

# Quality Discrimination on the Internet under Congestion Effects and Strategic Behavior by Users

혼잡효과가 있는 경우의 인터넷 백본서비스의 품질차별화 전략

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## 국문요약

이 논문은 인터넷 백본서비스 품질에 대해 서로 다른 반응도를 가진 사용자가 있는 경우, 백본을 보유한 독점적 기업은 자사의 서비스에 대해 가격을 어떻게 차별화하는가를 분석하고 있다. 특별히, 인터넷 사용에 있어 혼잡이 있는 경우, 비선형적인 가격책정이 어떻게 진행되는 것인가에 초점을 맞추고 있다. 인터넷 혼잡효과를 고려할 경우, 제공되는 서비스 품질은 특수한 상황 하에서 망사용자의 전략적 형태와 인터넷 상에 제공되는 어플리케이션의 특성과 무관하게 사회적 최적으로 제공할 수 있음을 보여주고 있다. 또한 사회적 최적품질의 서비스가 제공되지 않는 경우에 있어서 파라미터의 조건들을 제시하고 있다.

**핵심어** : 인터넷백본서비스, 혼잡, 품질차별, 전략적 행동

## Abstract

This paper is related to the optimal pricing of Internet Backbone Services that are differentiated according to the size of the bandwidth required for applications with different quality sensitivities. It discusses IBP (Internet Backbone Provider) optimal nonlinear pricing process in the presence of a congestion externality in addition to strategic behavior by users. In this scenario, the quality of service provided may reach its socially optimal level irrespective of strategic behavior by users and may be independent of the characteristics of the applications delivered over the Internet.

**Key words:** Internet Backbone Service, congestion, quality differentiation, strategic behavior

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## I . Introduction

At present, each different communication service is provided by a distinct type of network, such as a telephone network, a private network, a data network, etc. However, various types of services can be provided through one public computer network, as it is possible to convert, transmit, and receive any type of data in digital form, due to the remarkable development of computer network functions. In the near future, it is expected that different service qualities, which are currently differentiated with respect to bandwidth requirements and provided separately, will be provided on one IP based network (for example, Internet). However, the network provider must consider the pricing of congestible resources since different service qualities need different network capabilities.

Shenker (1994) addressed the integration of services on the Internet and the resulting impact on pricing policies, and showed that the Internet, in order to be efficient, must employ per-user, quality-of-service-sensitive, and usage-based pricing policies. Cocchi, Estrin, Shenker, and Zhang (1993) used simulation to analyze user incentives provided under a network of multi-class services. They showed that it is necessary to have a fee system that discriminates according to the class of service, in order to obtain desirable results for every such class, and that it is possible that all tested users may be satisfied with such a billing method. That is, for a user of a less performance-sensitive application, the performance loss resulting from the use of the lower quality service may be offset by the monetary gain obtained from paying less similarly, for a user of a more performance-sensitive application, the performance gain resulting from the use of the higher class of service may offset the monetary loss from paying more for the service. Thus, providing a variety of QoS (quality of service) classes plays a role in the optimal allocation of network resources among different types of users.

However, the studies mentioned above introduced usage-based pricing and did not consider monopoly Internet backbone market, where the price discrimination can

be practiced. The Internet market is dominated by a few firms in both the local and global access network. Regarding local access, the "last mile into homes," the local telephone lines and the cable-TV lines are alternatives for private users (Clark 1999). Obviously, local access has to be offered locally. Due to their dominant position, the local access providers (LAPs) are typically subject to regulation of price and quality for local access as an input component. As to global access network, a few U.S. IBPs (Internet Backbone Providers) provide connection to the global access to the regional ISPs all over the world. Access to the top-level of global infrastructure is controlled by U.S. firms such as MCI WorldCom, Sprint, Genuity (formerly GTE), and AT&T. Even if no IBP separately is in position to use market power, a group of co-operating IBPs may be in position to do so (Cremer et al; Milgrom et al, 2000). Therefore, we assume one IBP provides global access and makes use of nonlinear tariffs.

On the other hand, since the internet is rather new, there are relatively few studies dealing with both multi-class service provision and congestion effects. Inspired by Mackie-Mason and Varian (1994, 1995a, 1995b), there exist some analyses of the congestion problem in the Internet and price setting. It has been shown that setting the price according to marginal congestion expenses is the most efficient means of allocating network resources.

Most studies dealing with congestion problem and Internet pricing have been concerned with user-based pricing. However, under the two-part, or non-linear, tariff, consumers may no longer reveal the true nature of their application requirements unlikely as in user-based pricing system.

As Shenker (1994) pointed out, clients (users) must request the appropriate service for their application in order for efficiency to be achieved. If the network provider has market power, he would like to extract all consumer surplus by discriminating via price. It is possible that a user with high-quality service requirements declares that he/she uses a less sensitive application, for strategic reasons. This involves a false declaration by the user of his type, as he uses the lower-quality service in order to save money, despite the fact that he would actually be willing to pay more for the higher-quality service. For details, see

Maskin and Riley (1984) and Srinagesh and Bradburd (1989). For example, suppose that there are two types of applications that have different sensitivities with respect to service quality. The user who employs a more sensitive application and thus has a higher demand for quality has a higher marginal and total utility from quality than does a user who employs a less sensitive application and thus has a lower demand for service quality. In this case, it is possible to increase profit, in contrast to the case of uniform quality provision, by providing a different quality of service to each consumer. Furthermore, assume that the monopolist charges the total amount that each user is willing to pay for the quality provided. In this case, an individual with high demand, as a result of monetary incentives, may attempt to increase his net surplus by selecting the lower quality of service intended for the low-demand consumer. Such results are similar to those derived in the case of the CATV service of the U.S., which is divided into a basic service and a high-quality service. The difference between this case and the case of Internet quality provision is that the service quality is related to the number of available channels in the former, while it is related to the number of packets actually used in the latter. The larger the size of the bandwidth used, the greater the service quality becomes. The selection of service quality by any one user (application) has an affect on other users (applications). If higher-quality service is selected by any one user, the congestion of the entire network becomes greater as a result. Economists express this in terms of a "congestion externality," or a "tragedy of the commons." Sometimes the Internet becomes congested, and there is simply too much traffic for the routers and lines to handle. At present, the only two ways the Internet can deal with congestion are to drop packets, so that some information must be re-sent by the application, or to delay traffic. Considering this network congestion effect, the resulting quality level of service may be different to that in previously examined self-selection models See, for example, Chiang and Spatt (1982), Mussa and Rosen (1979), Cooper (1984), Maskin and Riley (1984), Matthews and Moore (1987), Srinagesh and Bradburd (1989), and Srinagesh, Bradburd and Koo (1992), among others.. Strategic action by users may have different affects on the pricing and quality of service in the presence of the congestion effect.

This paper presents a second-degree price-discrimination model with a congestion effect and discusses the way in which the quality of service in this context differs from that in previously examined self-selection models, as well as from the socially optimal level. Section 2 presents the basic model. Section 3 derives the procedure by which a monopolist provides multi-quality network services and reviews its implications. Section 4 concludes.

## II. The Basic Model

Service quality is related to how many packets are transmitted per hour. Quality intensive services, such as videoconferences, require a very large number of packets to be transmitted per hour, as compared to the packet transmission requirements for a lower quality service, such as electronic mail. For the purposes of this model, the services provided are divided into two types: a high-quality service, which requires a greater number of packets per hour, and a low-quality service. Generally, high-quality services, such as visual conferences, require a considerable number of packets, while low-quality services, such as electronic mail, require a smaller number of packets. Furthermore, it is assumed that there are two types of users who are grouped according to the quality demand resulting from the sensitivity of their applications: users with high demand (for example, users of video conferences) and users with low demand (for example, users of E-mail). The user types are classified in terms of  $\theta_i (i=1,2)$ , where  $\theta_2 > \theta_1$ . The number of users of type  $\theta_i$  is  $n_i$ . There may be number of types more than three or continuous types of consumer, but we assume two types consumer for explanatory convenience. We assume that  $n_i$  is sufficiently large that no type of user is excluded. If the number of high demand type is sufficiently, the monopolist may exclude the low-types to make more profits. This makes our problem uninteresting. These network services are provided in a monopoly environment, and the network provider charges the aggregation price for each packet required for the provided service. The menu  $(q_1, p_1)$  is provided for the

low-demand user, while the menu  $(q_2, p_2)$  is provided for the high-demand user. (Here,  $q$  is the quantity of packets transmitted per unit of time, and  $p$  is the price.) The monopolist has incomplete information about consumers' identity. This means that the monopolist is not able to force consumers to choose the options in the way he desires.

The following net user surplus of type  $\theta_j$  user is obtained if the menu  $(q_j, p_j)$  is selected by type  $\theta_j$ :

$$U_j(q_j, p_j) = \theta_j (V(q_j) - a(n_1 q_1 + n_2 q_2)) - p_j \quad (j = 1, 2 \quad n_i > 0) \quad (1)$$

, where  $V$  is the utility of the user of a given service quality ( $V' > 0, V'' \leq 0$ ), and a high-demand user gets a greater utility (both marginal and total) from a given level of quality than does a low-demand user. The parameter  $a (\geq 0)$  is the marginal congestion cost incurred when many users use the Internet at the same time. Therefore, equation (1) means that high demand users are characterized by higher congestion costs.

The profit maximization problem of a monopolist is as follows: The operator naturally incurs the cost of securing facility capacity, but this cost is not taken into consideration, for the sake of convenience, as it does not affect the results of the analysis.

$$\max \Pi = \sum_{i=1}^2 n_i p_i \quad (2)$$

subject to

$$\theta_i (V(q_i) - a(n_1 q_1 + n_2 q_2)) - p_i \geq 0 \quad (2-1)$$

$$\theta_i(V(q_i) - a(n_1q_1 + n_2q_2)) - p_i \geq \theta_i(V(q_j) - a(n_1 + n_2)q_j) - p_j \quad (2-2)$$

$$\theta_i(V(q_i) - a(n_1q_1 + n_2q_2)) - p_i \geq \theta_i(V(q_j) - a(n_1 + n_2)q_i) - p_j \quad i, j = 1, 2 \quad i \neq j \quad (2-3)$$

,where equations (2-1) are participation constraints that ensure that it is more profitable for the user to use the Internet than not to do so. Equations (2-2) and (2-3) are incentive compatibility constraints that ensure that it is more advantageous for the user of each type to select his type-appropriate menu than to select that of the other type. If any one user selects the menu intended for his/her own type, that user should be guaranteed his maximum payoff, regardless of the quality level or menu selected by other users of the same or different types. When a user belonging to a specific type selects a specific quality, all cases of mixing and selecting of  $q_1, q_2$  by the  $n_1 + n_2 - 1$  other users should be taken into consideration. However, in view of the fact that the considered case is either one of  $q_1 < q_2$  or  $q_1 > q_2$ , the remaining equations, which are not taken into consideration in this model, will be automatically satisfied by the above equation when only the case of same-quality selection is considered. Therefore, the second item on the right side of the incentive constraint is  $a(n_1 + n_2)q_i$ . In addition, there are four incentive constraints, two with respect to  $q_1 < q_2$ , and two with respect to  $q_1 > q_2$ .

The next analysis comprises the benchmark case of the socially optimal outcome. The socially optimal network quality is later used as a reference by which to evaluate the monopoly outcome. Assume that two types of quality are provided throughout the society. Then, the social welfare function is given by:

$$W = \sum_{i=1}^2 n_i \theta_i \left( V(q_i) - a \left( \sum_{j=1}^2 n_j q_j \right) \right) \quad (3)$$

If let the quality of maximizing the above social welfare denoted by  $q_i^s$ , then the optimal solution is as follows:

$$\theta_i (V'(q_i^s) - a(n_1 + n_2)) = 0 \quad i = 1, 2 \quad (4)$$

It can be seen from equation (4) that the socially optimal quality is determined at the level at which the marginal utility of the service quality is the same as the marginal congestion cost. Also, the quality of service is independent of the type of user because the congestion effects are proportional to  $\theta_i$  (that is,  $q_i^s = q^s$ ). The marginal congestion cost is the sum of two marginal congestion effects experienced by users of type  $\theta_1$  and type  $\theta_2$ . This is similar to the general problem of sharing external public resources it is also similar to the results derived by Mackie-Mason and Varian (1994).

### III. Determination of the optimal quality of service and its implications

In solving the problem of the monopolist, some constraint equations are redundant, according to the relative size of  $q_i$ . To show this, the following equation can be derived from the two incentive constraints (2-2) and (2-3):

$$(\theta_2 - \theta_1)(V(q_2) - V(q_1)) + an_1(\theta_2 + \theta_1)(q_2 - q_1) \geq 0 \quad (5)$$

It can be seen that equation (5) implies that  $q_2 \geq q_1$ . If  $q_1 > q_2$ , equation (5) is not satisfied, since the right side of equation (5) is always negative. Therefore, in order for equation (5) to be satisfied, it must be that  $q_1 \leq q_2$ , and that the first constraint of equations of (2-2) and the second constraint of (2-3) can be ignored, as they are redundant. In view of this, the profit maximization problem of the monopolist may be reformulated as follows:



$$\max \Pi = \sum_{i=1}^2 n_i p_i \quad (6)$$

subject to

$$\theta_i (V(q_i) - a(n_1 q_1 + n_2 q_2)) - p_i \geq 0 \quad (6-1)$$

$$\theta_i (V(q_i) - a(n_1 q_1 + n_2 q_2)) - p_i \geq \theta_j (V(q_j) - a(n_1 q_1 + n_2 q_1)) - p_j \quad (6-2)$$

$$q_2 \geq q_1 \quad i, j = 1, 2 \quad i \neq j \quad (6-3)$$

From equation (6-2), the following equation can be derived.

$$(\theta_2 - \theta_1)((V(q_2) - V(q_1)) - a n_2 (\theta_2 + \theta_1)(q_2 - q_1)) \geq 0 \quad (7)$$

It can be seen that the first constraint of (6-1) and the second constraint of equations (6-2) are binding if the second constraint of equations (6-1) and the first constraint of equations (6-2) are ignored. Then, the following price schedule can be obtained:

$$\begin{aligned} P_1 &= \theta_1 (V(q_1) - a(n_1 q_1 + n_2 q_2)) \\ P_2 &= \theta_2 (V(q_2) - a(n_1 q_1 + n_2 q_2)) - (\theta_2 - \theta_1) V(q_1) + a(n_1 (\theta_2 - \theta_1) q_1 - n_2 (\theta_1 q_2 - \theta_2 q_1)) \end{aligned}$$

Substituting these into the ignored constraints, the problem of profit maximization can be reformulated as follows: It can be easily seen that  $P_2 \geq P_1$  under constraints (8-1) and (8-2).

$$\max \Pi = n_1 \{ \theta_1 (V(q_1) - a(n_1 q_1 + n_2 q_2)) \} + n_2 \left\{ \begin{array}{l} \theta_2 (V(q_2) - a(n_1 q_1 + n_2 q_2)) - (\theta_2 - \theta_1) V(q_1) + \\ a(n_1 (\theta_2 - \theta_1) q_1 - n_2 (\theta_1 q_2 - \theta_2 q_1)) \end{array} \right\} \quad (8)$$

subject to

$$q_2 \geq q_1 \quad (8-1)$$

$$(\theta_2 - \theta_1)(V(q_2) - V(q_1)) - an_2(\theta_2 + \theta_1)(q_2 - q_1) \geq 0 \quad (8-2)$$

Note that the ignored constraints (the second constraint of equations (6-1) and the first constraint of equations (6-2)) are automatically satisfied by equations (6-3) and (7). Assuming an interior solution and ignoring constraints (8-1) and (8-2) for the time being, the first-order conditions for the above problem are given by:

$$n_1 \{ \theta_1 (V'(q_1) - a(n_1 + n_2)) \} - n_2 \{ (\theta_2 - \theta_1) V'(q_1) - a(n_1(\theta_2 - \theta_1) + n_2\theta_2) \} = 0 \quad (9)$$

$$n_2 (\theta_2 (V'(q_2) - a(n_1 + n_2)) - an_2\theta_1) = 0 \quad (10)$$

It can be seen from equations (9) and (10) that the constraints (8-1) and (8-2), which are ignored for the moment, are not automatically satisfied, as they depend on the size of the congestion effect  $a$ . It can also be seen that  $q_1 < q^s, q_2 = q^s$  from equations (9) and (10), if  $a = 0$ , and, therefore, that equations (8-1) and (8-2) are satisfied automatically. However, the form of the optimal solution is complicated, because the value of  $a$ , if  $a > 0$ , will affect the optimal level of quality. Furthermore, it can be seen intuitively that  $q_1 > q^s, q_2 < q^s$ , from equations (9) and (10), if  $a$  is sufficiently large. In addition, the larger the value of  $a$  is, the closer the value of  $q_1$  is to the value of  $q_2$ . This problem is analyzed mathematically as shown below. The method used in the present analysis is that we first solve the problem with constraint (8-1) ignored, and then check whether the ignored constraint is actually satisfied.

Then, we get following lemma 1:

Lemma 1

Case I:  $\frac{\theta_1}{\theta_2} \leq \frac{n_1}{n_1 + n_2}$

i) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} < a$ , then equation (8-1) should be binding and so  $q_1 = q_2 = q^s$

ii) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} \leq a \leq \frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)}$ , then equation (8-1) should be binding, and so  $q_1 = q_2 = q^s$

iii) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} > a$ , then  $q_1 < q^s, q_2 < q^s$  or  $q_1 = q_2 = q^s$  where the ignored constraint (8-1) is not satisfied.

Case II:  $\frac{\theta_1}{\theta_2} > \frac{n_1}{n_1 + n_2}$

i) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a$ , then  $q_1 = q_2 = q^s$

ii) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} \leq a \leq \frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)}$ , then  $q_1 < q^s, q_2 < q^s$  or  $q_1 = q_2 = q^s$  where the ignored constraint (8-1) is not satisfied.

iii) If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} > a$ , then  $q_1 < q^s, q_2 < q^s$  or  $q_1 = q_2 = q^s$  where the ignored constraint (8-1) is not satisfied.

**Proof)** Refer to Appendix 1.

Lemma 1 characterizes the optimal solution of quality depending on congestion effects. As previously mentioned, the value of  $q_1$  approaches the value of  $q_2$  gradually as the congestion effect becomes larger. However, from Lemma 1, it can be seen that the incentive of the high-demand user to obtain a surplus may disappear completely, due to the congestion effect resulting from the consumption of high-quality services, as the ignored constraint equation (8-1) is binding if this effect is very large. This effect is higher when the relative size of utility indicator

$\frac{\theta_1}{\theta_2}$  is smaller than relative number of low-user ( $\frac{n_1}{n_1 + n_2}$ ). It can also be seen that the level of service quality provided is lower than the socially optimal level when congestion effects are sufficiently low, irrespective of whether the user has a high or low demand.

Lemma 1 implies that the optimal quality of network service is not influenced by the ratio of the numbers of each specific type of user (i.e.,  $n_1/n_2$ ), but that it is influenced by the degree of congestion and  $n_2$  (the number of high-demand users). From Lemma 1, we get following proposition:

**Proposition 1**

*i)* In the case that congestion effects from common use exist but are not large, the monopolistic network operator provides a level of quality that is inferior to the socially optimal quality, for both types of users, because of service-quality discrimination. In this case, it can be shown that the constraint equation (8-1) is not binding.

*ii)* The same quality is provided to both high- and low-demand users when the congestion effect is sufficiently large.

*iii)* The possibility of offering same quality for both type of users is higher when

the relative size of utility indicator for low-user ( $\frac{\theta_1}{\theta_2}$ ) is smaller than relative number of low-user ( $\frac{n_1}{n_1 + n_2}$ ).

The proof of Proposition 1 is straightforward from Lemma 1. Part *i*) of Proposition 1 shows that the monopolist provides the high-demand user with less than his socially optimal quality; this result contrasts with the result of the standard self-selection models, in which the monopolist provides the socially optimal quality to the high-demand user. In the standard self-selection model, this latter result is obtained because the monopolist, who would like to obtain higher profits from the high-demand user, must ensure that the high-demand user receives a certain level of surplus in order to discourage him from strategic behavior. In contrast to that benchmark model, the additional congestion variable in this model changes the optimal quality of service provided to each type of user. The monopolist still wishes to discourage strategic behavior on the part of the high-demand user, but an additional gain results from the reduction of congestion that may be as great as the surplus received by the high-demand user who pretends to have low demand. This gain is affected by not only by the service quality provided to the low-demand user, but also by the service quality provided to the high-demand user. Furthermore, this additional gain increases as the quality provided to the high-demand user increases. When considering congestion effects, the monopolist should leave a high surplus to the high-demand user in order to prevent strategic behavior on his part. Therefore, the monopolist could increase its profit by providing a quality lower than the socially optimal level for the high-demand user.

Part *ii*) of Proposition 1 indicates that the monopolist may gain more by providing the same level of service to both types of user than by quality discriminating. This result differs from the standard conjecture that the monopolist gains more by providing different service qualities. The monopolist may gain more by providing the same quality to both types of users, if the surplus that must be

left to high-demand users in order to prevent strategic behavior is sufficiently large.

Part *iii*) of Proposition 1 implies that the monopolist should guarantee the high-demand user more surplus when the utility ratio ( $\frac{\theta_1}{\theta_2}$ ) is higher than number  $\frac{n_1}{n_1 + n_2}$  of low-demand user even if the congestion effects ( $a$ ) is smaller.

It can be seen from equation (8) that, if congestion effects are very large, the surplus left to high-demand users is also large, and that this surplus increases in the level of congestion. Therefore, if congestion effects are very large, the monopolist has no incentive to discriminate the quality. It can also be seen easily from equation (8) that, if congestion effects exist, the quality provided to high-demand users is less than that provided in the absence of congestion effects, and that the service quality provided to low-demand users is relatively better. In the case of high-demand users, as shown in equation (10), the additional factors that affect the level of service quality provided, as compared to the standard model without congestion effects, are the congestion effect ( $a$ ) and the utility indicator ( $\theta_i$ ), and the number of each type of users ( $n_i$ ). What differs from the case of the high-demand users is that such factors now work as incentives that raise the service quality provided to low-demand users, when the congestion effect is sufficiently large. If congestion effects exist, the monopolist can increase its profit resulting from the difference in the degree of congestion across quality levels by raising the quality it provides to the low-demand users. This is because, if the difference between the two service qualities provided is small, the surplus left to the high-demand user is greatly reduced.

In summary, the monopolist's incentives to lower the quality it provides to high-demand users and to raise the quality it provides to low-demand users should increase with the level of congestion until the two levels of quality eventually coincide. At this coincident level of quality, the degree of congestion is so great that it is more profitable for the monopolist to raise the quality it provides to low-demand users than to alter that of high-demand users.

#### IV. Concluding remarks

This paper analyzes the incentives of a monopolist network provider to offer multiple qualities of service and shows that the incentive to quality discriminate among users of differing demands by means of nonlinear tariffs varies according to the magnitude of congestion effects, which are frequently present when resources are publicly shared. Several previous studies have shown that, if a different price is offered for each service quality, according to the characteristics of the service provided, all users gain furthermore, the network operator gains. Other studies have demonstrated that, if price is determined by the marginal congestion cost when many users jointly use a network, resources are allocated efficiently. Moreover, this method of per-unit packet pricing is efficiently provided by the market.

The additional contributions of this study are twofold. The first main contribution concerns the derivation of the tariff structure and the strategic actions of the users. In the case that the monopolist uses its market power to impose a nonlinear tariff that varies according to service quality, users have incentives to select the menu that is inappropriate for their types. The second contribution is the finding that the service quality provided to each type of user is affected by the usage of the other party, as resources are shared in common. If both of the above aspects are taken into consideration, the monopolist may not discriminate the quality and may provide socially optimal quality. Therefore, it is possible that the monopolist prefers to provide the same quality of service to all types of users rather than use uniform pricing or priority pricing based on usage. This means that it may be desirable to employ the first-in-first-out (FIFO) packet-scheduling algorithm that is, the first packet to arrive at a network switch is the first one sent.

We assumed that there were two types of consumers. However, the number of types might be more than three, it may be interesting. We assumed only the monopoly market, and therefore the investigation for the competitive market will be valuable.

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## 정충영

KAIST에서 “자기선택요금제도와 비대칭적 정보하에 있는 다생산물 독점기업에의 응용에 관한 연구” 로 학위를 받았으며, ETRI에서 선임연구원으로 근무하다 현재 한남대학교 경영학과에서 재직중이다. 연구분야는 통신망간 상호접속 및 요금, 정보통신정책, e-business 등이다.



### 〈Appendix 1〉

$$\text{Case I: } \frac{\theta_1}{\theta_2} \leq \frac{n_1}{n_1 + n_2}$$

First, consider the optimal solution to the problem without either of the constraint equations (8-1) and (8-2), which are as follows:

$$q_1 < q^s, q_2 < q^s \text{ if } \frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} \geq a \quad (\text{A1-1})$$

$$q_1 > q^s, q_2 < q^s \text{ if } \frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a \quad (\text{A1-2})$$

If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} < a$ , then  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a$  and we get  $q_1 > q_2$  that violates the ignored constraint (8-1) from equation (A1-2). Therefore, if

$$\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} < a, \text{ the equation (8-1) must be binding and so, where}$$

$q_1 = q_2 = q^s$  the ignored constraint (8-2) is automatically satisfied.

Next, if  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} > a$ , then  $q_1 < q^s, q_2 < q^s$ , from equation (A1-1), and

it can be established that  $(\theta_2 - \theta_1)V'(q_2) > an_2(\theta_1 + \theta_2)$ . Therefore, the ignored constraint (8-2) is automatically satisfied, as follows:

$$\frac{an_2(\theta_1 + \theta_2)}{\theta_2 - \theta_1} < V'(q_2) \leq \frac{V(q_2) - V(q_1)}{q_2 - q_1}$$

In this case, the optimum solution is  $q_1 < q^s, q_2 < q^s$ .

Finally, if  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} \leq a \leq \frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)}$ , we get  $q_1 > q^s, q_2 < q^s$

that violate the ignored constraint (8-1) from equation (A1-2). Therefore, if

$\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a$ , the equation (8-1) must be binding and so

$q_1 = q_2 = q^s$ , where the ignored constraint (8-2) is automatically satisfied.

Case II:  $\frac{\theta_1}{\theta_2} > \frac{n_1}{n_1 + n_2}$

If  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a$ , we get  $\frac{(\theta_2 - \theta_1)V'(q_1)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} < a$  and solutions

$q_1 > q^s, q_2 < q^s$  from equation (A1-2) because  $V'' \leq 0$ .

However, because this equation violates the ignored constraint (8-1), the optimum solution is  $q_1 = q_2 = q^s$ , where the ignored constraint (8-2) is automatically satisfied.

Next, if  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} > a$ , then  $\frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)} > a$

and so  $q_1 < q^s, q_2 < q^s$ , from equation (A1-1). Meanwhile, it can be established

that  $(\theta_2 - \theta_1)V'(q_2) > an_2(\theta_1 + \theta_2)$ . Therefore, the ignored constraint (8-2) is automatically satisfied, as follows:

$$\frac{an_2(\theta_1 + \theta_2)}{\theta_2 - \theta_1} < V'(q_2) \leq \frac{V(q_2) - V(q_1)}{q_2 - q_1}$$

In this case, the optimum solution is  $q_1 < q^s, q_2 < q^s$  or  $q_1 = q_2 = q^s$  where the

ignored constraint (8-1) is not satisfied.

Finally,  $\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} \leq a \leq \frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)}$  can be solved by taking equation (8-2) into consideration, as follows:

$$n_1\theta_1(V'(q_1) - a(n_1 + n_2)) - n_2((\theta_2 - \theta_1)V'(q_1) - a(n_1(\theta_2 - \theta_1) + n_2\theta_2)) + \lambda(an_2(\theta_2 + \theta_1) - (\theta_2 - \theta_1)V'(q_1)) = 0$$

$$n_2\theta_2(V'(q_2) - a(n_1 + n_2)) + \lambda((\theta_2 - \theta_1)V'(q_2) - an_2(\theta_2 + \theta_1)) - an_2^2\theta_1 = 0$$

If  $\lambda > 0$ , then constraint (8-2) is binding. Therefore, it must be established that

$$\frac{(\theta_2 - \theta_1)V'(q_2)}{n_2(\theta_1 + \theta_2)} \leq a \leq \frac{(\theta_2 - \theta_1)V'(q_1)}{n_2(\theta_1 + \theta_2)} \quad \text{for any } q_1, q_2 \tag{A1-3}$$

because  $V'(q_2) \leq \frac{V(q_2) - V(q_1)}{q_2 - q_1} = \frac{an_2(\theta_1 + \theta_2)}{\theta_2 - \theta_1} \leq V'(q_1)$  from binding of constraint (8-2). However, if  $q_1 > q^s$ , equation (A1-3) contradicts

$$\frac{(\theta_2 - \theta_1)V'(q^s)}{n_2(\theta_1 + \theta_2)} \leq a. \quad \text{Therefore, optimal solution is } q_1 < q^s, q_2 < q^s \text{ or}$$

$q_1 = q_2 = q^s$  where the ignored constraint (8-1) is not satisfied.

If  $\lambda = 0$ , then optimal solution is  $q_1 < q^s, q_2 < q^s$ ,

because  $a \leq \frac{(\theta_2 - \theta_1)V'(q^s)}{(n_1(\theta_2 - \theta_1) + n_2\theta_2)}$  or  $q_1 = q_2 = q^s$  where the ignored constraint (8-1) is not satisfied. (End of proof)