JOURNAL OF THE KOREAN SOCIETY OF AGRICULTURAL ENGINEERS DEC. 2006, pp. 35-43

JOURNAL OF THE KOREAN SOCIETY
OF AGRICULTURAL ENGINEERS,
Vol. 48, No. 7

Liquefaction Behaviour and Prediction of Deviator Stress for Unsaturated Silty Sand

Lee, Dal Won*

Abstract

This study was carried out to investigate the liquefaction behaviour and predict deviator stress with matric suction, of unsaturated silty sand. The unsaturated soil tests were conducted using a modified triaxial cell and specimens were prepared using the moisture tamping method. The axis translation technique was used to create the desired matric suctions in the specimen. Undrained triaxial compression tests were carried out at matric suction of 0, 2, 5, 10 and 25 kPa. The specimens were sheared to axial strains of about 20% to obtain steady state conditions. The results showed that liquefaction of silty sand only occurs at matric suction of 0 kPa and 2 kPa. The results also show that at matric suctions of 5, 10 and 25 kPa, the resistance to liquefaction increases. As the suction increases, the undrained effective stress path approached the drained stress path. Also, the predicted and measured maximum deviator stress for unsaturated soils using the effective stress concept showed good agreement as matric suction increases. The deviator stress increase is nonlinear as matric suction increases.

Keywords : Liquefaction, Unsaturated soils, Matric suction, Deviator stress

---

I. Introduction

The ground in a natural state is divided into saturated and unsaturated regions according to the degree of saturation. To understand the deformation behaviour of saturated and unsaturated soils and perfect coupled flow progression, we have to use numerical analysis and verify through experimental studies.

The soil of effective stress state is changed into unique stress as the variation of total stress, pore water pressure and air pressure at given yield surface. Especially, the effective stress concept can be significant simplifying effect on the constitutive modeling of saturated and unsaturated soils.

Many failures of sea dike, earth dam, embankment and cut slope, foundation and other earth structures have been caused by the liquefaction of soils. Especially, liquefaction of saturated underwater slopes such as sea dike is

---

* Dept. of Rural Infrastructure Eng., Chungnam National University
* Corresponding author. Tel.: +82-42-821-5793
Fax: +82-42-821-8877
E-mail address: dwlee@cnu.ac.kr

a major concern of study because of its frequent occurrence and its effect on the safety of coastal structures. The underwater slopes of the saturated noncohesive soil is easily deformed caused by liquefaction due to contractive volumetric strains and reduction in effective stresses. The factors which affect liquefaction are earthquake activity, rapid sedimentation, gas, tidal currents, etc.

Previous studies for static liquefaction were carried out for uniform clean sand or nonplastic silt sand (Yamamuro et al., 1997, 1998; Vaid et al., 1995; Lade et al., 1997). However, earth dam and embankment slope which experienced liquefaction have soil including larger silt content than clean sand, but most prior studies on static liquefaction of silty sand assumed and designed under such condition that silty sand and clean sand had almost the same behaviour. But silty sand shows quite different liquefaction behaviour from clean sand and effective stress concept is appeared differently at yield surface. Therefore, to analyse the flow of water and air passing inner unsaturated soil, stress-strain relationship with the variation of matric suction has to be considered.

In effective stress concept, it was verified that volumetric change, shear strength and stress path are appeared differently with the variation of matric suction (Bishop & Blight, 1963; Blight, 1967; Fredlund & Morgenstern, 1977). Since then, there were many studies to investigate unique effective stress equation in unsaturated soil. Eventually, it has to be applied not unique but more than two independent stress variables, and it depended on stress path (Fredlund et al., 1977, 1985a). Also, many studies were carried out to compare and analyse failure envelope line and internal friction angle (ϕ^b) in unsaturated soil, critical state line (CSL) and deviator stress-mean effective pressure with matric suction and the effect of suction-stress coefficient (Ho et al., 1982; Gan et al., 1988; Wheeler et al., 1995; Estabragh et al., 2004; Lu et al., 2004).

There were many studies to investigate static liquefaction behaviour in saturated soil, but not many studies performed experimentally about liquefaction behaviour of silty sand in unsaturated condition, and evaluated the suction effect about the resistance of liquefaction.

The main objective of this paper is to compare and analyses experimentally the liquefaction behaviour of unsaturated silty sand with matric suction and verify the application of predicted deviator stress by using effective stress concept.

Ⅱ. Approach Method of Effective Stress in Unsaturated Soil

The determination methods of the shear strength in unsaturated soil are the method of effective stress approach (Bishop, 1959) and the independent state variables approach (Fredlund, 1978). The independent state variables approach requires extensive and time consuming laboratory testing to determine the material parameters, due to nonlinear of internal friction angle (ϕ^b) with matric suction, and the predictive capacity of the approach is limited to situations where the suction range used in the laboratory to establish ϕ^b is the same as that expected in the field. The failure envelope line appears unique value if it is assumed that shear strength obtained by the effective stress approach and the independent state variables approach is the same.

For unsaturated soils, the effective stress concept is expressed as follows (Bishop, 1959);

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \cdots\cdots\cdots (1)$$

where σ: total stress,
$(\sigma - u_a)$: net normal stress,
$(u_a - u_w)$: matric suction,
χ: effective stress parameter (saturated soils: 1, dry soils: 0),
u_a: pore air stress,
u_w: pore water stress

χ is the effective stress parameter related to the total stress, air pressure, pore water pressure and degree of saturation. The advantage of the effective stress approach is a unique stress variable rather than two or three of independent stress variables that the change of the shear strength with variation in total stress, air pressure and pore water pressure, but there is a major difficulty to make determination of the effective stress parameter (χ). There were many studies to investigate relationships between the effective stress parameter (χ) and the degree of saturation (Sr) in unsaturated soil, but they approached by only macroscopic concept because of difficulties to express forces in interparticle contacts into the form of a stress. Theoretical models were hardly expressed at microscopic concept.

Khalili et. al. (1998) compared and analysed the relationship between the effective stress parameter (χ) and suction ratio (s/s_e) through data from 14 cases in the literature, and it may be expressed as a unique relationship verified by experiments.

$$\chi = \begin{cases} (s/s_e)^{-0.55} & \text{for } s \geq s_e \\ 1 & \text{for } s \leq s_e \end{cases} \cdots\cdots\cdots (2)$$

where s_e: air entry value, s: matric suction

The air entry values were obtained from the soil characteristic curves. To predict deviator stress with the variation of suction using effective stress concept, applied theoretical equation is expressed as followed (Khalili et. al., 2004).

$$q = Mp' = a + [(\sigma - u_a) + \chi(u_a - u_w)]M \cdots\cdots (3)$$

where $a = 6\ c'\cos\phi'/(3 - \sin\phi')$,
$M = 6\ \sin\phi'/(3 - \sin\phi')$
c', ϕ': effective cohesion, effective internal friction angle
$\sigma - u_a$: net mean stress $[(\sigma_1 + 2\sigma_3)/3 - u_a]$

To predict the stress path in unsaturated soil, this equation is validated that net normal stress $(\sigma - u_a)$ is only constant during the shearing because the change of air pressure is equal to a change in the effective stress.

Ⅲ. Materials and Methods

1. Soil Properties

The used soils were silty sand obtained at Lyell dam site near Sydney in Australia, the grain size distribution curve and physical properties were given in Fig. 1 and Table 1, and it was passed No. 50 sieve size (0.3 mm) to remould specimens in uniform grain size.

Table 1 Physical properties of the used soil

Gs	PI (%)	Maximum dry density (KN/m^3)	Optimum moisture content (%)	USCS
2.61	N.P	19.63	10.1	SM

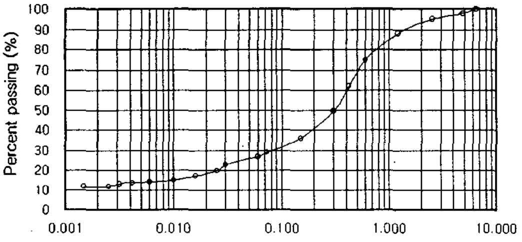

Fig. 1 Grain size distribution curve

2. Specimens Preparation and Saturation Procedure[14)]

The preparation of specimens at low density are generally created by the moist tamping method and funnel deposition method. The tests were created by using the moist tamping method. The specimens was remoulded at 7% of water content and ten layers were used to achieve the required specimen height, and then the saturation procedure was performed by purging the specimen with carbon dioxide (CO_2) before adding de-aired water. A minimum of 300 kPa back pressure was used to ensure complete saturation, a minimum B value of 0.95 was obtained for all specimens. All specimens were isotropically consolidated to their desired consolidation pressure before shearing, volumetric changes during saturation and consolidation were determined by measuring the quantity of water by a calibrated burette and corrected to axial stress.

3. Test Equipment

The unsaturated tests were conducted using a Bishop-Wesley hydraulic triaxial cell, modified for testing unsaturated soils, and the maximum confining pressure can be measured up to 1,000 kPa. During saturation and consolidation, the corrections for triaxial cell cylinder and the specimen area were obtained from measured volume changes.

The total measuring equipment which measured axial displacement, cell pressure, load, pore water pressure, air pressure transducer and volume change were connected to computer through data acquisition system. The modified triaxial cell is given in Fig. 2.

The control of suction was made by axis translation technique used generally for prevention of cavitation (Hilf, 1956). In the triaxial cell, each specimens were allowed to reach equilibrium under an applied net confining stress ($\sigma_3 - u_a$) of 200 kPa. Undrained and drained triaxial compression tests were sheared at a strain rate of 0.05 mm/min and 0.005 mm/min, respectively. The suction controlled tests were carried out at matric suction of 0, 2, 5, 10 and

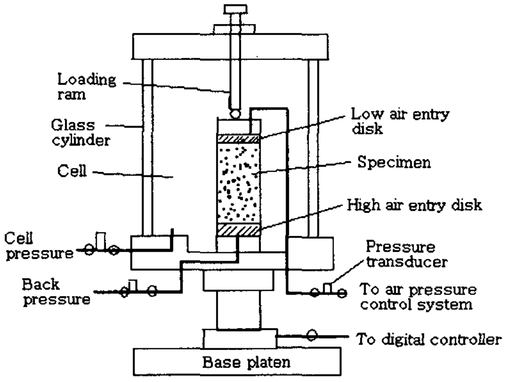

Fig. 2 Modified triaxial cell

25 kPa for undrained triaxial compression tests, and 0, 2, 5, 10 and 15 kPa for drained triaxial compression tests. The specimens were sheared to axial strains of about 20% to obtain steady state conditions.

Ⅳ. Results and Discussion

1. Stress-Strain Behaviour with Matric Suction

Fig. 3 shows undrained stress-strain behaviour with suction of 0, 2, 5, 10 and 25 kPa at net confining pressure of 200 kPa.

When specimens were tested in a saturated state and at a suction of 2 kPa, the deviator stress achieved an initial peak after which it declines to a minimum value and it appeared strain softening as deviator strain increases. But, when specimens were tested at higher values of suction, it appeared strain hardening as suction increases.

The basic mechanism for static liquefaction behaviour is related to the interaction between the silt grains and the larger sand grains. During the compression and shearing, the silt grains slide into the void spaces, this would cause a volumetric contraction at initial condition and a greater potential for liquefaction under undrained conditions. This fact may explain why natural alluvial field deposits of silty sands easily liquefy at higher relative densities than clean sands in the laboratory.

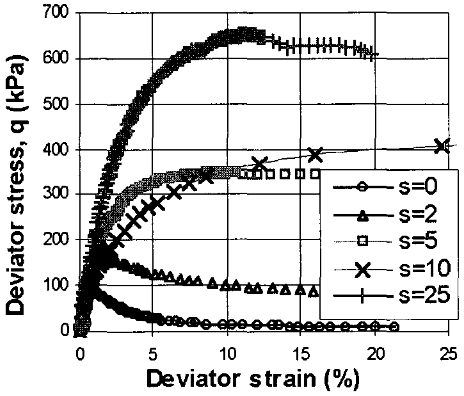

Fig. 3 Undrained stress-strain behaviour with matric suction

Ho et al. (1982) reported that the shear strength was increased as suction increases at constant net confining pressure, the failure envelope line was constant to both saturated and unsaturated states, and the effect of suction became different by internal friction angle(ϕ^b). Loret et al. (2002) reported that the elastic-plastic stress-strain relationship in unsaturated soils was appeared softening in undrained condition and hardening in drained condition. The deviator stress in drained condition was larger than undrained condition and undrained stress path was approximately the same with saturated soils. Cunningham et al. (2003) reported that the deviator stress increases as suction increases, but it was a little decreased at suction over 900 kPa.

2. Liquefaction Behaviour with Matric Suction

To investigate the air and water flow through inner unsaturated soils, it is needed to consider the stress-strain relationship with the variation of suction. When the degree of saturation decreases and the suction increases, the permeability coefficient decreases and shear strength increases, resulting in continuously increasing the amount of air.

Fig. 4 shows the relationship between the mean effective stress [$p' = (\sigma_1' + 2\sigma_3')/3$] and deviator stress ($q=\sigma_1-\sigma_3$) with variation of suction.

The undrained effective stress path appeared a drop after an initial peak and complete static liquefaction occurred at a saturated state (s=0) and at a suction of 2 kPa. But, when specimens were tested at higher values of suction, there was continuously increasing deviator stress caused by increased dilation. Also, the stress paths exhibited characteristic of increasing stability or resistance against liquefaction as the suction increased. As the suction increases, the undrained effective stress path approaches the drained stress path.

In the field, real soils are inevitably in an unsaturated state. It is expected that this study would provide a better understanding and analysis of the liquefaction behaviour of existing soil structures and in general, the behaviour of unsaturated soils. Also, this would undoubtedly assist in the development of more precise and simple modeling of any current soil structure.

Wheeler et al. (1995) identified that the slope of the critical state line and cohesion for an unsaturated clay were a function of suction, most of unsaturated shear strength approached to the critical state line (CSL) in saturation state, confirming the uniqueness of the CSL for both saturated and unsaturated states in the effective stress concept. Rampino et al. (2000) reported that the compressibility and shear strength of compacted silty sand were under the strong influence of suction, the mechanical properties of the soil improved according to an exponential law.

3. Prediction of Deviator Stress with Matric Suction

Fig. 5 shows the results of the predicted deviator stress carried out on the undrained (CU) and drained test (CD) using effective stress concept at the mean effective stress (p')-maximum deviator stress (q) plane with suction, and relate the results to critical state line (CSL) at saturation state.

The maximum deviator stress by the undrained tests (CU) in unsaturated state approaches to the critical state line, defined using the saturated test

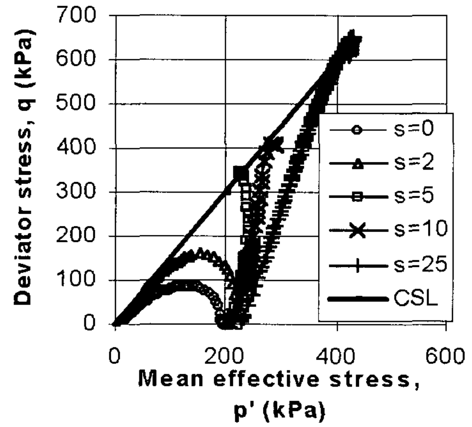

Fig. 4 Undrained effective stress path with variation of suction

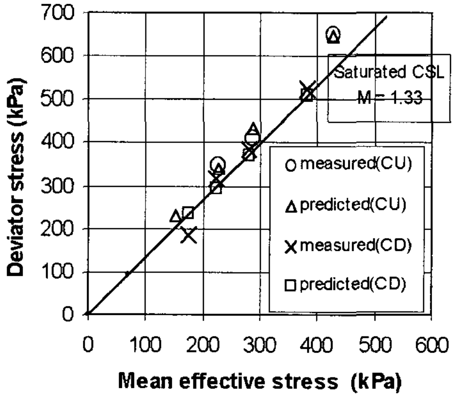

Fig. 5 Evolution of critical state line with suction in $p'-q$ plane

data.

However, some points are slightly above the line, suggesting an increase in the maximum deviator stress with suction. Also, the predicted and measured maximum deviator stress for unsaturated soils using the suggested equation and the unsaturated parameters showed generally in good agreement as matric suction increases.

The suggested equation may be only applied which the net confining pressure held constant during the shearing process because a change in the initial pore water pressure and air pressure in the undrained state is different to a change in failure state. Nevertheless, the results of the predicted deviator stress were in good agreement with the measured value and approached to the critical state line at saturated state.

Also, the deviator stress by the drained tests (CD) approaches to the critical state line (CSL), the predicted and measured maximum deviator stress for unsaturated soils using the saturated parameters showed generally in good agreement as matric suction increases. Therefore, it is clear that a unique critical state line (CSL) exists for both saturated and unsaturated states in the effective stress concept.

Fig. 6 shows the results of the predicted and measured deviator stress in the undrained (CU) and drained test (CD) on different values of suction with net confining pressure of 200 kPa.

As can be observed from Fig. 6, the predicted and measured maximum deviator stress for undrained (CU) and drained tests (CD) using the unsaturated and saturated parameters showed generally in good agreement as matric suction increases, although the predicted suction at small suction (CD) appears to the grater than measured value. The deviator stress increase is nonlinear as matric suction increases.

Fig. 7 presents the relationship between the pore water pressure and air pressure at different values of suction for unsaturated tests (CU).

The pore water and air pressure rapidly increases from initial strain as soon as shearing commences at suction of 2, 5 and 10 kPa, and it was appeared decreases as strain increases at suction of 25 kPa. Also, it is observed that the change of the pore water and air pressure

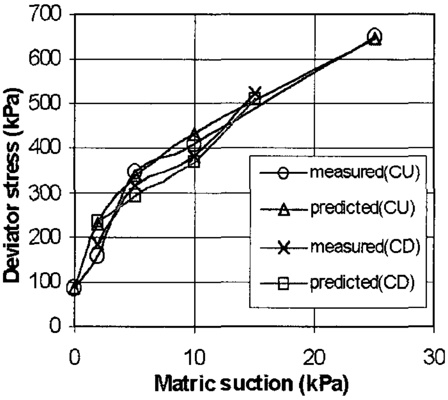

Fig. 6 Comparison between predicted and measured deviator stress in CU and CD tests.

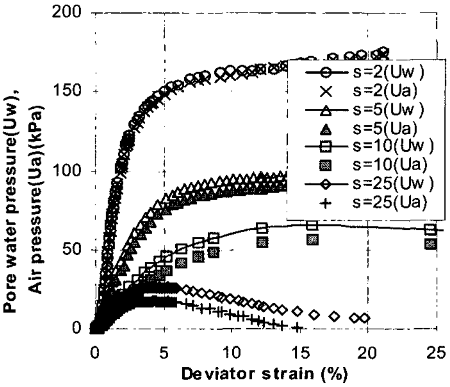

Fig. 7 Comparison between the pore water pressure and air pressure with suction (CU)

increases as the suction increases.

Fredlund et al. (1985a) reported that the internal friction angle in unsaturated soil was always appeared less than or the same value the internal friction angle in saturated soil. Gan et al. (1988) and Rassam et al. (1999) reported that the failure envelope line in relation to suction and shear strength increased linearly up to air entry value as increasing suction, it appeared nonlinearly increasing the degree of unsaturation beyond that.

Khalili et al. (2004) investigated the shear strength in unsaturated soils using effective stress concept, and the uniqueness of the critical state line(CSL) for both saturated and unsaturated soils in the deviatoric stress- effective mean stress plane existed.

V. Conclusions

This study compared and analysed experimentally the liquefaction behaviour of silty sand with matric suction and verified the application of predicted deviator stress by using the effective stress concept. The following results were obtained:

1. The deviator stress appeared strain softening at suction of 0 kPa and 2 kPa, and there was continuously increasing deviator stress and it appeared strain hardening as suction increases.
2. The undrained effective stress path appeared to drop after an initial peak, and complete static liquefaction only occurred at a suction of 0 kPa and 2 kPa. The results also showed that the stress paths were exhibited characteristic of increasing stability or resistance against liquefaction as the suction increases.
3. The predicted and measured maximum deviator stress for undrained (CU) and drained tests (CD) using the effective stress concept showed generally in good agreement and the deviator stress increase was nonlinear as matric suction increases. It was shown that the critical state line (CSL) was unique in deviator stress-effective mean stress plane for both saturated and unsaturated state of soil. It is considered that this has a significant simplifying effect on the constitutive modeling of unsaturated soils.

Acknowledgements

This study has been done during a visit to the University of New South Wales (UNSW) in Sydney. Special acknowledgements are given to Professor Nasser Khalili and Dr. Eduardo Rojas for their assistance in the laboratory.

References

1. Bishop, A. W., 1959, The principle of effective stress. Teknish Ukeblad, 106(39): 1-5. quoted *J. of the Soil Mech. and Found. Div. ASCE*, 93(SM2): 125-148.
2. Bishop, A. W. and G. E. Blight, 1963, Some aspects of effective stress in saturated and unsaturated soils, *Geotechinque*, 13(3): 177-197.
3. Blight, G. E., 1967, Effective stress evaluation for unsaturated soils, *J. of the Soil Mech. and Found. Div. ASCE*, 93(SM2): 125-148.
4. Cunningham, M. R., A. M Ridley., K. Dineen. and J. B. Burland, 2003, The mechanical behaviour of a reconstituted unsaturated silty clay, *Geotechinque*. 53(2): 183-194.
5. Estabragh, A. R., A. A. Javad, and J. C. Boot, 2004, Effect of compaction pressure

on consolidation behaviour of unsaturated silty soil, *Can. Geotech. J.* 41(3): 540–550.

6. Fredlund. D. G. and N. R. Morgenstern, 1977, Stress state variables for unsaturated soils, *J. of Geotech. Engrg. ASCE*, 103(GT5): 447–466.
7. Fredlund. D. G., N. R. Morgenstern, and R. A. Widger, 1978, The shear strength of unsaturated soils, *Can. Geotech. J.*, 15(3): 313–321.
8. Fredlund. D. G., 1985a, Theory formulation and application for volume change and shear strength problems in unsaturated soils, *In Proc. 11th Int. Conf. Soil Mech. Found. Engrg.(San Francisco)*.
9. Gan, J. K. M., D. G. Fredlund, and H. Rahardjo, 1988, Determination of the shear strength parameters of an unsaturated soil using the direct shear test, *Can. Geotech. J.*, 25(3): 500–510.
10. Ho, Y. F. and D. G. Fredlund, 1982, A muti–stage triaxial test for unsaturated soils, *ASTM, Geotech. Testing J.*, 5(2): 18–25.
11. Khalili, N. and M. H. Khabbaz, 1998, A unique relationship for χ for the determination of the shear strength of unsaturated soils, *Geotechinque*, 48(5): 681–687.
12. Khalili, N., F. Geiser, and G. E. Blight, 2004, Effective stress in unsaturated soils: Review with new evidence, *International J. of Geomechanics, ASCE*, 4(2):115:126.
13. Lade, P. V. and J. A. Yamamuro, 1997, Effects of nonplastic fines on static liquefaction of sands, *Can. Geotech. J.*, 34(6): 918–928.
14. Lee, D. W., 2006, Liquefaction Behaviour of saturated silty sand under monotonic loading conditions, *J. of the Korean Society of Agricultural Engineers*, 48(4): 67–74
15. Loret, B. and N. Khalili, 2002, An effective stress elastic–plastic model for unsaturated porous media, *Mechanics of Materials*, 34: 97–116.
16. Lu, N. and D. V. Griffiths, 2004, Profiles of steady–state suction stress in unsaturated soils, *J. of Geotech. and Geoenviron. Engrg. ASCE*, 130(10): 1063–1076.
17. Rassam, D. W. and D. J. Williams, 1999, A relationship describing the shear strength of unsaturated soil, *Can. Geotech. J.*, 36(2): 363–368.
18. Rampino, C., C. Mancuso, and F. Vinale, 2000, Experimental behaviour and modelling of an unsaturated compacted soil, *Can. Geotech. J.*, 37(4): 748–763
19. Vaid, Y. P. and J. Thamas, 1995, Liquefaction and postliquefaction behavior of sand, *Journal of Geotechnical Engineering. ASCE*, 121(2): 163 –173.
20. Wheeler, S. J. and V. S. Ivakumar, 1995, An elasto–plastic critical state framework for unsaturated soils, *Geotechinque*, 45: 35–53.
21. Yamamuro, J. A. and P. V. Lade, 1997, Static liquefaction of very loose sands, *Can. Geotech. J.*, 34(6): 905–917
22. Yamamuro, J. A. and P. V. Lade, 1998, Steady–state concepts and static liquefaction of silty sands, *J. of Geotechnical and Geoenvironmental Engineering. ASCE*, 124(9): 868–877.