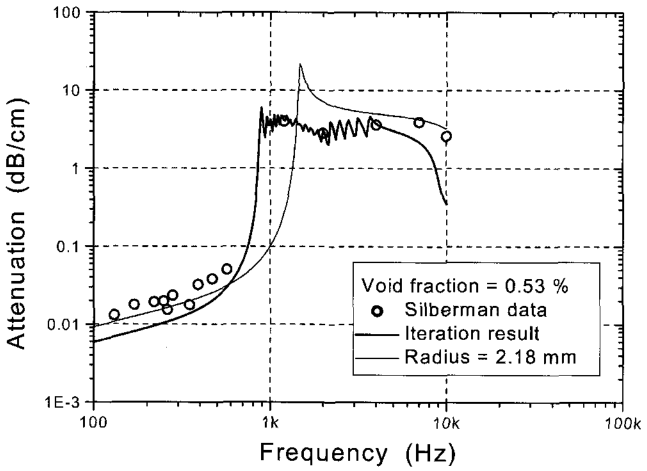

Fig. 6. Sound attenuation coefficient against incident sound frequency in bubbly water (void fraction: 0.53%).: data of Silberman, heavy solid line: results of iterative inverse method, solid line: bubble radius of 2.18 mm.

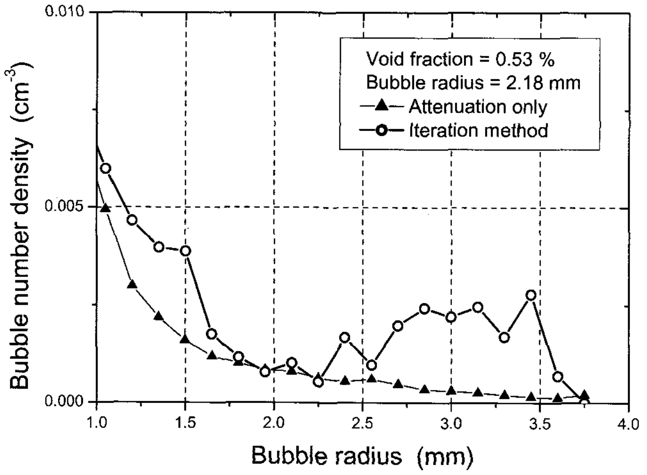

Fig. 7. Bubble size distribution estimated from measured sound attenuation (void fraction: 0.53%). -▲- : the case considering only sound attenuation, -○- : the case of iterative inverse method considering sound attenuation with sound speed variation.

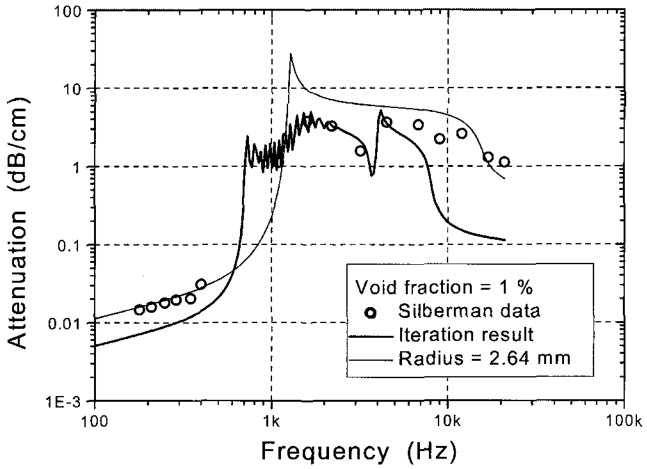

Fig. 8. Sound attenuation coefficient against incident sound frequency in bubbly water (void fraction: 1%).: data of Silberman, heavy solid line: results of iterative inverse method, solid line: bubble radius of 2.64 mm.

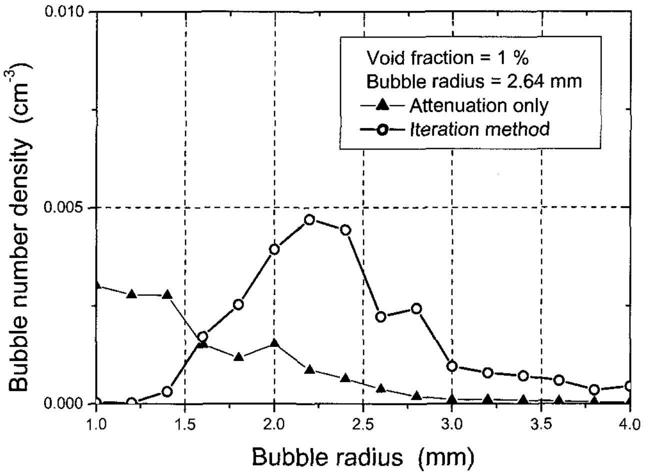

Fig. 9. Bubble size distribution estimated from measured sound attenuation (void fraction: 1%). -▲- : the case considering only sound attenuation, -○- : the case of iterative inverse method considering sound attenuation with sound speed variation.

The case of high void fraction of 1% is shown in Figs. 8 and 9. Although the high void fraction condition is over the limit of iterative method, the iterative result is not so bad as shown in Fig. 9. The estimated center radius of bubble distribution is about 2.2 mm rather than 2.64 mm by Silberman.

As the results of bubble size distributions, the averaged bubble radii are summarized in Table 1.

Table 1. The estimated bubble radii from Silbermann and the iterative inverse method

Void fraction (%)	Bubble radius (mm) (Silberman)	Bubble radius (mm) (iterative method)
0.0377	1.03	0.75
0.22	2.1	1.7
0.53	2.18	3.1
1	2.64	2.2

5. Conclusions

An iterative inverse acoustic bubble sizing method was introduced for estimating bubble size distribution in bubbly water that considers the effect of sound speed variation from the bubbles. Using the estimated sound speed with the given sound attenuation, a better estimate of the bubble size distribution is calculated. Numerical iterative inverse results with the measured data set from Silberman show that the iterative method estimates better than the traditional inverse method. The benefit of this new iterative method is that sound speed information can extract the attenuation data although sound speeds are not given initially.

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