Doubly-Selective Channel Estimation for OFDM Systems Using a Pilot-Embedded Training Scheme

Lidong Wang · Dongmin Lim

Abstract

Channel estimation and data detection for OFDM systems over time- and frequency-selective channels are investigated. Relying on the complex exponential basis expansion channel model, a pilot-embedded channel estimation scheme with low computational complexity and spectral efficiency is proposed. A periodic pilot sequence is superimposed at a low power on information bearing sequence at the transmitter before modulation and transmission. The channel state information(CSI) can be estimated using the first-order statistics of the received data. In order to enhance the performance of channel estimation, we recover the transmitted data which can be exploited to estimate CSI iteratively. Simulation results show that the proposed method is suitable for doubly-selective channel estimation for the OFDM systems and the performance of the proposed method can be better than that of the Wiener filter method under some conditions. Through simulations, we also analyze the factors which can affect the system performances.

Key words : Channel Estimation, Doubly-Selective Fading Channel, Iterative Operation, MIMO, Pilot-Embedded Scheme.

Ⅰ. Introduction

In orthogonal frequency division multiplexing(OFDM) systems, the quality of channel estimation has a major impact on the overall system performance, especially when the channel is time and frequency selective (or doubly selective). Many techniques have been proposed in the literature to estimate and equalize channel characteristics for OFDM transmission. Generally, channel estimation for the OFDM systems may be classified into training approaches, and blind or semi-blind approaches. Training approaches are based on training symbols that are a priori known to the receiver. For time-varying channels, one has to send training signal frequently and periodically to keep up with the changing channel. Pilot-symbol assisted modulation (PSAM) method is based on training symbols which are inserted periodically in the time and/or frequency domain. However, these training pilots consume bandwidth and reduce the data rate. The blind channel estimation approach is based solely on noisy signal exploiting statistical and other properties of transmitted signals. Relative to the training approaches, blind approaches typically require longer data records and entail higher complexity.

In order to overcome the shortcomings of the above approaches, recently a pilot-embedded(PE) approach for channel estimation via first-order statistics has been discussed in [1]~[6]. A periodic training sequence is added at a low power to the information bearing sequence at the transmitter before modulation and transmission. At the receiver, the first-order statistics are used for channel estimation. The training pilots are added onto information signals so the channel is estimated without sacrificing the bandwidth efficiency. For example, if a pilot symbol is inserted every r data symbols in the PSAM method, then the bandwidth efficiency improvement of the PE method over the PSAM method will be $1/r$. The strongpoint of the PE scheme is its high bandwidth efficiency, low computational complexity, and possibility of improved power efficiency at the transmitter. But the shortcoming is that some useful power is wasted in the PE sequence, which affects the effective signal-to-noise ratio and the bit-error-rate at the receiver.

In this paper, we extend the PE channel estimation approach of [1], which is for time-invariant or slow fading single-input multiple-output channel estimation, to the situation of doubly-selective channel estimation in the OFDM systems and use complex exponential basis model to approximate the time-varying coefficients of the OFDM channels. Thus, the time-varying channel estimation is reduced into estimating the coefficients of the basis functions. In order to enhance the channel estimation performance, we also adopt iterative operation in which the recovered information sequence can be exploited.

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Ⅱ. System Model

We consider a zero padding(ZP) single-input single-output OFDM system. At the transmitter, a predetermined periodic pilot sequence $\{C(n)\}$ is added to the information symbol sequence $\{B(n)\}$, that is $X(n)=B(n)+C(n)$. The resulting block $X(n)$ of length N is then transformed into a time-domain sequence using an N-point inverse discrete Fourier transform(IDFT). To avoid inter-block interference(IBI), a ZP of length L equal to the channel order is inserted at the end of each block. The transmitted time-domain symbol $x(k)=b(k)+c(k)$, where $b(k)$ and $c(k)$ are IDFT and ZP adding of $B(n)$ and $C(n)$ respectively, and the received symbol $y(k)$ can be written as

$$x(k)=\begin{cases}\sum_{n=0}^{N-1} X(n)e^{j2\pi kn/N}, & k=(0,1,\ldots,N-1)\\ 0, & k=(N,N+1,\ldots,N+L-1)\end{cases} \tag{1}$$

$$y(k)=\sum_{l=0}^{L} h(k,l)x(k-1)+v(k) \tag{2}$$

where $v(k)$ is additive white Gaussian noise(AWGN) with zero mean, and $h(k,l)$ is the channel impulse response of the lth path at time k. In a period of R symbol duration T_s, $h(k,l)$ can be expressed by the complex exponential basis expansion model(BEM)[7] as

$$h(k,l)=\sum_{q=-Q/2}^{Q/2} h_q(l)e^{j\omega_q k} \tag{3}$$

where $h_q(l)$ is the coefficient of the lth tap and qth BEM, which is kept time-invariant over the period of R sample duration. Even numbered Q is the order of BEM, and $\omega_q=2\pi q/R$, where R determines the frequency resolution with $R\geq N+L$. Q should be selected such that $(Q+1)/(2RT_s)\geq f_{\max}$, where $f_{\max}$ is the maximum Doppler frequency which is assumed to be known at the receiver beforehand. For the sequel, we rewrite (2) in the matrix form as

$$\boldsymbol{y}=(1/N)\boldsymbol{H}\boldsymbol{F}^{-1}\boldsymbol{X}+\boldsymbol{v}_y \tag{4}$$

where $\boldsymbol{X}=[X(0),X(1),\cdots,X(N-1)]^T$, $\boldsymbol{v}_y=[v_y(0),v_y(1),\cdots,v_y(N+L-1)]^T$ is the received noise vector, $\boldsymbol{F}$ is an N-point discrete Fourier transform(DFT) matrix, the entry of which $[\boldsymbol{F}]_{k,n}=\exp(-j2\pi kn/N)$, $\boldsymbol{y}=[y(0),y(1),\cdots,y(L+N-1)]^T$, and $\boldsymbol{H}$ is the $(N+L)\times N$ channel matrix

$$\boldsymbol{H}=\begin{bmatrix} h(0,0) & & \\ \vdots & \ddots & \\ h(0,L) & & h(N-1,0) \\ & \ddots & \vdots \\ & & h(l-1,L)\end{bmatrix} \tag{5}$$

If the channel state information(CSI) is known at the receiver, the frequency response of the received block $\boldsymbol{Z}=[Z(0),Z(1),\cdots,Z(N-1)]^T$ can be expressed as

$$\boldsymbol{Z}=\boldsymbol{F}\boldsymbol{H}^{\dagger}\boldsymbol{y}=(1/N)\boldsymbol{F}\boldsymbol{H}^{\dagger}\boldsymbol{H}\boldsymbol{F}^{-1}\boldsymbol{X}+\boldsymbol{v}_z \tag{6}$$

where $\boldsymbol{H}^{\dagger}$ is the pseudo inverse of $\boldsymbol{H}$ and $\boldsymbol{v}_z$ is a noise vector.

Ⅲ. Channel Estimation Algorithm

From (2) and (3), we have

$$y(k)=\sum_{q=-Q/2}^{Q/2}\sum_{l=0}^{L} h_q(l)e^{j\omega_q k}[b(k-l)+c(k-l)]+v(k) \tag{7}$$

We assume that $B(n)$ has zero mean value and $b(k)$ which is IDFT of $B(n)$ also has zero mean value, so we can have the expected value as

$$E[y(k)]=\sum_{q=-Q/2}^{L}\sum_{l=0}^{Q/2} h_q(l)e^{j\omega_q k}c(k-l) \tag{8}$$

The sequence $c(k)$ is periodic with period $P=N+L$, so $c(k)$ can be expressed as $c(k)=\sum_{p=0}^{P-1} c_p d^{j\alpha_p k}$ where $c_p=\frac{1}{P}\sum_{k=0}^{P-1} c(k)e^{-j\alpha_p k}$ and $\alpha_p=2\pi p/P$, then we have

$$E[y(k)]=\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_q(l)e^{j\omega_q k}\left(\sum_{p=0}^{P-1} c_p e^{j\alpha_p(k-l)}\right) \tag{9}$$

For convenience, we define $d_{pq}=\sum_{l=0}^{L} c_p h_q(l)e^{-j\alpha_p l}$, then (9) is given as

$$E[y(k)]=\sum_{q=-Q/2}^{Q/2}\sum_{p=0}^{P-1} d_{pq}e^{j(\omega_q+\alpha_p)k} \tag{10}$$

If P is chosen properly, then $\omega_q+\alpha_p$ are all distinct, and the estimation of d_{pq} can be expressed as

$$\tilde{d}_{pq}=\frac{1}{T}\sum_{k=1}^{T} y(k)e^{-j(\omega_q+\alpha_p)k} \tag{11}$$

We can see that $\tilde{d}_{pq}\rightarrow d_{pq}$ as $T\rightarrow\infty$. Now we estimate $h_q(l)$ from $\tilde{d}_{pq}$ with a linear method. Defining and $\boldsymbol{D}_p=[d_{p(-Q/2)}, d_{p(-Q/2+1)},\cdots, d_{p(Q/2)}]^T$ and $\boldsymbol{h}_l=[h_{-Q/2}(l), h_{-Q/2+1}(l),\cdots,h_{Q/2}(l)]^T$, then $\boldsymbol{D}_p=\sum_{l=0}^{L} c_p e^{-j\alpha_p l}\boldsymbol{h}_l$, or $\boldsymbol{CH}=\boldsymbol{D}$, where $\boldsymbol{H}=[\boldsymbol{h}_0^T, \boldsymbol{h}_1^T,\cdots, \boldsymbol{h}_L^T]^T$, $\boldsymbol{D}=[\boldsymbol{D}_0^T, \boldsymbol{D}_1^T, \cdots, \boldsymbol{D}_{P-1}^T]^T$ and

$$\boldsymbol{C}=\begin{bmatrix} c_0 I_Q & c_0 I_Q & \cdots & c_0 I_Q \\ c_1 I_Q & c_1 I_Q e^{-j\alpha_1} & \cdots & c_1 I_Q e^{-j\alpha_1 L} \\ \cdots & \cdots & \cdots & \cdots \\ c_{P-1} I_Q & c_{P-1} I_Q e^{-j\alpha_{P-1}} & \cdots & c_{P-1} I_Q e^{-j\alpha_{P-1} L}\end{bmatrix} \tag{12}$$

Suppose that $P\geq L+1$ and $c_p\neq 0\,\forall p$, then the rank of $\boldsymbol{C}$ is $(Q+1)(L+1)$ and $h_q(l)$ can be determined uniquely.

The CSI can be estimated by

$$\tilde{H}=(C^H C)^{-1} C^H D \tag{13}$$

As $c(k)$ is known at the receiver, pseudo inverse of C can be calculated offline and this will not give rise to any additional computational complexity for channel estimation. Because pilot sequence and information bearing sequence pass through the same channel, the algorithm introduced above views the information bearing sequence as interference to channel estimation. The detected information bearing sequence can be exploited to enhance channel estimation performance. The performance enhancement algorithm which we call iterative PE(IPE) is introduced below.

Assuming that we have collected N_s samples of the received symbols $y=[y(0), y(1), \cdots, y(N_s-1)]^T$, substitute $x(k)$ for $[b(k)+c(k)]$ in (7) and rewrite (7) in matrix form as

$$\boldsymbol{y}=\sum_{q=-Q/2}^{Q/2} \Psi_q S_x \boldsymbol{h}_q + \boldsymbol{v} = \boldsymbol{G}\boldsymbol{h}_g + \boldsymbol{v} \tag{14}$$

where $\boldsymbol{x}=[x(0), x(1), \cdots, x(N_s-1)]^T$, $\boldsymbol{v}=[v(0), v(1), \cdots, v(N_s-1)]^T$, $\Psi_q=\mathrm{diag}\{1, e^{j\omega_q}, \cdots, e^{j\omega_q(N_s-1)}\}$, $\boldsymbol{h}_q=[h_q(0), \cdots, h_q(L)]^T$, $\boldsymbol{h}_g=[\boldsymbol{h}^T_{-Q/2}, \cdots, \boldsymbol{h}^T_{Q/2}]^T$, $\boldsymbol{G}=[\Psi_{-Q/2} S_x, \cdots, \Psi_{Q/2} S_x]$ and

$$S_x=\begin{bmatrix} x_0 & 0 \cdots & 0 \\ \vdots & \ddots & \vdots \\ x_L & \cdots & x_0 \\ \vdots & & \vdots \\ x_{N_s-1} & \cdots \cdots & x_{N_s-(L+1)} \end{bmatrix}_{N_s \times (L+1)} \tag{15}$$

There are $(Q+1)(L+1)$ coefficients of exponential basis functions unknown, to estimate the channel uniquely the total number of adopted symbols must satisfy $N_s \geq (Q+1)(L+1)$. The channel can be estimated by

$$\tilde{\boldsymbol{h}}_g=(G^H G)^{-1} G^H \boldsymbol{y} \tag{16}$$

The IPE channel estimation can be operated as follow 5:

a) Use (13) to estimate the channel roughly. We can get the CSI $\tilde{h}(k, l)$ by (3) from the estimated BEM coefficient $\tilde{h}_q(l)$.
b) Use (6) to equalize the channel and estimate $B(n)$ as $\tilde{B}(n)$($\tilde{B}(n)=Z(n)-C(n)$). Recover the transmitted information symbols by quantizing $\tilde{B}(n)$ into $\bar{B}(n)$ with the knowledge of the symbol alphabet.
c) Define $\bar{X}(n)=\bar{B}(n)+C(n)$, replace $X(n)$ in (1) by $\bar{X}(n)$ and use (1) to make IDFT and ZP adding operation, then we collect N_s samples denoting $\bar{\boldsymbol{x}}=[\bar{x}(0), \bar{x}(1), \cdots, \bar{x}(N_s-1)]^T$. Substitute $\bar{\boldsymbol{x}}$ for $\boldsymbol{x}$ in (14) and use (16) to estimate the channel $\boldsymbol{h}_g$ as $\tilde{\boldsymbol{h}}_g=(\bar{G}^H \bar{G})^{-1} \bar{G}^H \boldsymbol{y}$, where $\bar{G}=[\Psi_{-Q/2} S_{\bar{x}}, \cdots, \Psi_{Q/2} S_{\bar{x}}]$.
d) Repeat b)~c) if necessary.

Ⅳ. Simulations

In this section, we show some simulation results of the proposed channel estimation algorithm. We consider an OFDM system with doubly-selective fading channel of order L=3, and the channel taps are assumed to be i.i.d.. We consider N=16 subcarriers, and a ZP of length 3. Information bearing data are chosen from QPSK signalling with symbol rate $f_s=1/T_s$ and pilot sequence is pseudo-random series, where T_s is sampling time interval. The adopted number of symbols T in (11) is set as 200, R and N_s are all set as 200. We define signal-to-noise ratio(SNR) as $\mathrm{SNR}=10\log(\sigma_x^2/\sigma_v^2)$, where $\sigma_x^2=E(|x(k)|^2)$, σ_v^2 is the variance of AWGN, and define power loss factor β as $\beta=-10\log(\sigma_b^2/(\sigma_b^2+\sigma_c^2))$ where $\sigma_b^2=E(|b(k)|^2)$ and $\sigma_c^2=E(|c(k)|^2)$. β is used to measure the information data power loss introduced by pilot sequence. We adopt normalized mean square error (MSE) value as the measurement of channel estimation performance, that is

$$\mathrm{MSE}=\left(\sum_{k=0}^{N_s}\sum_{l=0}^{L}|h(k,l)-\tilde{h}(k,l)|^2\right)\left(\sum_{k=0}^{N_s}\sum_{l=1}^{L}|h(k,l)|^2\right). \tag{17}$$

We use the complex exponential basis expansion model to approximate the channel. The doubly-selective fading channel in mobile wireless communications can be modelled as wide-sense-stationary uncorrelated-scattering(WSSUS) channel. The autocorrelation function of the WSSUS channel is $E(h(k_1, l_1)h^*(k_2, l_2))=\phi_h((k_1-k_2)T_x, l)\delta(l_1-l_2)$. In a rich-scattering environment, $\phi_h((k_1-k_2)T_s, l)=\phi_k((k_1-k_2)T_s)\phi_l(l)$, where ϕ_k is time correlation function and ϕ_l is power delay profile function. The Fourier transform of the $\phi_k((k_1-k_2)T_s)$ gives the Doppler power spectrum $\phi_D(f)$. In the Jakes' fading channel model, $\phi_k((k_1-k_2)T_s)=J_0(2\pi f_{\max}(k_1-k_2)T_s)$ and $\phi_D=1/(\pi f_{\max}\sqrt{1-(f/f_d)^2}, |f|<f_{\max}$ where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind and $f_{\max}$ is the maximum Doppler frequency. For the lth tap of the channel, the complex exponential basis expansion model coefficient vector $\boldsymbol{h}(l)=[h_1(l), H_2(l), \cdots, h_Q(l)]^T)$ is obtained from the Jakes' model over R symbol periods. The simulation results are obtained from the average of 500 Monte Carlo runs.

In Fig. 1 and 2, we run the simulations for the different Doppler spreads and the maximum Doppler frequencies are $f_{\max}$=100, 200 Hz, so the normalized Do-

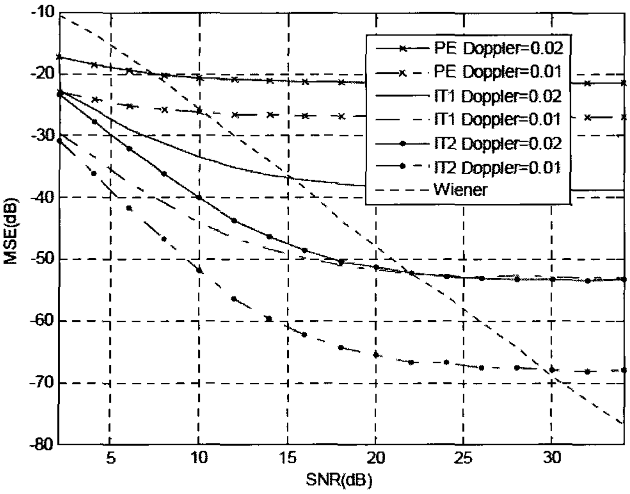

Fig. 1. Channel estimation MSE as a function of SNR.

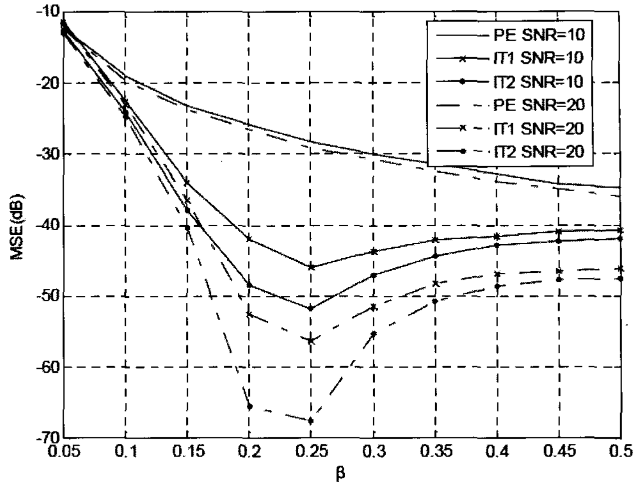

Fig. 3. Channel estimation MSE as a function of β.

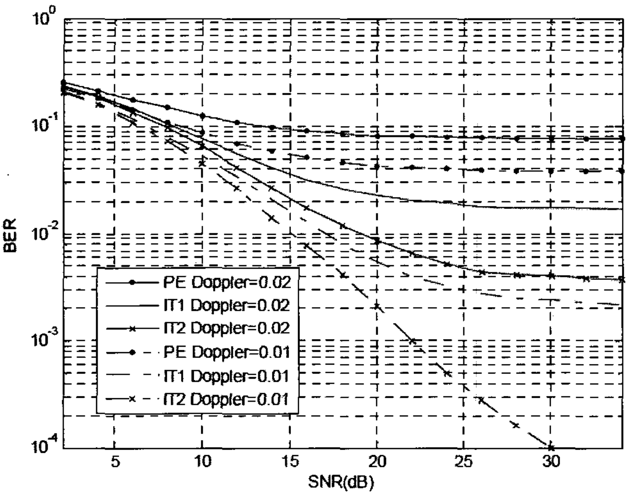

Fig. 2. BER performance as a function of SNR.

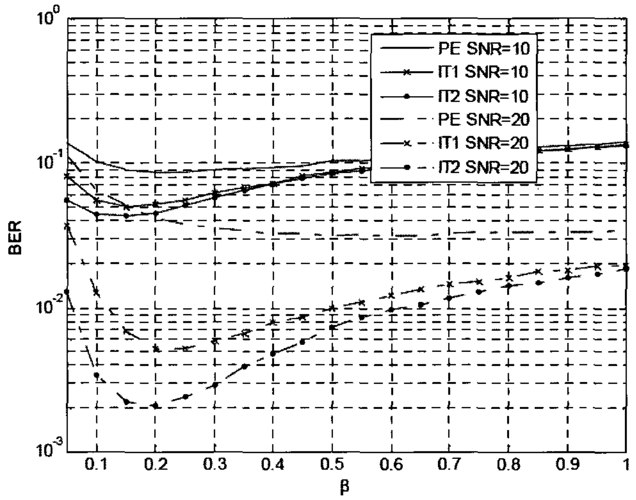

Fig. 4. BER performance as a function of β.

ppler frequency of time variant channel $f_{\max}T_s$ is 0.01 and 0.02 respectively, Q for 0.01 Doppler is set as 4 and for 0.02 as 8, so $(Q+1)/(2RT_s)\geq f_{\max}$ is satisfied. Power loss factor $\beta=0.2$. Figs. 1 and 2 show the channel estimation MSE and bit error rate(BER) vs. SNR using only step a) (denoted as PE), first iteration specified by step b) (denoted as IT1) and second iteration specified by step b) (denoted as IT2) respectively. For comparison, we also give out the MSE performance of the Wiener estimation for PSAM introduced in [8]. For the Wiener estimation, pilots are inserted in the time domain. The parameters for the Wiener estimation are set as follows: frame size is 19, number of pilot symbols involved in Wiener filtering is 5, the normalized Doppler frequency of time variant channel $f_{max}T_s$ is 0.01 and other parameters are the same as in pilot-embedded method introduced in the paper. It can be seen that the performance of iterative enhancement approach is much better than the pilot embedded approach(only step a) and Doppler frequency affects MSE obviously. For the Wiener method, the channel estimation MSE decreases linearly with increasing values of SNR, for pilot embedded method MSE also decreases with increasing values of SNR, but at high values of SNR, MSE decreases slowly and suffers from error floor. It is because that at high values of SNR, the main interference to channel estimation comes from the information bearing symbols not from the white Gaussian noise. Although at high values of SNR the performance of pilot-embedded method and its enhancement algorithm are worse than that of the Wiener estimation, training pilots have to be used in the Wiener estimation which will reduce the data rate.

In Fig. 3 and 4, normalized Doppler frequency of time variant channel $f_{max}T_s$ is set as 0.01, Q is set as 4.

Figs. 3 and 4 show the channel estimation MSE and BER vs. SNR using only step a), first iteration specified by step b) and second iteration specified by step b) respectively. The channel estimation MSE decreases with the increasing values of β for the pilot-embedded approach using step a) only, but for other method the MSE and BER decrease at low values of β but increase at high values of β. The addition of pilot sequence to the transmitted signal decreases the power available for conveying data. As a result, as the power allocated to the known sequence is increased, the power available for the information bearing symbols is decreased, hence the BER is to increase, which can affect conversely the channel estimation accuracy of iterative algorithm. If the power allocated to the pilot sequence is too small, then the channel estimate is less accurate and the BER can also be expected to increase.

V. Conclusions

Pilot embedded channel estimation and its iterative performance enhancement scheme for the doubly-selective OFDM channel is proposed in this paper. In OFDM systems, time-varying fading effect of wireless channel can impair the orthogonality of the subchannel, resulting in the increase of error floor. First-order statistic method which is with computational simplicity is adopted to estimate CSI roughly, and then the estimated CSI and the recovered information carrying symbols can be used to estimate CSI conversely. We can use the method proposed in the paper to estimate the doubly-selective OFDM channel only assuming the knowledge of the maximum Doppler frequency beforehand which is rather easy to obtain in practice. Performance of the proposed algorithm is showed by simulations, and the influences of some parameters such as SNR and power loss factor are also demonstrated. The performance of the pilot embedded scheme is better than Wiener method for PSAM when enough number of symbols T and power loss factor β are adopted, especially for low SNR. The scheme weproposed is suitable for the estimation of time-variant and time-invariant channel as well. The effectiveness of the algorithm is confirmed by numerical simulations.

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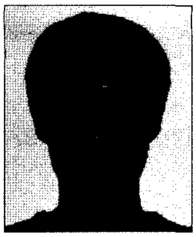

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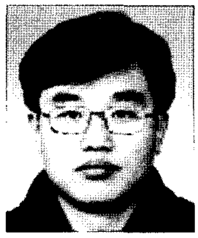

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