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CHAIN DEPENDENCE AND STATIONARITY TEST FOR TRANSITION PROBABILITIES OF MARKOV CHAIN UNDER LOGISTIC REGRESSION MODEL

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ABSTRACT

To identify whether the sequence of observations follows a chain dependent process and whether the chain dependent or repeated observations follow a stationary process or not, alternative procedures are suggested in this paper. These test procedures are formulated on the basis of logistic regression model under the likelihood ratio test criterion and applied to the daily rainfall occurrence data of Bangladesh for selected stations. These test procedures indicate that the daily rainfall occurrences follow a chain dependent process, and the different types of transition probabilities and overall transition probabilities of Markov chain for the occurrences of rainfall follow a stationary process in the Mymensingh and Rajshahi areas, and non-stationary process in the Chittagong, Faridpur and Satkhira areas.

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1. INTRODUCTION

Markov chain provides probability models under stochastic process to describe different types of transition probabilities for chain or time dependent data. The logistic regression model is also a probabilistic model for analyzing binary data. By utilizing logistic regression model Muenz and Rubinstein (1985) developed different types of covariate dependent transition probabilities of Markov chain.

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To determine the chain dependence, order and stationary of transition probabilities of Markov chain model Goodman (1955), and Anderson and Goodman(1957) developed and suggested test procedures as score chi-square test statistic under the likelihood ratio criterion (Bartlett, 1951; Hoel, 1954). Further, Tong (1975), Gates and Tong (1976) and Katz (1981) developed and suggested the Akaike's Information Criterion (AIC) method using the minimum AIC estimate (MAICE) technique for identifying the order and stationarity of transition probabilities of Markov chain. However, Shibata (1976) and Schwarz (1978) showed that the AIC procedure provides an inconsistent estimator to estimate the order of auto-regressive processes and to determine the order of transition probabilities of Markov chain. In this context, Schwarz (1978) developed the Bayesian Information Criterion (BIC) method as an alternative to the AIC method for determining the order. Similarly, to determine the stationarity of transition probabilities of Markov chain, Schwarz suggested the minimum BIC estimate (MBICE) technique. Furthermore, Katz (1981) showed that the BIC method is not only consistent, but also asymptotically optimal. However, Sinha (1997) displayed that these test procedures may not be suitable for identifying the chain dependence, order and stationarity of different types of transition probabilities of Markov chain, and indicating that these test procedures are suitable for aggregate transition counts. Therefore, we require to develop another test procedure for identifying the chain dependence, order and stationarity of different types of transition probabilities for Markov chain as an alternative to the traditional test procedures.

The main objectives of this paper are:

(i) to develop the test procedure for testing whether the observations follow a chain dependent process or not by employing logistic regression model; and

(ii) to develop the test procedure for testing the stationarity of different types of transition probabilities, and overall transition probabilities of Markov chain for chain dependent observations employing logistic regression model.

As an application of the test procedures for different types of transition probabilities and overall transition probabilities of Markov chain, the data of daily rainfall occurrences for five selected stations of Bangladesh are used in this paper.

2. LOGISTIC REGRESSION MODELS FOR TRANSITION PROBABILITIES OF MARKOV CHAIN

Following the argument of Sinha (1997), to identify whether the sequence of observations follows a chain dependent process and a stationarity process for different types of transition probabilities of Markov chain, in this study we develop test procedures as an alternative to the traditional test procedures (Anderson and Goodman, 1957) under the likelihood ratio test criterion by employing logistic regression model (Cox, 1970; Muenz and Rubinstein, 1985). Procedures for testing the hypotheses concerning chain dependence and stationarity are developed following Cox (1970) and Islam (1994). To assess the effect of transition counts on transition probabilities the logistic regression model is employed.

2.1. Markov chain

Let us assume that $x_1, x_2, \ldots, x_n$ be a set of chain dependent repeated observations and $n_{jk}(t)$ be the first order transition count for these observations. The term $n_{jk}(t)$ indicates that an individual is in state j at time $t-1$ and in state k at time t. Similarly, $n_{ijk}(t)$ indicates the second order transition count. This transition implies that the number of individuals in state i at $t-2$, in state j at $t-1$ and in state k at t. Let $p_{jk}(t)$ and $p_{ijk}(t)$ be the first and second order transition probability matrices respectively, and for stationary chains these are denoted by $p_{jk}(t) = p_{jk}$ and $p_{ijk}(t) = p_{ijk}$ for all i, j, $k = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, T$. The ML estimates (Anderson and Goodman, 1957) of transition probabilities for the transition probability matrix ($t.p.m$) are:

$$\hat{p}_{jk} = \frac{n_{jk}}{n_{j\cdot}} = \frac{n_{jk}(t)}{n_j(t-1)} \quad \text{and} \quad \hat{p}_{ijk} = \frac{n_{ijk}}{n_{ij\cdot}} = \frac{n_{ijk}(t)}{n_{ij}(t-1)},$$

where $n_{j\cdot} = \sum_{k=1}^{m} n_{jk}$ and $n_{ij\cdot} = \sum_{k=1}^{m} n_{ijk}$. Therefore, the first and second order two-state transition probability matrices, p_{jk} and p_{ijk} $(i, j, k = 0, 1)$, respectively are defined as

$$\begin{array}{cc} & \text{Actual Day} \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \text{Previous Day} \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = p_{jk} \end{array} \tag{2.1}$$

and

$$\begin{array}{cc} & \text{Actual Day} \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \text{Previous Days} \begin{array}{c} 00 \\ 10 \\ 01 \\ 11 \end{array} & \begin{bmatrix} p_{000} & p_{001} \\ p_{100} & p_{101} \\ p_{010} & p_{011} \\ p_{110} & P_{111} \end{bmatrix} = p_{ijk}. \end{array} \tag{2.2}$$

Similarly, higher order transition probability matrix may be defined.

2.2. Test procedure under the likelihood ratio criterion for logistic regression

Islam (1994) has developed multistate survival models for transitions and reverse transitions for a proportional hazard model. In this study, we reformulate Islam's approach by using logistic regression model for describing different types of transition probabilities of Markov chain. Thus, to develop logistic regression model for Markov chain, we assume that for all m, n, $p = 0, 1$, where m, n and p indicate different states of the Markov chain. Here 0 and 1 indicate dry and wet days respectively. Then

$$f_{ml}(t) = \begin{cases} 1, \text{ if } m \to n(=1) \\ 0, \text{ otherwise} \end{cases}$$

for the first order Morkov model and

$$f_{mnl}(t) = \begin{cases} 1, \text{ if } m \to n \to p(=1) \\ 0, \text{ otherwise} \end{cases}$$

for the second order Markov model.

The terms $f_{m1}(t)$ and $f_{mn1}(t)$ are the transition counts for first and second order Markov model respectively. If a transition is made from state m to state $n(=1)$ and then a transition is made to state $p(=1)$, *i.e.*, $m \to n(=1)$ and $m \to n \to p(=1)$, then these are denoted by 1 and otherwise 0. The logistic regression model for transition of the type m to $n(=1)$ at time t for the first order Markov model is defined as

$$\lambda_1\{t, n(=1)|m, f_{ml}(t) = 1, X_{ml}(t)\} = \frac{\exp(\beta_{ml} + \sum_{z=1}^{Z} \beta_{ml(z)} X_{ml(z)})}{1 + \exp(\beta_{ml} + \sum_{z=1}^{Z} \beta_{ml(z)} X_{ml(z)})}. \tag{2.3}$$

Such a model for the transition of the type $m \to n \to p(=1)$ at time t for the second order Markov model becomes

$$\lambda_2\{t, p(=1)|n, m, f_{mnl}(t) = 1, X_{mnl}(t)\} = \frac{\exp(\beta_{mnl} + \sum_{z=1}^{Z} \beta_{mnl(z)} X_{mnl(z)})}{1 + \exp(\beta_{mnl} + \sum_{z=1}^{Z} \beta_{mnl(z)} X_{mnl(z)})}. \tag{2.4}$$

Here $z = 1, 2, \dots, Z$ and $\beta_{m1(z)}$ is the regression coefficient for covariate $X_{m1(z)}$ for the transition probabilities of the first order Markov model. The term $\beta_{mn1(z)}$ is the regression coefficient for covariate $X_{mn1(z)}$ for the transition probabilities of the second order Markov model. To identify whether the effects of covariates on the first and second order transition counts are equal or not, let us consider the null hypothesis

$$H_0 : \beta_{m1(z)} = \beta_{mn1(z)}.$$

Following Islam (1994), we may develop another model instead of Model (2.3) for testing the hypothesis. This model is constructed by the replacement of transition of type $m \to n \to p(=1)$ by the transition of type $m \to n(=1)$ at time t. This model is defined as

$$\lambda_3\{t, n(=1)|m, f_{mnl}(t) = 1, X_{ml}(t)\} = \frac{\exp(\beta_{ml} + \sum_{z=1}^{Z} \hat{\beta}_{mnl(z)} X_{ml(z)})}{1 + \exp(\beta_{ml} + \sum_{z=1}^{Z} \hat{\beta}_{mnl(z)} X_{ml(z)})}. \tag{2.5}$$

Here $\hat{\beta}_{mnl(z)}$ is the estimated regression parameter for the model (2.4). To test the null hypothesis, the likelihood ratio test statistic $\chi^2 = -2 \log \lambda$ with 1 degree of freedom (d.f.) is considered, where λ is the likelihood ratio which becomes

$$\lambda = \frac{L\left[\lambda_3\{t, n(=1)|m, f_{mnl}(t) = 1, X_{ml}(t) = 1\}\right]}{L\left[\lambda_1\{t, n(=1)|m, f_{ml}(t) = 1, X_{ml}(t) = 1\}\right]}.$$

On the basis of this logical extension, a test procedure is developed for testing the stationarity of different types of transition probabilities and also for testing the overall transition probabilities of Markov model for chain dependent observations.

2.3. Test for chain dependence

Let us consider a sequence of observations $x_1, x_2, \dots, x_n$ at time t ($t = 1, 2, \dots, T$). To identify whether the sequence of observations follow a chain dependent process or not, let us consider the null hypothesis that the observations do not follow a chain dependent process against the alternative that the observations follow a chain dependent process. In order to test the hypothesis we assume that sequence of observations follows the transition count n_{jk} $(j, k = 1, 2, \ldots, m)$ from j to k and the transition probabilities p_{jk} in which an individual is in state j at time $t-1$ and in state k at time t.

Let the transition probability matrix P consist of m identical rows, *i.e.* $P = (p_{jk})$, where $j, k = 1, 2, \ldots, m$. To estimate the transition probabilities p_{jk} of matrix P, Anderson and Goodman (1957) suggested the maximum likelihood estimation procedure. The MLE of transition probabilities is shown in Section 2.1. For testing the above hypothesis, we consider the two-state transition probability matrices (*t.p.m*) p_{jk} for all $j, k = 0, 1$. Based on this *t.p.m.*, Cox (1970) suggested that the process is completely random iff $p_{01} = p_{11}$, otherwise the process is non-random, *i.e.*, observations follow a chain dependent process. To test the null hypothesis, a test procedure is developed by employing logistic regression under the likelihood ratio criterion. For the development of this test statistic, we consider the following assumptions:

(i) the sequence of observations follows the logistic form; and

(ii) the immediately previous state of the initial state is non-random.

The likelihood function for the first order two-state transition probability matrix (2.1) is defined as

$$p_{00}^{n_{00}} p_{01}^{n_{01}} p_{10}^{n_{10}} p_{11}^{n_{11}}.$$

For testing the hypothesis, let us consider the functions (Cox, 1970)

$$p_{01} = \frac{\exp(\alpha)}{1+\exp(\alpha)} \quad \text{and} \quad p_{11} = \frac{\exp(\alpha+\Delta)}{1+\exp(\alpha+\Delta)}, \tag{2.6}$$

where α and Δ are the parameters. Employing these logistic forms, the above likelihood function is defined as

$$\frac{\exp(\alpha\, n_{.1} + \Delta\, n_{11})}{(1+\exp(\alpha))^{n_{0.}}(1+\exp(\alpha+\Delta)^{n_{1.}}}. \tag{2.7}$$

The maximum likelihood estimate of parameters α and Δ are given by

$$\hat{\alpha} = \log\left\{\frac{n_{.1} - n_{11}}{n_{0.} - n_{.1} + n_{11}}\right\} = \log\left\{\frac{n_{01}}{n_{00}}\right\}, \tag{2.8}$$

$$\hat{\Delta} = \log\left\{\frac{n_{11}}{n_{.1} - n_{11}}\right\} - \log\left\{\frac{n_{.1} - n_{11}}{n_{0.} - n_{.1} + n_{11}}\right\} = \log\left\{\frac{n_{11}}{n_{10}}\right\} - \log\left\{\frac{n_{01}}{n_{00}}\right\}, \tag{2.9}$$

where $n_{.1} = n_{01} + n_{11}$, $n_{.0} = n_{00} + n_{10}$, $n_{0.} = n_{00} + n_{01}$ and $n_{1.} = n_{10} + n_{11}$. Here n_{00}, n_{01}, n_{10} and n_{11} are the transition counts for transition of the types

$0 \to 0,\ 0 \to 1,\ 1 \to 0$ and $1 \to 1$ respectively. To justify the above null hypothesis under logistic regression model, we suppose that

$$H_0 : \Delta = 0 \text{ against } H_1 : \Delta \neq 0.$$

To test the hypothesis, the likelihood ratio test statistic $\chi^2 = -2\log\lambda$ is defined, where λ is likelihood ratio, *i.e.*,

$$\lambda = \frac{L(\hat{\alpha}, \Delta_0)}{L(\hat{\alpha}, \hat{\Delta})}, \tag{2.10}$$

where $\hat{\alpha}$ and $\hat{\Delta}$ are the ML estimate of α and Δ respectively. Taking the natural logarithm of λ, we obtain

$$\log\lambda = \log L(\hat{\alpha}, \Delta_0) - \log L(\hat{\alpha}, \hat{\Delta}),$$

where

$$L(\hat{\alpha}, \Delta_0) = \frac{\exp(\hat{\alpha}\, n_{.1})}{(1+\exp(\hat{\alpha}))^{n_{0.}}(1+\exp(\hat{\alpha}))^{n_{1.}}} = \frac{\exp(\hat{\alpha}\, n_{.1})}{(1+\exp(\hat{\alpha}))^{n_{0.}+n_{1.}}}$$

and

$$\log L(\hat{\alpha}, \Delta_0) = \hat{\alpha} n_{.1} - (n_{0.} + n_{1.}) \log(1+\exp(\hat{\alpha})).$$

Further

$$L(\hat{\alpha}, \hat{\Delta}) = \frac{\exp(\hat{\alpha}\, n_{.1} + \hat{\Delta}\, n_{11})}{(1+\exp(\hat{\alpha}))^{n_{0.}}(1+\exp(\hat{\alpha}+\hat{\Delta}))^{n_{1.}}}$$

and

$$\log L(\hat{\alpha}, \hat{\Delta}) = \hat{\alpha}\, n_{.1} + \hat{\Delta}\, n_{11} - n_{0.} \log(1 + \exp(\hat{\alpha})) + n_{1.} \log(1+\exp(\hat{\alpha}+\hat{\Delta})).$$

Therefore,

$$-2\log\lambda = 2\{n_{1.}\log(1+\exp(\hat{\alpha})) + \hat{\Delta} n_{11} - n_{1.}\log(1+\exp(\hat{\alpha}+\hat{\Delta}))\} \tag{2.11}$$

which is asymptotically distributed as χ^2 (Kendall and Stuart, 1973) with p (d.f.), where p is the number of parameters ($p = 1$).

2.4. Test for stationarity

To develop a procedure for testing the stationarity of chain dependent observations instead of existing test procedure (Anderson and Goodman, 1957), let us consider the transition matrix P which contains $n_{jk}(t)$ transition count for individuals in state j at time $t-1$ moving to state k at time t, for all $j,k = 1,2,\ldots,m$ and $t = 1,\ldots,T$. For this transition count $p_{jk}(t)$ indicates the transition probability for state k at time t given that state j at time $t-1$. To test the stationarity of chain dependent observations, we consider the null hypothesis that the different types of transition probabilities and overall transition probabilities of Markov chain are stationary, against the alternative that the transition probabilities are not stationary as shown below:

$$H_0 : p_{jk}(t) = p_{jk} \quad \text{against} \quad H_1 : p_{jk}(t) \neq p_{jk}.$$

The ML estimate of transition probabilities are shown in Section 2.1.

To test the null hypothesis, we consider logistic regression models for transition probabilities under the following assumptions:

(i) observations of chain dependent process follow the logistic form;

(ii) the individual $n_j(0)$ and $n_{jk}(1)$ are non-random; and

(ii) each row of transition probability matrix is independent.

Thus, for two-state transition probabilities let us suppose that

$$p_{j1} = \frac{\exp(z_{j1})}{1+\exp(z_{j1})} \tag{2.12}$$

and

$$p_{j1}(t_1) = \frac{\exp(z_{j1}(t_1))}{1+\exp(z_{j1}(t_1))}, \tag{2.13}$$

where $z_{j1} = \alpha_{j1} + \beta_{j1}\, n_{j1}(t)$ and $z_{j1}(t_1) = \alpha_{j1}(t_1) + \beta_{j1}(t_1)\, n_{j1}(t_1)$. Here $t = 1,2,\ldots,T$ and $t_1 = 1,2,\ldots,T_1$, the independent variables $n_{j1}(t)$ and $n_{j1}(t_1)$ are the transition counts of $k^{th}(=1)$ state of first order transition matrix at time t and t_1 respectively, α_{j1} and β_{j1}, and $\alpha_{jl}(t_1)$ and $\beta_{j1}(t_1)$ are the regression parameters of model (2.12) and (2.13) respectively. To develop the test procedure for testing the above null hypothesis, we proceed to constuct another logistic form following Islam's extension (see, Section 2.2), instead of (2.13), on the basis of estimated regression parameter $\hat{\beta}_{j1}(t_1)$ of (2.13) and which is defined as

$$p_{j1} = \frac{\exp(x_{j1})}{1+\exp(x_{j1})}, \tag{2.14}$$

where $x_{j1} = \gamma_{j1} + \hat{\beta}_{j1}(t_1)\, n_{j1}(t)$. Here $t = 1, 2, \ldots, T$ and $t_1 = 1, 2, \ldots, T_1$, the independent variable $n_{j1}(t)$ is the transition count of $k^{th}(= 1)$ state of first order transition matrix at time t. The term $\gamma_{j1}(t)$ is the regression parameter. Therefore, employing the equations (2.12) and (2.14), we may develop the likelihood ratio test statistic for testing the previously stated null hypothesis.

For higher order transition probabilities of Markov chain to identify these equations, let us consider the transition probability of state l at time t, given that the state k at time $t-1, \ldots$, the state j at time $t-s+1$ and state i at time $t-s$, where $t = s, s+1, \ldots, T$ which is denoted by $p_{ij\cdots kl}$ for all $i, j, \ldots, k, l = 0, 1$. Similarly, we also consider another transition probability $p_{ij\cdots kl}(t_1)$ of state l at time t_1 given that the state k at time $t_1 - 1, \ldots$, state j at time $t_1 - s + 1$, and state i at time $t_1 - s$, where $t_1 = s, s+1, \ldots, T_1$ for all $i, j, \ldots, k, l = 0, 1$. To determine the stationarity of chain dependent observations, we consider the null hypothesis that the different types of transition probabilities for Markov chain are stationary against the alternative that different types of transition probabilities are not stationary:

$$H_0 : \beta_{ij\cdots r-1,1}(t_1) = \beta_{ij\cdots r-1,1} \quad \text{against} \quad H_1 : \beta_{ij\cdots r-1,1}(t_1) \neq \beta_{ij\cdots r-1,1}$$

Thus, on the basis of logistic regression we suppose that

$$p_{ij\cdots r-1,1} = \frac{\exp(z_{ij\cdots r-1,1})}{1 + \exp(z_{ij\cdots r-1,1})} \tag{2.15}$$

and

$$p_{ij\cdots r-1,1}(t_1) = \frac{\exp(z_{ij\cdots r-1,1})(t_1)}{1 + \exp(z_{ij\cdots r-1,1})(t_1)}, \tag{2.16}$$

where $z_{ij\cdots r-1,1} = \alpha_{ij\cdots r-1,1} + \beta_{ij\cdots r-1,1}\, n_{ij\cdots r-1,1}(t)$ and $z_{ij\cdots r-1,1}(t_1) = \alpha_{ij\cdots r-1,1}(t_1) + \beta_{ij\cdots r-1,1}(t_1) n_{ij\cdots r-1,1}(t_1)$. Here $t = 1, 2, \ldots, T$ and $t_1 = 1, 2, \ldots, T_1$, the independent variables $n_{ij\cdots r-1,1}(t)$ and $n_{ij\cdots r-1,1}(t_1)$ are the transition counts of $r^{th}(= 1)$ state of $(r-1)^{th}$ order transition matrix at time t and t_1 respectively. The terms $\alpha_{ij\cdots r-1,1}$ and $\beta_{ij\cdots r-1,1}$, $\alpha_{ij\cdots r-1,1}(t_1)$ and $\beta_{ij\cdots r-1,1}(t_1)$ are the regression parameters of model (2.15) and (2.16) respectively. To develop the test procedure for testing the above hypothesis, another logistic form has been constructed by using $\hat{\beta}_{ij\cdots r-1,1}(t_1)$ of model (2.16) which can be shown as follows

$$p_{ij\cdots r-1,1} = \frac{\exp(x_{ij\cdots r-1,1})}{1 + \exp(x_{ij\cdots r-1,1})}\,, \tag{2.17}$$

where $x_{ij\cdots r-1,1} = \gamma_{ij\cdots r-1,1} + \hat{\beta}_{ij\cdots r-1,1}(t_1)\, n_{ij\cdots r-1,1}(t)$. Here $t = 1, 2, \ldots, T$ and $t_1 = 1, 2, \ldots, T_1$, $n_{ij\cdots r-1,1}(t)$ is the independent variable which is considered as the transition count that an individual in state $r(=1)$ at time t_1 given that in state $r-1$ at time $t_1-1, \ldots,$ in the state j at time t_1-s+1, and in state i at time t_1-s, where $t_1 = s, s+1, \ldots, T_1$ for all $i, j, \ldots, k, r = 0, 1$. The term $\gamma_{ij\cdots r-1,1}$ is the regression parameter. Therefore, for testing the null hypothesis, the likelihood ratio test statistic is defined utilizing the models (2.15) and (2.17).

2.5. Estimation of parameters for transition probabilities of logistic regression model

To estimate the parameters α_{j1} and β_{j1}, $\alpha_{j1}(t_1)$ and $\beta_{j1}(t_1)$ and γ_{j1} of models (2.12), (2.13) and (2.14) respectively and the parameters $\alpha_{ij\cdots r-1,1}$ and $\beta_{ij\cdots r-1,1}$, $\alpha_{ij\cdots r-1,1}(t_1)$ and $\beta_{ij\cdots r-1,1}(t_1)$ and $\gamma_{ij\cdots r-1,1}$ for models (2.15), (2.16) and (2.17) respectively, the maximum likelihood estimation method under Newton-Raphson iteration procedure is employed (Muenz and Rubinstein, 1985; Islam, 1994). Therefore, the likelihood function for model (2.12) is defined as

$$L = \Pi_{t=1}^{T}\{(1-p_{j1})^{n_{j0}(t)} {p_{j1}}^{n_{j1}(t)}\}, \tag{2.18}$$

where $n_{j0}(t)$ and $n_{j1}(t)$ are the numbers of transitions for $j, k^{th}(=0)$ state and $j, k^{th}(=1)$ state respectively at time $t(=1, 2, \ldots, T)$. To substitute the expression (2.12) in (2.18) we obtain

$$L = \Pi_{t=1}^{T}\left\{\frac{1}{1+\exp(z_{j1})}\right\}^{n_{j0}(t)}\left\{\frac{\exp(z_{j1})}{1+\exp(z_{j1})}\right\}^{n_{j1}(t)} \tag{2.19}$$

The log likelihood function for model (2.12) becomes

$$\begin{aligned}\log L =& \sum_{t=1}^{T} n_{j1}(t)\left(\alpha_{j1}+\beta_{j1}n_{j1}(t)\right)\\ &-\sum_{t=1}^{T}\left(n_{j0}(t)+n_{j1}(t)\right)\log\left(1+\exp(\alpha_{j1}+\beta_{j1}n_{j1}(t))\right).\end{aligned} \tag{2.20}$$

Similarly, the log likelihood function for models (2.13), (2.14), (2.15), (2.16) and (2.17) are given by

$$\begin{aligned}\log L =& \sum_{t_1=1}^{T_1} n_{j1}(t_1)\left(\alpha_{j1}(t_1)+\beta_{j1}(t_1)\,n_{j1}(t_1)\right)\\ &-\sum_{t_1=1}^{T_1}\left(n_{j0}(t_1)+n_{j1}(t_1)\right)\log\left(1+\exp(\alpha_{j1}(t_1)+\beta_{j1}(t_1)\,n_{j1}(t_1))\right)\end{aligned} \tag{2.21}$$

for model (2.13),

$$\log L = \sum_{t=1}^{T} n_{j1}(t) \left(\gamma_{j1} + \hat{\beta}_{j1}(t_1)\, n_{j1}(t)\right) - \sum_{t=1}^{T} (n_{j0}(t) + n_{j1}(t)) \log \left(1 + \exp(\gamma_{j1} + \hat{\beta}_{j1}(t_1) n_{j1}(t))\right) \quad (2.22)$$

for model (2.14),

$$\log L = \sum_{t=1}^{T} n_{ij\cdots r-1,1}(t)\, (\alpha_{ij\cdots r-1,1} + \beta_{ij\cdots r-1,1}\, n_{ij\cdots r-1,1}(t)) - \sum_{t=1}^{T} (n_{ij\cdots r-1,0}(t) + n_{ij\cdots r-1,1}(t)) \times \log (1 + \exp(\alpha_{ij\cdots r-1,1} + \beta_{ij\cdots r-1,1}\, n_{ij\cdots r-1,1}(t)) \quad (2.23)$$

for model (2.15),

$$\log L = \sum_{t_1=1}^{T_1} n_{ij\cdots r-1,1}(t_1)\, (\alpha_{ij\cdots r-1,1}(t_1) + \beta_{ij\cdots r-1,1}(t_1)\, n_{ij\cdots r-1,1}(t_1)) - \sum_{t_1=1}^{T_1} (n_{ij\cdots r-1,0}(t_1) + n_{ij\cdots r-1,1}(t_1)) \times \log (1 + \exp(\alpha_{ij\cdots r-1,1}(t_1) + \beta_{ij\cdots r-1,1}(t_1)\, n_{ij\cdots r-1,1}(t_1)) \quad (2.24)$$

for model (2.16) and

$$\log L = \sum_{t=1}^{T} n_{ij\cdots r-1,1}(t) \left(\gamma_{ij\cdots r-1,1} + \hat{\beta}_{ij\cdots r-1,1}(t_1)\, n_{ij\cdots r-1,1}(t)\right) - \sum_{t=1}^{T} (n_{ij\cdots r-1,0}(t) + n_{ij\cdots r-1,1}(t)) \times \log \left(1 + \exp(\gamma_{ij\cdots r-1,1} + \hat{\beta}_{ij\cdots r-1,1}(t_1)\, n_{ij\cdots r-1,1}(t))\right) \quad (2.25)$$

for model (2.17).

To perform above estimation procedure, the information matrix is denoted by I, where I^{-1} is the variance-covariance matrix with respect to parameters (see Appendix for detail explanation).

2.6. Test of hypothesis for the determination of stationarity of Markov chain

To test whether the different types of transition probabilities are stationary for Markov model, a test procedure is developed under the logistic regression model (Islam, 1994) (see, Section 2.2). For this test procedure, let us consider the null hypothesis that the different types of transition probabilities are stationary for the specified order of Markov model against the alternative that the transition probabilities are non-stationary. That is,

$$H_0 : \beta_{j1} = \beta_{j1}(t_1) \quad \text{against} \quad H_1 : \beta_{j1} \neq \beta_{j1}(t_1),$$

where β_{j1} and $\beta_{j1}(t_1)$ are the logistic regression parameters for $j, k^{th}(= 1)$ transition count at time $t\,(t = 1, \ldots, T)$ and $t_1\,(t_1 = 1, \ldots, T_1)$ respectively, of two-state transition matrix. Similarly, for higher order transition probabilities, the null hypothesis can be written as

$$H_0 : \beta_{ij\cdots r-1,1} = \beta_{ij\cdots r-1,1}(t_1) \quad \text{against} \quad H_1 : \beta_{ij\cdots r-1,1} \neq \beta_{ij\cdots r-1,1}(t_1),$$

where $\beta_{ij\cdots r-1,1}$ and $\beta_{ij\cdots r-1,1}(t_1)$ are the logistic regression parameters for $i, j, \ldots r-1, r^{th}(= 1)$ transition count at time $t\,(t = 1, \ldots, T)$ and $t_1\,(t_1 = 1, \ldots, T_1)$ respectively. In order to determine the stationarity of overall transition probabilities of Markov chain, the null hypothesis is defined as

$$H_0 : p_{ij\cdots r-1,1}(t) = p_{ij\cdots r-1,1}(t_1) \quad \text{against} \quad H_1 : p_{ij\cdots r-1,1}(t) \neq p_{ij\cdots r-1,1}(t_1),$$

where $p_{ij\cdots r-1,r}(t)$ and $p_{ij\cdots r-1,r}(t_1)$ are $(r-1)^{th}$ order transition probability matrix at time $t\,(t = 1, \ldots, T)$ and $t_1\,(t_1 = 1, \ldots, T_1)$ respectively. The likelihood ratio for $j, k^{th}(= 1)$ transition count is

$$\lambda_{j1} = \frac{\text{Likelihood function (conditional)}}{\text{Likelihood function (unconditional)}} = \frac{L_1}{L_2}.$$

Here

$$\begin{aligned} \log L_1 = & \sum_{t=1}^{T} n_{j1}(t)\left(\hat{\gamma}_{j1} + \hat{\beta}_{j1}(t_1)\, n_{j1}(t)\right) \\ & - \sum_{t=1}^{T} (n_{j0}(t) + n_{j1}(t)) \log\left(1 + \exp(\hat{\gamma}_{j1} + \hat{\beta}_{j1}(t_1)\, n_{j1}(t))\right) \end{aligned} \tag{2.26}$$

and

$$\log L_2 = \sum_{t=1}^{T} n_{j1}(t) \left(\hat{\alpha}_{j1} + \hat{\beta}_{j1} n_{j1}(t)\right) - \sum_{t=1}^{T} (n_{j0}(t) + n_{j1}(t)) \log \left(1 + \exp(\hat{\alpha}_{j1} + \hat{\beta}_{j1} n_{j1}(t))\right). \quad (2.27)$$

Then

$$-2 \log \lambda_{j1} = 2 (\log L_2 - \log L_1), \quad (2.28)$$

which is asymptotically distributed as χ^2_{j1} with 1 d.f. In addition, to identify the stationarity of overall transition probabilities of Markov chain, let us consider χ^2 statistic with 2^r d.f.,
where

$$\chi^2 = \sum_j \chi^2_{j1}, \quad (2.29)$$

and for 2^r d.f., 2 is the number of states for the Markov chain and r is the order of Markov chain. Similarly, for higher order transition probabilities, the likelihood ratio test statistic is stated as

$$-2 \log \lambda_{ij \cdots r-1,1} = 2 (\log L_2 - \log L_1), \quad (2.30)$$

which is asymptotically distributed as $\chi^2_{ij \cdots r-1,1}$ with 1 d.f. Here $\log L_1$ and $\log L_2$ are defined as

$$\log L_1 = \sum_{t=1}^{T} n_{ij \cdots r-1,1}(t) \left(\hat{\gamma}_{ij \cdots r-1,1} + \hat{\beta}_{ij \cdots r-1,1}(t_1) n_{ij \cdots r-1,1}(t)\right) - \sum_{t=1}^{T} (n_{ij \cdots r-1,0}(t) + n_{ij \cdots r-1,1}(t)) \times \log \left(1 + \exp(\hat{\gamma}_{ij \cdots r-1,1} + \hat{\beta}_{ij \cdots r-1,1}(t_1)\, n_{ij \cdots r-1,1}(t))\right) \quad (2.31)$$

and

$$\log L_2 = \sum_{t=1}^{T} n_{ij \cdots r-1,1}(t) \left(\hat{\alpha}_{ij \cdots r-1,1} + \hat{\beta}_{ij \cdots r-1,1} n_{ij \cdots r-1,1}(t)\right) - \sum_{t=1}^{T} (n_{ij \cdots r-1,0}(t) + n_{ij \cdots r-1,1}(t)) \times \log \left(1 + \exp(\hat{\alpha}_{ij \cdots r-1,1} + \hat{\beta}_{ij \cdots r-1,1}\, n_{ij \cdots r-1,1}(t))\right). \quad (2.32)$$

To identify the stationarity of overall transition probabilities of Markov chain, the χ^2 statistic with 2^r d.f. is considered, which is expressed as

$$\chi^2 = \sum_{ij\cdots r-1,1} \chi^2_{ij\cdots r-1,1} \tag{2.33}$$

for all $i, j, \ldots, r = 0, 1$ and for 2^r d.f., 2 is the number of states and r is the order of Markov chain.

3. DATA

The daily rainfall occurrence data of the rainy season in the period between 1964 and 1990 for five selected stations, namely Chittagong, Mymensingh, Rajshahi, Faridpur and Satkhira of Bangladesh, are utilized to identify the feasibility of the test procedures. These data are collected from the Department of Meteorology, Government of People's Republic of Bangladesh. The period between the months of April and October has been considered as the rainy season, because the major agricultural crops (Aus and Aman rice) under traditional system cultivated during this period, depend greatly upon the occurrences of rainfall due to scanty irrigation facilities.

4. DETERMINATION OF THE CHAIN DEPENDENCE AND STATIONARITY OF DAILY RAINFALL OCCURRENCE IN SELECTED STATIONS OF BANGLADESH

To identify whether the daily rainfall occurrence for the rainy season for selected stations of Bangladesh follows chain dependence or stationarity, Sinha (1997) utilized the likelihood ratio test statistic χ^2_{LR} and score test statistic χ^2 on the basis of aggregate data. However, he found that these test statistics do not provide apparently significant results for chain dependence and stationarity of rainfall occurrence. Therefore, to identify these criteria, we utilize the test procedures which are developed in this study using logistic regression model under likelihood ratio criterion. To test the null hypothesis by the utilization of χ^2 statistic it is always observed that the value of χ^2 increases with the increase of sample size. For overcoming this problem, in this study we consider p−value up to 0.001 as the cut-off point.

4.1. Chain dependent test for daily rainfall occurrences of Bangladesh

To identify whether the successive observations of daily rainfall occurrences for some selected stations in Bangladesh, namely Chittagong, Mymensingh, Rajshahi, Faridpur and Satkhira follow a chain dependent process or not, let us consider the null hypothesis that the successive observations of daily rainfall occurrences do not follow chain dependent process against the alternative that the daily rainfall occurrences follow chain dependent process.

In order to test the null hypothesis we consider a test procedure which is developed in this study (see, Section 2.3) according to the suggestions made by Cox (1970) on the basis of logistic regression model under likelihood ratio criterion. This likelihood ratio test statistic for logistic regression is denoted by χ^2_{LRLR}, *i.e.* $-2\log\lambda = \chi^2_{LRLR}$, where λ is the likelihood ratio. To perform this test statistic, Cox (1970) suggested that if $\Delta = 0$ for equation (2.6) then the null hypothesis is true.

For testing the null hypothesis to calculate the value of test statistic χ^2_{LRLR} we consider the equation (2.11), here for all $j, k = 0, 1$ and $n = 5,778$, where 0 (zero), 1 (one) and n indicate the dry day, wet day and total number of days under consideration respectively. In this equation $\hat{\alpha}$ and $\hat{\Delta}$ are the estimated parameters (see, equations (2.8) and (2.9) respectively), n_{11} is the transition count for the actual day is wet given that the previous day was also wet and $n_{1.}$ is the number of previous days which were wet. The estimated values of $\hat{\alpha}$ and $\hat{\Delta}$, and the calculated values of χ^2_{LRLR} for Chittagong, Mymensingh, Rajshahi, Faridpur and Satkhira stations for Bangladesh are shown in Table 4.1. This table implies that the values of χ^2_{LRLR} are significant (p−value < 0.001) for all the selected stations, *i.e.* this test statistic indicates that the successive observations of daily rainfall occurrences follow a chain dependent process. Clearly, from Table 4.1 it appears that the daily rainfall occurrences follow a Markov model of order one, *i.e.* the occurrence of rainfall on a particular day for all the selected stations of Bangladesh mainly depends upon the rainfall of the immediate preceding day.

4.2. Stationarity test for daily rainfall occurrences for different stations of Bangladesh

The daily rainfall occurrences for consecutive days for selected stations of Bangladesh follow a chain dependent process (see, Section 4.1). In order to examine the stationarity of different type transition probabilities of Markov chain for rainfall occurrences, the proposed test procedure (see, Section 2.4) using lo-

TABLE 4.1 *Estimated values of $\hat{\alpha}$ and $\hat{\Delta}$, and the values of χ^2_{LRLR} for testing whether the daily rainfall occurrences follow chain dependent process or not for selected stations of Bangladesh*

Name of stations	*Estimated values of $\hat{\alpha}$*	*Estimated values of $\hat{\Delta}$*	*Observed values of χ^2_{LRLR}*
Chittagong	−0.39794	0.78166	940.21*
Mymensingh	−0.45346	0.73839	751.51*
Rajshahi	−0.52458	0.70375	580.12*
Faridpur	−0.38688	0.63609	582.21*
Shatkhira	−0.55721	0.84266	872.81*

NOTE : *The d.f. for χ^2_{LRLR} is* 1. $*$ *denotes $p - value < 0.001$.*

gistic regression model under lokelihood ratio criterion is employed. Sinha (1997) showed that the likelihood ratio test statistic χ^2_{LR} (Goodman, 1955) and score test statistic χ^2 (Anderson and Goodman, 1957) are not suitable for identifying the stationarity of different types of rainfall transitions for Markov chain. To identify the stationarity of transitions, the test procedure under logistic regression is employed on the basis of order of different types of rainfall transitions. The order of different types of daily rainfall transitions and overall transitions for Chittagong and Faridpur stations follow transition of order one, and for Mymensingh, Rajshahi and Satkhira stations follow transition of order two (Sinha, 1997).

In order to test the stationarity for different types of rainfall transitions, we consider equations (2.12), (2.13) and (2.14) for first order, and the equations (2.15), (2.16) and (2.17) for higher order Markov model. On the basis of these equations we may define the following null hypotheses for testing the stationarity of rainfall:

H_0 : The transitions for daily rainfall occurrences for r^{th} order Markov model are stationary of order t_1, *i.e.*, $\beta_{j1} = \beta_{j1}(t_1)$ for first order and $\beta_{ij\cdots r-1,1} = \beta_{ij\cdots r-1,1}(t_1)$ for higher order Markov model;

H_1 : The transitions for daily rainfall occurrences for r^{th} order Markov model are not stationary, *i.e.*, $\beta_{j1} \neq \beta_{j1}(t_1)$ for first order and $\beta_{ij\cdots r-1,1} \neq \beta_{ij\cdots r-1,1}(t_1)$ for higher order Markov model. Here r is the order of rainfall transitions, $t_1 (= 1, 2, \ldots, T_1)$ is the time, α_{j1} and $\beta_{j1}, \alpha_{j1}(t_1)$ and $\beta_{j1}(t_1)$, γ_{j1} are the parameters of equations (2.12), (2.13) and (2.14) respectively. Further $\alpha_{ij\cdots r-1,1}$ and $\beta_{ij\cdots r-1,1}$, $\alpha_{ij\cdots r-1,1}(t_1)$ and $\beta_{ij\cdots r-1,1}(t_1)$, $\gamma_{ij\cdots r-1,1}$ are the parameters of equations (2.15), (2.16) and (2.17) respectively. To estimate these parameters we adopt ML estimation method under Newton-Raphson iteration procedure

(Muenz and Rubinstein, 1985; Islam, 1994) using standard computer language FORTRAN 77.

The time T_1 is identified on the basis of figures constructed by Sinha (1997) for year wise total number of daily rainfall transition (actual day is wet) counts for specified order of rainfall transition of Markov chain. From these figures he observed that the transitions Wet/Dry and Wet/Wet for first order Markov model for Chittagong station follow approximately a cycle of 4 years. For Mymensingh station the transitions $Wet/Dry/Dry$ and $Wet/Wet/Dry$, and the transitions $Wet/Dry/Wet$ and $Wet/Wet/Wet$ for second order Markov model follow a cycle of 4 and 3 years respectively. The transitions $Wet/Dry/Wet$, $Wet/Wet/Dry$ and $Wet/Wet/Wet$ and $Wet/Dry/Dry$ for second order Markov model for Rajshahi station follow a cycle of 2, 3 and 6 years respectively. For Faridpur station the transitions Wet/Wet and Wet/Dry for first order Markov model follow a cycle of 2 and 5 years respectively. The transitions for second order Markov model $Wet/Wet/Wet$ follow a cycle of 3 years, and $Wet/Dry/Dry$, $Wet/Dry/Wet$ and $Wet/Wet/Dry$ also follow a cycle of 4 years for Satkhira station. However, these computations are not presented here owing to space constraint. To calculate the value of test statistic for the identification of stationarity, the cycle T_1 is considered as the order of stationarity for respective transitions of daily rainfall occurrences for specified order of Markov model for each station.

For identifying the stationarity of rainfall transitions, the order of different types of rainfall transition for Markov Chain is considered. On the basis of these orders of rainfall transitions, we estimate the parameters of models (2.12), (2.13) and (2.14) for first order, and (2.15), (2.16) and (2.17) for higher order. The estimated values of parameters for these equations are shown in Tables 4.2 and 4.3 for first and second order rainfall transitions respectively. To estimate the parameters, we select $T_1 = 2$ years (1964 to 1965) and $T = 25$ years (1966 to 1990) according to figures constucted for the transition probabilities p_{11} and p_{101} for Faridpur and Rajshahi stations respectively. The time $T_1 = 3$ years (1964 to 1966) and $T = 24$ years (1967 to 1990) for the transition probabilities p_{011} and p_{111} for Rajshahi station, p_{101} and p_{111} for Mymensingh station, and p_{111} for Satkhira station. Further, $T_1 = 4$ years (1964 to 1967) and $T = 23$ years for the transition probabilities p_{01} and p_{11} for Chittagong, p_{001} and p_{011} for Mymensingh, p_{001}, p_{101} and p_{011} for Satkhira station. Similarly, for Faridpur station T_1 and T are considered 5 years (1964 to 1968) and 22 years (1969 to 1990) respectively for the transition probability p_{01}, and for Rajshahi station these are considered 6 years (1964 to 1969) and 21 years (1970 to 1990) respectively for p_{001}.

TABLE 4.2 *Estimated parameters α_{j1}, β_{j1}, $\alpha_{j1}(t_1)$, $\beta_{j1}(t_1)$ and γ_{j1} of first order Markov model for testing the null hypothesis $\beta_{j1}(t_1) = \beta_{j1}$ for analyzing the daily rainfall occurrences of Chittagong and Faridpur stations of Bangladesh*

Parameters	*Name of stations*	
	Chittagong	*Faridpur*
α_{01}	−2.68872 (0.35767)	−2.66779 (0.36669)
β_{01}	0.05665 (0.01113)	0.05174 (0.01022)
$\alpha_{01}(t_1)$	−3.31068 (0.68269)	−4.48060 (1.18050)
$\beta_{01}(t_1)$	0.08211 (0.02420)	0.11025 (0.03867)
γ_{01}	−3.50259 (0.04448)	−4.76367 (0.04389)
α_{11}	−0.28596 (0.30195)	−0.56138 (0.22491)
β_{11}	0.01525 (0.00402)	0.01803 (0.00353)
$\alpha_{11}(t_1)$	1.09947 (0.96092)	−0.28733 (0.45823)
$\beta_{11}(t_1)$	−0.00028 (0.01178)	0.01435 (0.00719)
γ_{11}	0.87165 (0.04443)	−0.33063 (0.04257)

NOTE : *Figures in parentheses indicate the standard error of estimated parameters. The daily rainfall transition follows first order Markov model for Chittagong and Faridpur stations and second order Markov model for Mymensingh, Rajshahi and Satkhira station (Sinha, 1997). Thus, the values of estimated parameters for Mymensingh, Rajshahi and Satkhira stations are not utilized in this table.*

To identify the stationarity of different types of transition probabilities and overall transition probabilities of the Markov chain for daily rainfall occurrences, let us consider the test statistic $\chi^2_{ij\cdots r-1,1}$ (equation 2.30) and χ^2 (equation 2.33) respectively. From the values of $\chi^2_{ij\cdots r-1,1}$ (Table 4.4), it is evident that different types of transition Wet/Dry for order 4 for Chittagong station, $Wet/Dry/Dry$, $Wet/Dry/Wet$, $Wet/Wet/Dry$ and $Wet/Wet/Wet$ for order 4, 3, 4, and 3 respectively for Mymensingh station, and for order 6, 2, 3 and 6 respectively for Rajshahi station, Wet/Wet for order 2 for Faridpur station, and $Wet/Dry/Dry$, $Wet/Wet/Dry$ and $Wet/Wet/Wet$ for order 4, 4 and 3 respectively for Satkhira station are stationary. Furthermore, for long run, the transition Wet/Wet for Chittagong station is also observed stationary of order 8. Because the value of χ^2_{11} (3.18042) is non-significant ($p-$value > 0.01) for a cycle of 8 years.

For overall transition probabilities of the Markov model for daily rainfall occurrences, the values of χ^2 (Table 4.4) indicate that occurrences are non-stationary in Chittagong, Faridpur and Satkhira stations. However, for the long run the overall transition probabilities of Markov chain in Chittagong station is observed stationary of order 8, since χ^2 (8.35935) is insignificant ($p-$value $>$ 0.01) for a cycle of 8 years. This table also shows that the overall transitions of

TABLE 4.3 *Estimated parameters α_{ij1}, β_{ij1}, $\alpha_{ij1}(t_1)$, $\beta_{ij1}(t_1)$ and γ_{ij1} of second order Markov model for testing the null hypothesis $\beta_{ij1}(t_1) = \beta_{ij1}$ for analyzing the daily rainfall occurrences for Mymensingh, Rajshahi and Satkhira stations of Bangladesh*

Parameters	*Name of stations*		
	Mymensingh	*Rajshahi*	*Satkhira*
α_{001}	−3.19870 (0.20982)	−3.69746 (0.28896)	−3.91524 (0.27621)
β_{001}	0.10196 (0.11480)	0.11487 (0.01313)	0.13557 (0.01389)
$\alpha_{001}(t_1)$	−2.71846 (1.49395)	−3.81010 (0.37473)	−4.58152 (0.46509)
$\beta_{001}(t_1)$	0.07844 (0.06898)	0.10859 (0.01752)	0.167780 (0.02660)
γ_{001}	−2.79032 (0.05738)	−3.56225 (0.05549)	−4.54814 (0.05680)
α_{101}	−1.19846 (0.22893)	−1.83996 (0.30913)	−1.79337 (0.29600)
β_{101}	0.06432 (0.01339)	0.10132 (0.02282)	0.10433 (0.02118)
$\alpha_{101}(t_1)$	−0.82994 (1.46867)	−1.88062 (0.80025)	−0.79247 (0.96156)
$\beta_{101}(t_1)$	0.03450 (0.09543)	0.09887 (0.06712)	0.03274 (0.08324)
γ_{101}	−0.71683 (0.07382)	−1.80771 (0.07503)	−0.83546 (0.07834)
α_{011}	−1.11975 (0.31052)	−1.15538 (0.37964)	−1.34817 (0.44810)
β_{011}	0.07611 (0.01527)	0.07036 (0.02010)	0.08797 (0.02328)
$\alpha_{011}(t_1)$	−2.28420 (1.03663)	−0.88620 (0.80768)	−0.69708 (0.67305)
$\beta_{011}(t_1)$	0.12513 (0.05027)	0.05956 (0.04761)	0.04714 (0.04435)
γ_{011}	−2.08297 (0.08034)	−0.95530 (0.07358)	−0.57361 (0.07761)
α_{111}	−0.03600 (0.16181)	−0.06551 (0.24999)	0.12365 (0.26485)
β_{111}	0.01700 (0.00304)	0.01919 (0.00693)	0.01585 (0.00546)
$\alpha_{111}(t_1)$	0.25190 (0.65472)	−0.43340 (0.50769)	−0.21148 (0.59655)
$\beta_{111}(t_1)$	0.01062 (0.01409)	0.03325 (0.01703)	0.02597 (0.01531)
γ_{111}	0.28326 (0.05727)	−0.55417 (0.06244)	−0.35209 (0.06016)

NOTE : *Figures in parentheses indicate the standard error of estimated parameters. The daily rainfall transition follows first order Markov model for Chittagong and Faridpur stations and second order Markov model for Mymensingh, Rajshahi and Satkhira station (Sinha, 1997). Thus, the values of estimated parameters for Chittagong and Faridpur stations are not shown in this table.*

daily rainfall occurrences for second order Markov model in Rajshahi station is stationary and in Mymensingh station is non-stationary. However, the values of Chi-square (χ^2_{001}, χ^2_{101}, χ^2_{011} and χ^2_{111}) for different types of transition probabilities of the second order Markov model in Mymensingh station for daily rainfall occurrences provide sufficient evidence for stationarity. Therefore, we may conclude that the daily rainfall occurrences for the second order Markov model of Mymensingh station are stationary.

TABLE 4.4 *Observed* χ^2_{j1}, χ^2_{ij1} *and* χ^2 *for testing the null hypothesis* $\beta_{j1}(t_1) = \beta_{j1}$, $\beta_{ij1}(t_1) = \beta_{ij1}$, *and* $p_{jk}(t_1) = p_{jk}$ *and* $p_{ijk}(t_1) = p_{ijk}$ *respectively for analyzing the daily rainfall occurrences for five selected stations of Bangladesh*

χ^2 *Statistic*	*Name of stations*				
	Chittagong	*Mymensingh*	*Rajshahi*	*Faridpur*	*Satkhira*
χ^2_{01}	5.17883	–	–	32.02515*	–
χ^2_{11}	14.77246*	–	–	1.09265	–
χ^2_{001}	–	4.36987	0.23236	–	4.83734
χ^2_{101}	–	5.09122	0.01160	–	11.84853*
χ^2_{011}	–	9.85214	0.28964	–	3.14993
χ^2_{111}	–	4.46436	4.06384	–	3.42993
χ^2	19.95129*	23.77759*	4.59744	33.11780*	23.26574*

NOTE : *Degrees of freedom for* χ^2_{01}, χ^2_{11}, χ^2_{001}, χ^2_{101}, χ^2_{011} *and* χ^2_{111} *is* 1 *and for* χ^2 *is* 2 *and* 4 *for first and second order Markov model respectively.* $*$ *denotes* $p - value < 0.001$.

5. DISCUSSION AND CONCLUSSION

A test procedure has been developed in this study to determine whether the sequence of observations follows a chain dependent process. In addition to a test procedure for stationarity of different types of transition probabilities of Markov chain, an alternative test procedure has also been developed for chain dependence observations. We developed this test procedure following Islam (1994) as an alternative to the existing test procedures. These test procedures are formulated for logistic regression model under likelihood ratio criterion. Employing the existing test procedures, we may identify the stationarity of Markov chain for aggregate data. However, utilizing this stationarity test procedure, we may identify the stationarity of different types of transition probabilities of Markov model for chain dependent observations.

The proposed test procedures have been employed to determine the patterns of daily rainfall occurrences for selected stations in Bangladesh. These applications reveal that the proposed test procedures can be useful for identifying chain dependence and stationarity. From the results of these test procedures, we can summarize that the daily rainfall occurrences for selected stations of Bangladesh follow a chain dependent process. It is observed that the transition Wet/Dry is stationary of order 4 and Wet/Wet is not stationary for Chittagong station. It is revealed that Wet/Wet is stationary of order 2 and Wet/Dry is nonstationary for Faridpur station. However, for overall transition probabilities of daily rainfall occurrences are found non-stationary for these stations. The occurrences of rainfall for different types of second order transitions for Mymensingh, Rajshahi and Satkhira stations are found stationary except the transition $Wet/Dry/Wet$ of Satkhira station. The overall transition probabilities of rainfall occurrences for the second order Markov model are found stationary for Rajshahi station and non-stationary for Mymensingh and Satkhira stations. However, for Mymensingh station, different types of transition probabilities for daily rainfall occurrences appear to be stationary (Table 4.4). Hence, we may conclude that the daily rainfall occurrences for overall transition probabilities of the second order Markov model are stationary for Mymensingh station. We observed that the daily rainfall occurrences for Mymensingh and Rajshahi stations are stationary, and for Chittagong, Faridpur and Satkhira stations are nonstationary.

APPENDIX: ESTIMATION OF PARAMETERS

To estimate the parameters of logistic regression models (2.15), (2.16), (2.17), (2.18), (2.19) and (2.20) for different types of transition probabilities of the Markov model, the log likelihood function, estimation procedure and method are shown in Section 2.5.

For the execution of Newton Raphson iteration procedure under the method of MLE, the first and second derivatives of log likelihood functions with respect to corresponding parameter are needed to identify and set first derivative equal to zero. The first and second derivatives for (2.23) with respect to α_{j1} and β_{j1} are given by

$$\frac{\delta \log L}{\delta \alpha_{j1}} = \sum_{t=1}^{T} n_{j1}(t) - \sum_{t=1}^{T} \frac{(n_{j0}(t) + n_{j1}(t)) \exp(z_{j1})}{1 + \exp(z_{j1})}, \tag{A.1}$$

$$\frac{\delta \log L}{\delta \beta_{j1}} = \sum_{t=1}^{T} n_{j1}^2(t) - \sum_{t=1}^{T} \frac{(n_{j0}(t) + n_{j1}(t))\, n_{j1}(t) \exp(z_{j1})}{1 + \exp(z_{j1})}, \tag{A.2}$$

$$\frac{\delta^2 \log L}{\delta \alpha_{j1}^2} = -\sum_{t=1}^{T} \frac{(n_{j0}(t) + n_{j1}(t)) \exp(z_{j1})}{(1 + \exp(z_{j1}))^2}, \tag{A.3}$$

$$\frac{\delta^2 \log L}{\delta \beta_{j1}^2} = -\sum_{t=1}^{T} \frac{(n_{j0}(t) + n_{j1}(t))\, n_{j1}^2(t) \exp(z_{j1})}{(1 + \exp(z_{j1}))^2}, \tag{A.4}$$

For the solution of equations (A.1) and (A.2) under Newton-Raphson iteration procedure for the method of MLE to estimate the parameters the standard computer language FORTRAN 77 is utilized. The information matrix and vector I and U respectively may be defined as

$$U = \begin{bmatrix} \frac{\delta \log L}{\delta \alpha_{j1}} \\ \\ \frac{\delta \log L}{\delta \beta_{j1}} \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} \frac{\delta^2 \log L}{\delta \alpha_{j1}^2} & \frac{\delta^2 \log L}{\delta \alpha_{j1} \delta \beta_{j1}} \\ \\ \frac{\delta^2 \log L}{\delta \alpha_{j1} \delta \beta_{j1}} & \frac{\delta^2 \log L}{\delta \beta_{j1}^2} \end{bmatrix},$$

where

$$I^{-1} = \begin{bmatrix} var\,(\alpha_{j1}) & cov\,(\alpha_{j1}\,, \beta_{j1}) \\ \\ cov\,(\alpha_{j1}\,, \beta_{j1}) & var\,(\beta_{j1}) \end{bmatrix}$$

and

$$\frac{\delta \log L}{\delta \alpha_{j1} \delta \beta_{j1}} = -\sum_{t=1}^{T} \frac{(n_{j0}(t) + n_{j1}(t))\, n_{j1}(t) \exp(z_{j1})}{1 + \exp(z_{j1})}.$$

Similarly, we may estimate the parameters of logistic regression model for different types of transition probabilities of the Markov chain for first, second and higher orders.

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