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THE EXTENSION OF THREE-WAY BALANCED MULTI-LEVEL ROTATION SAMPLING DESIGNS

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ABSTRACT

The two-way balanced one-level rotation design, $r_1^m - r_2^{m-1}$, and the three-way balanced multi-level rotation design, $r_1^m(l) - r_2^{m-1}$, were discussed (Park *et al.*, 2001, 2003). Although these rotation designs enjoy balancing properties, they have a restriction of $r_2 = c \cdot r_1$ (c should be a integer value) which interferes with applying these designs freely to various situations. To overcome this difficulty, we extend the $r_1^m(l) - r_2^{m-1}$ design to new one under the most general rotation system. The new multi-level rotation design also satisfies tree-way balancing which is done on interview time, rotation group and recall time. We present the rule and rotation algorithm which guarantee the three-way balancing. In particular, we specify the necessary condition for the extended three-way balanced multi-level rotation sampling design.

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1. INTRODUCTION

We can classify the rotation sampling designs by number of respondent's recalls (called as level) in their rotation schemes. By this criterion, we consider l-level rotation designs where l is a positive integer. In this l-level rotation design, the respondent reports the information of current survey month and $l-1$ previous months. The design with $l = 1$ is called as one-level rotation design while the design with $l \geq 2$ is called as multi-level rotation design. The U.S. Current Population Survey (CPS), the Canadian Labor Force Survey (CLFS) belong to

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an one-level rotation sampling design and the U.S. Monthly Retail Trade Survey (MRTS), U.S. Survey of Income and Program Participation (SIPP), the U.S. Consumer Expenditure Survey (CES) and the U.S. National Crime Victimization Suvey (NCVS) belong to a multi-level rotation design.

Most of previous works for the rotation designs are done for one-level rotation design. The one-level rotation design has been studied extensively (Kumer and Lee, 1983; Fuller, 1990; Lee, 1990; Cantwell, 1990; Lent *et al.*, 1999; Park *et al.*, 2001). However, the multi-level aspect of this rotation design has not been investigated sufficiently before. Although Park *et al.* (2003) introduced three-way balanced multi-level rotation design, $r_1^m(l) - r_2^{m-1}$, their design has two limitations. The first is that r_1 and r_2, the intervals of rotation group in and out, should be fixed during surveying period. The second is that r_2 is defined multiple of r_1; $r_2 = c \cdot r_1$ and c should be an integer value. The two limitations cause $r_1^m(l) - r_2^{m-1}$ design to diminish its applicability.

In this paper, we extend the $r_1^m(l) - r_2^{m-1}$ design to new one under the most general rotation system by imposing a condition on rotation scheme and use an algorithm to satisfy this condition in constructing such design. The remainder of this paper is divided into 4 sections. In Section 2, we discuss the properties of the extended three-way balanced multi-level rotation sampling designs as well as its rotation pattern. The necessary condition for the extended multi-level rotation sampling designs to be balanced in three-ways is also provided in Section 3. In Section 4, we present an algorithm to construct the extended three-way balanced multi-level rotation sampling designs. Section 5 includes our concluding remarks. A theoretical proof is provided in the Appendix.

2. The Extended Three-Way Balanced Multi-Level Rotation Sampling Designs

Since Park *et al.* (2001, 2003) investigated balanced rotation design in detail, we discuss the extended three-way balanced multi-level rotation design directly without introducing the basic concept of the balanced rotation design. The "extended three-way balanced multi-level rotation design" is the conceptual name of our new rotation scheme for showing improvement. Hereafter, we use "l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design" rather than the extended three-way balanced multi-level rotation design for practical use.

In a one-level rotation designs, most general rotation system can be expressed as $r_{11} - r_{21} - r_{12} - r_{22} - \cdots - r_{1,m-1} - r_{2,m-1} - r_{1m}$. That is, a selected sample unit is interviewed for the first r_{11} months, out of the sample for the next r_{21} months, back to the sample for another r_{12} months and out of the sample for the next r_{22} months, and so on. This procedure is repeated until the sample unit is surveyed for $r_{11} + r_{12} + \cdots + r_{1m}$ times.

In l-level rotation design, each sample unit reports the information for the current month as well as for the $l-1$ previous months. When the sample unit returns to the sample for every l^{th} month, it again provides l months information. The $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ system in one-level rotation design is generalized to "multi-level" rotation system: once a sample unit is selected from each rotation group, the sample unit returns to the sample for every l^{th} month until its ${r_{11}}^{th}$ interview and is out of the sample for the next $r_{21}+l-1$ months. This procedure is repeated m cycles with r_{1i} months in sample and $r_{2i}+l-1$ months out of sample for the i^{th} cycle, $i = 1, 2, \ldots, m$, until this sample unit returns to the sample for its final $\sum_{i=1}^{m} {r_{1i}}^{th}$ interview. We denote this rotation system as l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ system.

When $l = 1$, $r_{11} = r_{12} = \cdots = r_{1m} = r_1$ and $r_{21} = r_{22} = \cdots = r_{2,m-1} = r_2$, the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ system is reduced to the one-level $r_1^m - r_2^{m-1}$ rotation system used for the two-way balanced one-level rotation design (Park *et al.*, 2001).

α	1					2					3					4					5					6				
g	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5					
t	u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''										
$t+1$		u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''									
M $t+2$			u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''								
O $t+3$				u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''							
N $t+4$					u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''						
T $t+5$						u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''					
H $t+6$							u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''				
$t+7$								u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''			
$t+8$									u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''		
$t+9$										u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''	
$t+10$											u_5	'	''	u_4	'	''						u_3	'	''	u_2	'	''	u_1	'	''

(i) recall time=0

Month	*Rotation Groups*				
	1	2	3	4	5
t	u_5	u_3	u_1	u_4	u_2
$t+1$	u_2	u_5	u_3	u_1	u_4
$t+2$	u_4	u_2	u_5	u_3	u_1
$t+3$	u_1	u_4	u_2	u_5	u_3
$t+4$	u_3	u_1	u_4	u_2	u_5
$t+5$	u_5	u_3	u_1	u_4	u_2
$t+6$	u_2	u_5	u_3	u_1	u_4
$t+7$	u_4	u_2	u_5	u_3	u_1
$t+8$	u_1	u_4	u_2	u_5	u_3
$t+9$	u_3	u_1	u_4	u_2	u_5
$t+10$	u_5	u_3	u_1	u_4	u_2

(ii) recall time=1

Month	*Rotation Groups*				
	1	2	3	4	5
t	u_2	u_5	u_3	u_1	u_4
$t+1$	u_4	u_2	u_5	u_3	u_1
$t+2$	u_1	u_4	u_2	u_5	u_3
$t+3$	u_3	u_1	u_4	u_2	u_5
$t+4$	u_5	u_3	u_1	u_4	u_2
$t+5$	u_2	u_5	u_3	u_1	u_4
$t+6$	u_4	u_2	u_5	u_3	u_1
$t+7$	u_1	u_4	u_2	u_5	u_3
$t+8$	u_3	u_1	u_4	u_2	u_5
$t+9$	u_5	u_3	u_1	u_4	u_2
$t+10$	u_2	u_5	u_3	u_1	u_4

(iii) recall time=2

Month	*Rotation Groups*				
	1	2	3	4	5
t	u_4	u_2	u_5	u_3	u_1
$t+1$	u_1	u_4	u_2	u_5	u_3
$t+2$	u_3	u_1	u_4	u_2	u_5
$t+3$	u_5	u_3	u_1	u_4	u_2
$t+4$	u_2	u_5	u_3	u_1	u_4
$t+5$	u_4	u_2	u_5	u_3	u_1
$t+6$	u_1	u_4	u_2	u_5	u_3
$t+7$	u_3	u_1	u_4	u_2	u_5
$t+8$	u_5	u_3	u_1	u_4	u_2
$t+9$	u_2	u_5	u_3	u_1	u_4
$t+10$	u_4	u_2	u_5	u_3	u_1

FIGURE 2.1 *The three-way balanced 3-level 3-5-2 design.*

The notation (α, g) in Figure 2.1 means the index for the α^{th} sample unit in the g^{th} rotation group, the u_i means the sample unit interviewed for the i^{th} time, and I and II indicate recall time 1 and 2, respectively. In Figure 2.1, we have $r_{11} = 3$, $r_{12} = 2$, $r_{21} = 5$, $m = 2$ and $l = 3$. Thus, the sample unit u_1 at month t returns to the sample at months, $t+3$ and is out of the sample for the next 7 months from month $t+4$ to month $t+10$. Then the same sample unit comes back to the sample at months $t+11$, $t+14$, $t+17$, $t+20$ and $t+23$, and retires from the sample completely. We call a l-level rotation design as l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design if the design has $\sum_{i=1}^{m} r_{1i}$ rotation groups for $m < \infty$ and follows the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ system. When $m = 1$, in particular, we call it l-level r_{11} in-then-out design denoted by l-level $r_{11} - 0$ which is the multi-level version of the one-level r_{11} in-then-out designs such as the Canadian Labor Force Survey and Australian Labor Force Survey.

Each sample unit reports all the information simultaneously for the l months in the l-level rotation design. Thus a sample unit interviewed at month t provides the information not only for month t but also $l-1$ previous months from month $t-1$ to month $t-l+1$. Now backtracking $l-1$ previous months from month t, we define that the sample unit interviewed at month t has the recall time j at month $t-j$, $j = 0, 1, \ldots, l-1$. With this recall time j, we formally define the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design as follows.

DEFINITION 2.1. *The l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design is balanced in three-ways if the following three properties are satisfied: Denote the number of rotation groups by $G = \sum_{i=1}^{m} r_{1i}$ (i.e., the total number of months for a sample unit to be in the sample).*

(a) *For each survey month, all G rotation groups are represented in the sample by their respective sample units, and each rotation group is represented by its l different sample units, one with recall time 0, another with recall time 1, ..., and the last with recall time $l-1$ (See Figure 2.1).*

(b) *For each survey month and for each recall time j, $j = 0, 1, \ldots, l-1$, the monthly sample is balanced in such a way that one of the G sample units is interviewed for the 1^{st} time, one for the 2^{nd} time, ..., and one for the G^{th} time (see Figure 2.1 (i), (ii) and (iii)).*

(c) *For every span of G survey months and each recall time, each of the G rotation groups contributes its G sample units in which one sample unit is interviewed for the first time, ..., one for G^{th} time.*

When we consider only one-level rotation design (*i.e.*, $l = 1$), the properties (a), (b) and (c) for three-way balancing are reduced to the properties requiring for the two-way balancing given by Park *et al.* (2001). The three-way balancing ensures that the replacement of sample units occurs in the same rotation group and the rotation scheme of a sample unit depends only on its interview time but not on its survey month, rotation group, and recall time. By the property (a) and the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ rotation design, the number of groups ($= G$) in monthly sample at any survey month is equal to the total number of months ($= G$) that any one sample unit is included in the sample. This implies that the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design balanced in three-ways is an multi-level extension of the previous two-way balanced design and the Cantwell's balanced design (Cantwell, 1990).

The U.S. CES uses 3-level 5-0 design and satisfies all properties of three-way balanced design given in Definition 2.1. Since the U.S. MRTS and SIPP use only one group in each survey month and repeat it for the life of the survey, these two surveys do not satisfy any of the three properties. Thus these surveys are not balanced in interview times, rotation groups, and recall times for monthly sample. On the other hand, as seen in Figure 2.1, the 3-level 3-5-2 rotation design satisfies all the properties. Therefore, this design is balanced in 3-ways. The one-level 4-8-4 design is balanced in two-ways and considered as a special case of the extended three-way balanced multi-level design.

3. A NECESSARY CONDITION FOR THREE-WAY BALANCING

We investigate the relationship among r_{1i}, r_{2i}, m and l for the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design. A necessary condition for the three-way balancing is presented by imposing certain rules on the numbers, r_{1i}, r_{2i}, m and l as follows.

THEOREM 3.1. *Suppose that the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design is balanced in 3-ways. For each given k, $k = 0, 1, \ldots, G-1$ and $G = \sum_{i=1}^{m} r_{1i}$, there is the unique integer m_k, $1 \le m_k \le G$ satisfying*

$$\text{mod}_G \left\{ m_k + k + (m_k - 1)(l - 1) + \sum_{i=1}^{m-1} r_{2i} I_{[m_k > \sum_{\xi=1}^{i} r_{1\xi}]} \right\} = 0, \quad (3.1)$$

where $I_{[m_k > \sum_{\xi=1}^{i} r_{1\xi}]} = 1$ if $m_k > \sum_{\xi=1}^{i} r_{1\xi}$ and $= 0$ otherwise.

The relationship among the numbers, r_{1i}, r_{2i}, m and l is formulated by observing m_k which satisfy (3.1) for all k. For example, in the 3-level 4-8-4 design with $r_{11} = r_{12} = 4$, $r_{21} = 8$, $m = 2$ and $l = 3$, we have $m_0 = 6$ for $k = 0$, $m_1 = 3$ for $k = 1$, $m_2 = 8$ for $k = 2$ and $m_3 = 5$ for $k = 3$, and so on. However, the 2-level 4-8-4 design does not have such m_k for all $k = 0, 1, \ldots, 7$. Similarly, the 2-level 2-3-1 design has $m_0 = 2$, $m_1 = 3$ and $m_2 = 1$, whereas in the 2-level 2-5-1 design, $m_0 = 2$, $m_2 = 1$ but m_1 is not obtainable. This implies that the necessary condition given in Theorem 3.1 depends on the number of cycles m, the levels of recall l, the number of in-sample months r_{1i} as well as the number of out-sample months r_{2i}.

In particular, when $r_{1i} = r_1$ and $r_{2i} = r_2$ for all $i = 1, 2, \ldots, m$ where r_2 is a multiple of r_1, the necessary condition given in (3.1) is reduced to

$$\text{mod}_{\,mr_1}\left\{m_k + k + (m_k - 1)(l - 1) + \left[\frac{m_k - 1}{r_1}\right] r_2\right\} = 0, \tag{3.2}$$

where $[\cdot]$ is the integer operator. This is the necessary condition for the previous three-way balanced design presented by Park *et al.* (2003). Furthermore, when we add a $l = 1$ to previous restrictions, the necessary condition given in (3.1) is reduced to

$$\text{mod}_{\,mr_1}\left\{m_k + k + \left[\frac{m_k - 1}{r_1}\right] r_2\right\} = 0, \tag{3.3}$$

where $[\cdot]$ is the integer operator. Since r_2 is a multiple of r_1, there is an integer m^* such that $m_0 = r_1 m^*$ for some $m^* = 1, 2, \ldots, m$. Plugging this $m_0 = r_1 m^*$ into (3.2), we have $\text{mod}_{mr_1}\{m^* r_1 + (m^* - 1) r_2\} = 0$. This is the necessary condition for the previous two-way balanced design presented by Park *et al.* (2001). These imply that the condition (3.1) produces more general class of two-way or three-way balanced designs than that of the previous two-way or three-way balanced designs because we allow different r_{1i} and r_{2i}.

4. An Algorithm for Constructing l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ Design

With r_{1i} months for a sample unit being in the sample, r_{2i} months for being out of sample, m cycles, and l recall times, we can construct the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design which is balanced in 3-ways whenever all m_k satisfying (3.1) exist for all k. The algorithm below provides the three-way balanced rotation design which is essentially the same as that of Park *et al.* (2001) except the extended recall time. This algorithm allocates not only sample units with the recall time 0 at each survey month, but also with other $l - 1$ recall times (*i.e.*,

$j = 0, 1, \ldots, l-1$). Using the three-way balanced 3-level 3-8-5 rotation design in Figure 4.1, we illustrate construction of the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design following the 3 steps algorithm below.

Step 1. To create the column labels for the units and groups on the top two rows, arrange sample units by their affiliation indices (α, g) in the order of $(1,1), (1,2), \ldots, (1,G), \ldots, (\alpha^*,1), (\alpha^*,2), \ldots, (\alpha^*,G)$ where $G = \sum_{i=1}^{m} r_{1i} = 8$ and α^* is a positive integer so that all sample units can last for the entire survey.

Step 2. Next fill in the third row for the initial month t. According to the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design, select every l^{th} index by the reverse order of the rotation pattern from Step 1 select every l^{th} index until we have the first r_{1m} indices; then leave the next $r_{2,m-1} + l - 1$ indices, repeating until the last r_{11} indices are selected. The G sample units with the selected indices are the sample of the initial month t. For the design with the 3-level 3-8-5 system in Figure 4.1, the first $r_{12}(=5)$ indices are $(1,1)$, $(1,4)$, $(1,7)$, $(2,2)$ and $(2,5)$ and the second $r_{11}(=3)$ indices are $(3,8)$, $(4,3)$ and $(4,6)$ after skipping $r_{21} + l - 1 = 10$ successive indices.

Step 3. To fill in the remaining rows, shift the row of month t one column to the right for each advancing month.

α	1								2								3								4								5								6	
g	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2
t	u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II										
$t+1$		u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II									
$M\,t+2$			u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II								
$O\,t+3$				u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II							
$N\,t+4$					u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II						
$T\,t+5$						u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II					
$H\,t+6$							u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II				
$t+7$								u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II			
$t+8$									u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II		
$t+9$										u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II	
$t+10$											u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I	II
$t+11$												u_8	I	II	u_7	I	II	u_6	I	II	u_5	I	II	u_4	I	II									u_3	I	II	u_2	I	II	u_1	I

FIGURE 4.1 *The three-way balanced 3-level 3-8-5 design.*

In Figure 4.1, constructing the 3-way balanced 3-level 3-8-5 design, the symbol u_i indicates the sample unit at the recall time 0 which is interviewed for the i^{th} time at each given month. By Step 1, we construct the first two rows for the affiliation indices for the sample unit α and group g. Step 2 creates the initial row for the sample of the starting survey month t. We interview the sample unit

(4,6) for the 1^{st} time, the sample unit (4,3) for the 2^{nd} time, the unit (3,8) for the 3^{rd} time. After skipping 10 indices, we continue to interview the units, (2,5) for the 4^{th} time, (2,2) for the 5^{th} time, ..., (1,1) for the 8^{th} time. Step 3 generates the sample for month $t + t_0$, $t_0 \geq 1$ by shifting t_0 steps to the right from the initial survey month t. Now all 8 rotation groups are represented by their respective sample units in each month.

Once the allocation of the sample units with the recall time 0 at each survey month is determined, then the sample units report their information for the previous $l - 1$ months. Therefore, the allocation of the sample units with the recall time more than 0 is straightforward. For example, consider the two months t and $t + 1$ in Figure 4.1. At month t, the sample units which are indexed by (α, g)=(1,2), (1,5), (1,8), (2,3), (2,6), (4,1), (4,4) and (4,7) have the same recall time of $j = 1$. These sample units have the recall time $j = 0$ at the survey month $t + 1$. We also observe that at any recall time 0, 1, or 2, each monthly sample contains all 8 rotation groups and interview times from 1 to 8, and each rotation group contains all 8 interview times for any perpendicular span of 8 months. This implies that the 3-level 3-8-5 design is balanced in 3-ways.

Similarly, for given r_{1i}, r_{2i}, m and l in l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design, one can check the existence of m_k satisfying the equation (3.1) given by Theorem 3.1. One of the easiest way to check the existence of such m_ks can be accomplished by Step 2 of Algorithm: when we obtain the G sample units by Step 2 from G different rotation groups, the corresponding l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design has such m_ks. Whenever such m_ks exist for all k, the algorithm given above always provides the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design enjoying three-way balancing.

TABLE 4.1 *The three-way balancing alternative designs to CPS and CES*

design	$m = 2$	$m = 3$	
CPS 1-level 4-8-4	1-level 1-8-7 1-level 2-8-6 1-level 3-8-5	1-level 1-3-4-1-3, 1-level 1-5-2-1-5, 1-level 2-1-5-2-1, 1-level 3-2-3-3-2,	1-level 1-4-5-2-2 1-level 1-6-4-2-3 1-level 2-3-4-3-2 1-level 3-4-2-4-3
CES 3-level 5-0	3-level 1-5-4 3-level 1-10-4 3-level 2-5-3 3-level 2-10-3	3-level 1-2-1-4-3, 3-level 1-4-3-5-2, 3-level 2-3-1-3-2, 3-level 3-2-1-1-1,	3-level 1-3-2-4-2 3-level 1-5-3-5-1 3-level 2-4-2-3-1 3-level 3-5-1-5-1

The 1-level 4-8-4 design using 8 rotation groups is the U.S. CPS, and the 3-level 5-0 design using 5 rotation groups is U.S. CES. By a necessary condition for three-way balancing and an algorithm, we can generate alternative designs to CPS and CES. In Table 4.1, we classify alternative designs by $g = \sum_{i=1}^{m} r_{1i}$ and m.

5. CONCLUSION

The l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ design that we propose in this paper is potentially usual tool to eliminate possible unbalancing problems in monthly sample. And our design is a most general form of the rotation sampling design. Observing all the previous publications, all these previous rotation designs can be expressed as special forms of our new design.

We have shown a necessary condition and have demonstrated a algorithm to create a three-way balanced system of the extended multi-level rotation design (l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$): balanced on interview time in monthly sample, rotation group and recall time. All rotation groups have equal opportunity to be represented in the sample for every recall time.

APPENDIX

The Proof of Theorem

Let $g_t(i)$ be the rotation group which contains the sample unit interviewed for the i^{th} time with its recall time 0 at survey month t. Since all G rotation groups are included in any monthly sample and each monthly sample is balanced on interview times by (a) and (b) of Definition 2.1,

$$g_t(m_{k_1}) \neq g_t(m_{k_2}) \text{ if and only if } m_{k_1} \neq m_{k_2} \quad \text{(A.1)}$$

and there is such that

$$g_t(m_k) = g_{t+k+1}(1), \quad \text{(A.2)}$$

where m_{k_1}, m_{k_2}, $m_k = 1, 2, \ldots, G$ and $k = 0, 1, \ldots, G-1$. The m_k given in (A.2) is unique. To see this, suppose $m_{k_1} \neq m_{k_2}$ for $k_1 = k_2$. Then $g_t(m_{k_1}) = g_t(m_{k_2})$ by (A.2) which contradicts (A.1) because of $m_{k_1} \neq m_{k_2}$. Now, suppose $m_{k_1} = m_{k_2}$ for $k_1 \neq k_2$. Then $g_t(m_{k_1}) = g_t(m_{k_2})$ by (A.1), and $g_t(m_{k_1}) = g_{t+k_1+1}(1)$ and $g_t(m_{k_2}) = g_{t+k_2+1}(1)$ by (A.2). Thus we have

$$g_{t+k_1+1}(1) = g_{t+k_2+1}(1). \quad \text{(A.3)}$$

Since the perpendicular balancing of interview time in rotation group given in (c) of Definition 2.1 implies that $g_t(i) = g_{t'}(i)$ only when $\text{mod}_G\{|t - t'|\} = 0$

for each $i = 1, 2, \ldots, G$, (A.3) means $\text{mod}_G\{|k_1 - k_2|\} = 0$ which contradicts to $0 < |k_1 - k_2| < G$ by assumption. Therefore, $m_{k_1} \neq m_{k_2}$ if and only if $k_1 \neq k_2$.

To end the proof, it suffices to show that the m_k satisfies the equation (3.1) if the m_k satisfies (A.2). Denote $m_k = \sum_{i=0}^{n_{1k}} r_{1i} + n_{2k}$ in (A.2) for $n_{1k} = 0, 1, \ldots, m-1$ and $n_{2k} = 1, \ldots, r_{1,n_{1k}+1}$ with $r_{10} = 0$. This is always possible since m_k is an interview time which ranges from 1 to G.

The l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ rotation system implies that each sample unit returns to the sample for every l^{th} month. Thus, from (A.2)

$$\begin{aligned} g_t(m_k) &= g_{t+k+1}(1) = g_{t+k+1+l}(2) = \cdots \\ &= g_{t+k+1+(r_{11}-1)l}(r_{11}) = g_{t+k+1+(r_{11}-1)l+r_{21}+l}(r_{11}+1) = \cdots \\ &= g_{t+k+1+(r_{11}-1)l+r_{21}+r_{12}l}(r_{11}+r_{12}). \end{aligned} \tag{A.4}$$

Repeat this procedure until we have

$$\begin{aligned} (A.4) &= g_{t+k+1+(r_{11}+r_{12})l+r_{21}+r_{22}}(r_{11}+r_{12}+1) = \cdots \\ &= g_{t+k+1+(r_{11}+r_{12}+r_{13})l+r_{21}+r_{22}-l}(r_{11}+r_{12}+r_{13}) = \cdots \\ &= g_{t+k+1+l\sum_{i=0}^{n_{1k}+1} r_{1i}+\sum_{i=0}^{n_{1k}} r_{2i}-l}\left(\sum_{i=0}^{n_{1k}+1} r_{1i}\right), \end{aligned} \tag{A.5}$$

where $r_{20} = 0$. Equation (A.5) means that the rotation group $g_t(m_k)$ will be in sample through its sample unit which is interviewed for the $\sum_{i=0}^{n_{1k}+1} r_{1i}{}^{th}$ time at month $t + k + 1 + l\sum_{i=1}^{n_{1k}+1} r_{1i} + \sum_{i=1}^{n_{1k}} r_{2i} - l$.

On the other hand, the l-level $r_{11} - r_{21} - \cdots - r_{2,m-1} - r_{1m}$ system also produces

$$\begin{aligned} g_t(m_k) =& g_t\left(\sum_{i=0}^{n_{1k}} r_{1i} + n_{2k}\right) = g_{t+l}\left(\sum_{i=0}^{n_{1k}} r_{1i} + n_{2k} + 1\right) \\ =& g_{t+2l}\left(\sum_{i=0}^{n_{1k}} r_{1i} + n_{2k} + 2\right) = \cdots \\ =& g_{t+(r_{1,n_{1k}+1}-n_{2k})l}\left(\sum_{i=0}^{n_{1k}+1} r_{1i}\right). \end{aligned} \tag{A.6}$$

That is, this particular rotation group $g_t(m_k)$ containing the sample unit which is interviewed for the $m_k{}^{th}$ time at month t is to be again in sample through its sample unit which is interviewed for its $\sum_{i=0}^{n_{1k}+1} r_{1i}{}^{th}$ time at survey month $t + (r_{1,n_{1k}+1} - n_{2k})l$.

By (A.5) and (A.6), the rotation group $g_t(m_k)$ is to be interviewed for the same times, $\sum_{i=0}^{n_{1k}+1} r_{1i}$ times, at both months $t+k+1+l\sum_{i=0}^{n_{1k}+1} r_{1i}+\sum_{i=0}^{n_{1k}} r_{2i}-l$ and $t+(r_{1,n_{1k}+1}-n_{2k})l$. Hence, $t+k+1+l\sum_{i=0}^{n_{1k}+1} r_{1i}+\sum_{i=0}^{n_{1k}} r_{2i}-l-\{t+(r_{1,n_{1k}+1}-n_{2k})l\}$ should be a multiple of $G=\sum_{i=1}^{m} r_{1i}$ by the perpendicular balancing of interview times in rotation group. Equivalently,

$$\bmod_G\left\{\left(\sum_{i=0}^{n_{1k}} r_{1i}+n_{2k}\right)l+\sum_{i=0}^{n_{1k}} r_{2i}-l+k+1\right\}=0. \tag{A.7}$$

Finally, since

$$m_k=\sum_{i=0}^{n_{1k}} r_{1i}+n_{2k} \text{ and } n_{2k}=1,\ldots,r_{1,n_{1k}+1},\ n_{1k}=\sum_{i=1}^{m-1} I_{[m_k>\sum_{\xi=1}^{i} r_{1\xi}]}.$$

This implies that $\sum_{i=0}^{n_{1k}} r_{2i}$ is equal to $\sum_{i=1}^{m-1} r_{2i} I_{[m_k>\sum_{\xi=1}^{i} r_{1\xi}]}$. Thus we have (3.1) by (A.7).

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