A Multi-Dimensional Radio Resource Scheduling Scheme for MIMO-OFDM Wireless Systems

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Abstract: **Orthogonal frequency division multiplexing (OFDM) and multiple input multiple output (MIMO) technologies provide additional dimensions of freedom with spectral and spatial resources for radio resource management. Multi-dimensional radio resource management has recently been identified to exploit the full dimensions of freedom for more flexible and efficient utilization of scarce radio spectrum while provide diverse quality of service (QoS) guarantees. In this work, a multi-dimensional radio resource scheduling scheme is proposed to achieve above goals in hybrid orthogonal frequency division multiple access (OFDMA) and space division multiple access (SDMA) systems. Cochannel interference (CCI) introduced by frequency reuse under SDMA is eliminated by frequency division and time division between highly interfered users. This scheme maximizes system throughput subjected to the minimum data rate guarantee for heterogeneous users and transmit power constraint. By numerical examples, system throughput and fairness superiority of the our scheduling scheme are verified.**

Index Terms: **Multiple input multiple output (MIMO), multiuser channel, orthogonal frequency division multiplexing (OFDM), radio resource management, scheduling.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been emerging as a promising technology to combat frequency selective fading, by dividing a broadband signal into multiple narrowband subcarriers (i.e., frequency diversity) [1], [2]. On the other hand, multiple input multiple output (MIMO) has also been attracting much attention due to the potential to increase spectral efficiency of wireless resource by using multiple transmit and multiple receive antennas [3]–[5] (i.e., spatial diversity). The combination of OFDM and MIMO has been recognized as one of the most promising way for broadband wireless communications both in B3G/4G and in IEEE 802.16 [6], [7] and 802.11n [8]. Researches have also shown that, if the OFDM-MIMO systems are fed with multiple users, multiuser diversity can also be exploited.

From the perspective of radio resource management, the OFDM-MIMO systems provide additional dimensions of radio resources in space and frequency domain, that is, the radio resources can be divided into subchannels in time region, frequency region, as well as space region. Thus, all kinds of diversity gains such as frequency diversity, space diversity, multiuser diversity, can be exploited simultaneously. We call this new topic as multi-dimensional radio resource management. How to divide and allocate these subchannels in efficient and flexible way is becoming a big challenge.

Manuscript received April 27, 2006.

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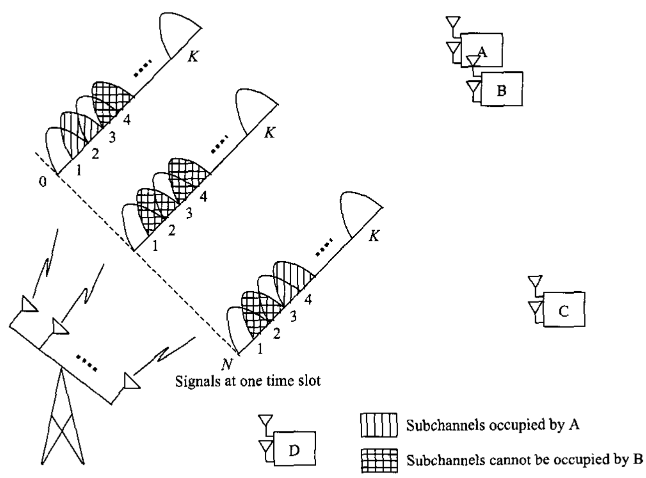

Fig. 1. An example of subchannel allocation in hybrid time division multiplexed OFDMA and SDMA when considering spatial correlation.

In multiuser OFDM systems, users' channels are always viewed as a set of ideally isolated parallel subcarriers, i.e., orthogonal frequency division multiple access (OFDMA) [9]. The bandwidth reuse by multiple spatially separable users in MIMO systems is through space division multiple access (SDMA). In SDMA, unlike in OFDMA, users' channels can not be completely orthogonal. To release the burden of multiuser detection, it is suggested to divide the users into groups by avoiding two highly correlated users being placed into a same group. Users in the same group can be multiplexed spatially, while groups must be multiplexed through orthogonal multiple access, say TDMA, OFDMA, etc. Fig. 1 shows an example in a single cellular downlink with N SDMA subchannels and K OFDMA subchannels. Users A and B are very close in location, thus they badly interference with each other when transmitting in a way of spatial division. They must communicate with the base station in different TDMA time slots or in different OFDM subcarriers. As the figure shows, since they are placed in the same time slot (perhaps due to data rate requirement or delay constraint), the OFDM subcarrier should be carefully allocated, as to B does not use the subcarrier occupied by A. In this circumstance, subchannel scheduling algorithm becomes more important.

So far, resource allocation in multiuser OFDM systems has been well studied. Typical subcarrier, power and bit allocation algorithms include maximizing the sum capacity subject to power constraint [10] and minimizing total transmitted power subject to certain quality of service (QoS) services [11]. Research in [12] extends the study in [11] by considering user spatial separability in multiuser MIMO channel. This research was under beamforming transmission uplink circumstance rather than the general SDMA approach. Another channel allocation algorithm considering user grouping is proposed in [13], a necessary constraint group size is introduced when solving the optimization problem.

In this paper, we propose a more general subchannel scheduling algorithm in adaptive OFDMA and SDMA broadcast channels. Cochannel interference (CCI) introduced by frequency reuse under SDMA is eliminated by user grouping algorithm. In comparison with the work in [12], our algorithm is a capacity maximal model and considers a more general linear coding SDMA circumstance. Compare to the work in [13], we take out the group size constraint in solving the optimization problem. We formulate the resource scheduling as a generalized processor sharing (GPS) type optimization problem across the subcarriers and spatial subchannels. The objective is to maximize system throughput while subjecting to the minimum data rate guarantee for heterogeneous users and transmission power constraint. To solve the optimization problem, a channel aware parallel weighted fair queueing (Cap-WFQ) is proposed. Simulation results show that the proposed algorithm achieves better performance in terms of system throughput and fairness.

The rest of this paper is organized as follows. In Section II, the system model of hybrid OFDMA and SDMA downlink system is described. In Section III. the optimized subchannel scheduling problem is formulated. The Cap-WFQ algorithm is proposed in Section IV. Simulation results are shown in Section V and conclusions are given in Section VI.

II. SYSTEM MODEL

We consider a MIMO-OFDM downlink system with one base station and multiple users. Suppose currently there are M active users waiting for service. The i-th ($i = 1, 2, \cdots, M$) user has the bandwidth requirement as R_i. The base station is equipped with N_T transmit antennas and user i ($i = 1, 2, \cdots, M$) is equipped with N_{R_i} receive antennas. Assume there are K independently allocatable subcarriers in the OFDM system and N independently allocatable spatial subchannels in the MIMO system.

Throughout this paper, we assume that the bandwidth of each subcarrier is less than the channel coherence bandwidth so that the channel appears to be flat fading on each subcarrier. Additionally, we suppose that the channel is block fading, i.e., the channel condition is constant over one time slot but varies among different time slots. The radio resource scheduling algorithm is carried out at the very beginning of each time slot.

In single input single output-OFDM (SISO-OFDM) system, the inter-carrier interference and inter-symbol interference can be conveniently eliminated by the cyclic prefix and perfect synchronization. Therefore, the OFDM channel is always modeled as a set of ideally parallel subchannels. However in a multiuser MIMO-OFDM system, let $\mathbf{H}_k^{(i)} \in \mathbf{C}^{N_{R_i} \times N_T}$ denote the MIMO channel matrix of user i on subcarrier k, $\mathbf{s}_k^{(i)} \in \mathbf{C}^{N \times 1}$ be the transmit signal on the transmit antenna array for user i, and $\mathbf{x}_k^{(i)} \in \mathbf{C}^{N_{R_i} \times 1}$ be the corresponding received vector

$$\mathbf{x}_k^{(i)} = \sqrt{\mathbf{p}_k^{(i)}}\, \mathbf{H}_k^{(i)}\, \mathbf{s}_k^{(i)} + \sqrt{\mathbf{p}_k^{(j)}}\, \mathbf{H}_k^{(i)} \sum_{j=1, j\neq i}^{M} \mathbf{s}_k^{(j)} + \mathbf{n}_k^{(i)} \qquad (1)$$

where $\sqrt{\mathbf{p}_k^{(i)}}$ indicates the transmit power on the antenna array and $\mathbf{n}_k^{(i)}$ is additive Gaussian noise.

In (1), the first item is useful information and the second is undesirable item denoting the CCI introduced by SDMA. CCI varies as a function of the scattering environment, the distance between the base station and users, the arrival and departure direction, the antenna geometrics, and so on. If some of two users' multipath channels are similar, for example they have a common scatterer, the two users will highly correlate.

For the multiuser MIMO-OFDM systems, orthogonal multiple accesses, such as OFDMA and TDMA, are popular options to avoid multiuser interference. This suggests a multiple access strategy where the users are divided into groups by avoiding place two highly correlated users into the same group and applying SDMA within each group and orthogonal multiple access between the groups. In the optimal resource allocation scheme formulation in the next section, we will take consideration of the user grouping effect.

III. OPTIMAL RESOURCE MANAGEMENT SCHEME

We formulate a GPS approach for joint spectral and spatial resource management. GPS is a flow-based ideally fair scheduling discipline. It assumes that multiple users can be served simultaneously according to the preestablished weights. The parallel character in fact is quite suitable for MIMO-OFDM systems where multiple users can be served simultaneously using different spatial and frequency subchannels. That was our motivation to study a GPS-type scheduling in MIMO-OFDM systems. [14] integrated GPS scheduling in OFDM subcarrier allocation, while our scheme is more general as it joints both spectral and spatial resource management. Similar work is also in [15].

In order to apply the GPS scheduling, we first assume that the number of subcarriers K is large enough such that the subchannel allocation can be carried out at any small frequency band. Therefore the subchannel distribution can be defined as continuous functions. We also assume that N times frequency reuse is feasible by SDMA. We introduce a time sharing factor $\rho_i(r, s)$ to indicate whether user i occupies the r-th spatial subchannel and s-th subcarrier, where $r = 1, 2, \cdots, N$, $s \in [0, W)$. W is the system bandwidth. The power allocating factor $P_i(r, s)$ indicates how much power allocated to user i on the r-th spatial subchannel and s-th subcarrier. It is subject to the total power constraint, that is

$$\sum_{i\in M} \sum_{r=1}^{N} \int_0^W P_i(r, s) ds \leq P_T. \qquad (2)$$

Suppose the M users in the system have divided into groups $G_1, \cdots, G_I, \cdots, G_J, \cdots, G_X$ according to their spatial compatibility. Users from different groups can not share the same subcarrier, i.e.,

$$\sum_{i\in G_I} \sum_{r=1}^{N} \rho_i(r, s) \sum_{j\in G_J} \sum_{r=1}^{N} \rho_j(r, s) = 0, \qquad i \neq j. \qquad (3)$$

At a GPS node, call admission control (CAC) interprets the i-th user's minimum tolerant data rate requirement R_i as the

$$\begin{cases} \max_{\rho_i(r,s),P_i(r,s)} \sum_{i\in M}\sum_{r=1}^{N}\int_0^W \log_2(1+\rho_i(r,s)P_i(r,s)\frac{\alpha_{r,s}^{(i)}}{N_0})ds \\ \text{subject to:} \\ \frac{\sum_{r=1}^{N}\int_0^W \log(1+\rho_i(r,s)P_i(r,s)\frac{\alpha_{r,s}^{(i)}}{N_0})ds}{\phi_i} = \frac{\sum_{r=1}^{N}\int_0^W \log(1+\rho_j(r,s)P_j(r,s)\frac{\alpha_{r,s}^{(j)}}{N_0})ds}{\phi_j}, & i,j\in M \\ \sum_{i\in G_I}\sum_{r=1}^{N}\rho_i(r,s)\sum_{j\in G_J}\sum_{r=1}^{N}\rho_j(r,s)=0, & i\neq j, \text{ for all } s \\ \sum_{i\in M}\rho_i(r,s)\leq 1, & \text{for all } r,s \\ \sum_{i\in M}\sum_{r=1}^{N}\int_0^W P_i(r,s)ds\leq P_T \\ P_i(r,s)\geq 0 \end{cases} \quad (4)$$

corresponding weight ϕ_i. Our objective is to maximize the total system throughput, subjecting to the predefined weights.

The optimal resource management is described in (4), where $\alpha_{r,s}^{(i)}$ is the channel gain for user i on the r-th spatial subchannel and s-th subcarrier and N_0 is noise power in a single subcarrier bandwidth. The first constraint in (4) is GPS discipline. The third constraint means that no more than one user transmits in the same subcarrier at one spatial subchannel.

This optimization is an ideal GPS scheduling. Although the parallel transmission property makes the joint MIMO-OFDM more fit for an ideal GPS scheduling, the real joint MIMO-OFDM system still transmits symbols as entities. Neither the bandwidth nor the transmission symbol is infinitely divisible. In the following, we propose a modified weighted fair queueing (WFQ) by applying the WFQ discipline in a channel aware parallel channel scheduling to fit for the environment in the joint MIMO-OFDM systems. WFQ has been considered as the best scheduling scheme among the ways to emulate GPS service in real packet systems. We call our modified WFQ as Cap-WFQ.

IV. CAP-WFQ IMPLEMENTATION

By Cap-WFQ, we decompose the optimization problem in (4) into two steps: User selection by WFQ discipline and mapping between the selected users and the subchannels.

WFQ discipline uses the notion of virtual finish time [16]. When the scheduler is ready to transmit the next packet at a time slot, it picks the packet that would be the first to complete service among all the packets queued in the system.

We implement the scheduling by the WFQ as follows.

1. Maintain a first in first out (FIFO) queue for symbols for user i. For the l-th arrival symbol, calculate the timestamp $t_i^{(l)}$
$$t_i^{(l)} = \max\{V,\ t_i^{(l-1)}\} + \frac{1}{\phi_i} \quad (5)$$
where $t_i^{(l-1)}$ is the timestamp of the $(l-1)$-th symbol and ϕ_i is used as the weight. V is the reference virtual time maintained by the scheduler, which keeps the timestamp of the last transmitted symbol from the scheduler. It is used to determine where the timestamp of a new or resumed user should start. Note that there is single variable V shared by all users in the scheduler. The timestamp $t_i^{(l)}$ specifies the expected transmission completion time for the l-th arrive symbol.
2. Schedule users based on the timestamps. Here the parallel WFQ, unlike traditional ones, NK symbols with the smallest timestamps among all queues are selected to form a selected symbol group (SSG) for transmission. It is possible that more than one symbol of a user are selected in a SSG under the condition that the user has relatively large minimum tolerant data rate requirement thus having relatively large weight or has starved for services for long time. It is also possible that none of the symbol for a user is selected, for relatively small weight the user possesses or have received excessive services before.

SSG contains symbols from different user groups. They cannot share a same subcarrier. If the users in a group cannot occupy all the spatial subchannels on a subcarrier, the unoccupied spatial subchannels will be empty. Therefore NK symbols in the SSG are unlikely accommodated in the total NK subchannels. Some schemes are required to pick up appropriate number of symbols in SSG to form another transmit symbol group (TSG), then try to refill TSG with symbols belonging to appropriate users.

We use $G'_I(I=1,2,\cdots,X)$ to denote the subset of G_I whose elements are symbols belonging to the users in G_I but selected to the SSG. The vacancy number of spatial subchannels for G'_I is

$$N_{G'_I}^{v} = N - (\mid G'_I \mid - \left\lfloor \frac{\mid G'_I \mid}{N} \right\rfloor N) \quad (6)$$

where $\mid \cdot \mid$ denotes the number of elements in a set. To refill TSG, we first search for vacancies of all groups. The number of vacancies is the number of symbols we must take out from the SSG. Since a group with largest number of elements can fill in as many spatial subchannels on the same subcarrier, it probably leaves least number of vacancies. Our search begins with the largest groups. The scheme is described as follows.

1. Look for the largest group G'_I in SSG.

2. Calculate $N^v_{G'_I}$ and take out $N^v_{G'_I}$ symbols with largest timestamps from the remaining groups except G'_I.
3. Add G'_I to TSG and delete it in SSG.
4. Reselect $N^v_{G'_I}$ symbols with smallest timestamps from G_I to TSG to refill the vacancies.
5. Go on with 1 until SSG is emptied.

The parallel WFQ scheme itself does not exploit diversity gains. It provides data rate guarantee for QoS services only. With the TSG the practical optimal subchannel allocation can be derived from (4) as (7).

$$\begin{cases} \max\limits_{\substack{\rho_j(r,c),\\ P_j(r,c)}} \sum\limits_{j\in TSG}\sum\limits_{r=1}^{N}\sum\limits_{c=1}^{K} \log_2(1+\rho_j(r,c)P_j(r,c)\frac{\alpha^{(j)}_{r,c}}{N_0}) \\ \text{subject to:} \\ \sum\limits_{i\in G''_I}\sum\limits_{r=1}^{N}\rho_i(r,c)\sum\limits_{j\in G''_J}\sum\limits_{r=1}^{N}\rho_j(r,c)=0,\ i\neq j,\ \text{for all } c \\ \sum\limits_{j\in TSG}\rho_j(r,c)\leq 1, \qquad \text{for all } r,c \\ \sum\limits_{j\in TSG}\sum\limits_{r=1}^{N}\sum\limits_{c=1}^{K}P_j(r,c)\leq P_T, \\ P_j(r,c)\geq 0 \end{cases} \quad (7)$$

where $P_j(r,c)$ is power allocation factor. Unlike the one in (4), it is a discrete version, with c and j representing subcarrier index and symbol index separately in the TSG. G''_I and G''_J in (7) are subsets of G_I and G_J, respectively, whose elements are symbols belonging to the users in G_I and G_J selected to the TSG.

The problem in (7) is a nonlinear optimization problem. The objective is an increasing function with $\alpha^{(j)}_{r,c}$, therefore the problem is to look for the best combination of larger $\alpha^{(j)}_{r,c}$s among all available users' channel gains in the TSG.

We seek a suboptimal solution. For each subcarrier, we look for N largest $\alpha^{(j)}_{r,c}$s in each G''_I. If we use X' to denote the number of groups in the TSG, a total number of KNX' values are selected from the overall KNM available $\alpha^{(j)}_{r,c}$ parameters. Then for KX' possible combinations of N selected $\alpha^{(j)}_{r,c}$s, we apply power allocation among NK subchannels. We rearrange the selected $\alpha^{(j)}_{r,c}$s in descending order as $\alpha_1\geq\alpha_2\geq\cdots\geq\alpha_{NK}$. The power allocation problem is formulated as

$$\begin{cases} \max\limits_{P_k}\prod\limits_{k=1}^{NK}(1+P_k\frac{\alpha_k}{N_0}) \\ \text{subject to:} \\ \sum\limits_{k=1}^{N}KP_k\leq P_T \\ P_k\geq 0. \end{cases} \quad (8)$$

Solution to (8) is provided in the Appendix in detail. If

$$P_T > NK(\frac{N_0}{\alpha_{NK}}-\frac{1}{NK}\sum_{l=1}^{NK}\frac{N_0}{\alpha_l}) \quad (9)$$

is satisfied, when $\mathbf{P}=\mathbf{P}^*=[p^*_1,p^*_2,\cdots,p^*_{NK}]^T$, where

$$p^*_k=\frac{1}{NK}P_T+\frac{1}{NK}\sum_{l=1}^{NK}\frac{N_0}{\alpha_l}-\frac{N_0}{\alpha_k},\quad k=1,2,\cdots,NK \quad (10)$$

the global maximum value of (8) is

$$C(\mathbf{P}^*)=(\frac{1}{NK}P_T+\frac{1}{NK}\sum_{l=1}^{NK}\frac{N_0}{\alpha_l})^{NK}\prod_{k=1}^{NK}\frac{\alpha_k}{N_0}. \quad (11)$$

After power allocation, we compare the KX' possible $C(\mathbf{P}^*)$s, then pick up the largest one. The corresponding $\mathbf{P}^*$ is the final power allocation scheme, and the according combination of $\alpha_1\geq\alpha_2\geq\cdots\geq\alpha_{NK}$ indicates the appropriate subchannel allocation.

V. NUMERICAL RESULTS

In this section, we evaluate performance of the proposed method under a zero forcing SDMA transmission system [17]. In the simulation, we considered 8 transmit antennas at the base station and 2 receive antennas at each user. Thus in zero forcing SDMA system, the number of maximum simultaneous users in SDMA was 4. An OFDM system with 16 subcarriers was considered. Suppose there were 24 active users in the system.

We first evaluated the influence of both multiuser diversity gain and CCI on the system capacity. The average transmit SNR was set to be 10 dB. In this simulation, we changed the threshold in the spatial compatible user grouping algorithm to divide the users into 1 to 24 groups. The proposed Cap-WFQ with and without user grouping algorithm were compared with round robin scheduling schemes. The capacity under round robin scheme with user grouping algorithm reflected the CCI elimination gain over the one without user grouping algorithm. While the Cap-WFQ with user grouping algorithm were influenced by both multiuser diversity gain and CCI elimination gain. As shown in Fig. 2, when the group number is small (from 1 to 3), the CCI elimination gain is not obvious. The system capacity increases slightly for round robin scheme. However for Cap-WFQ, the system capacity decreases because the increment of CCI elimination gain can not compensate the decrement in multiuser diversity loss, for the choice of users in spatial are constraint in the groups with the freedom of spatial diversity decreasing from 24 to 6 in average (average user number in each group). Then as the group number continues to increase, CCI elimination gain turns out to be the dominant factor. When the system capacity for round robin scheme increases 0.8 bps/Hz (group number increases from 4 to 12), the system capacity for Cap-WFQ begins to increase and exceeds the curve without user grouping. The increment is round 0.5 bps/Hz. Then when the group number reaches 24, that is all the users are not spatial compatible, the SDMA system changes into a single user MIMO, thus spatial diversity no longer exists. Again the multiuser diversity loss overcomes the CCI elimination gain. System capacity decreases rapidly. At this time, the gap between Cap-WFQ and round robin scheme is achieved from frequency diversity gain.

Fig. 3 depicts the system capacity versus average SNR. Group number in the spatial compatible user selection algorithm was set to be 12. This figure shows that the system capacity for all methods increase with the average SNR and the Cap-WFQ algorithms outperform round robin schemes for all the values of the average SNR.

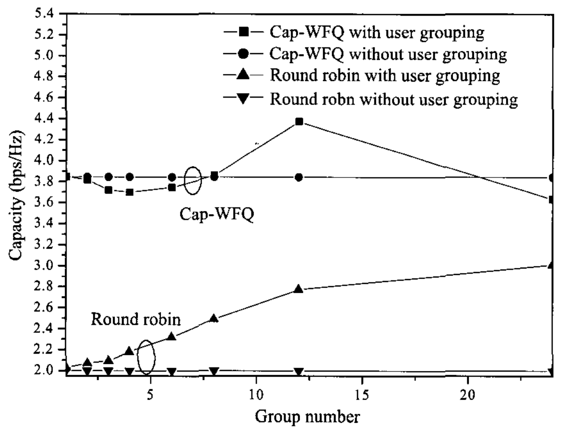

Fig. 2. Channel efficiency as a function of user group number.

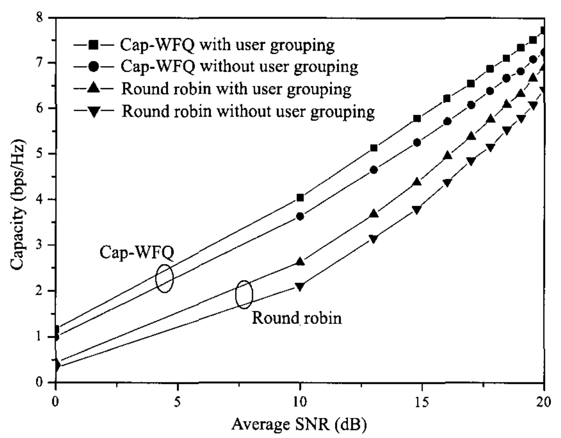

Fig. 3. Channel efficiency as a function of average SNR.

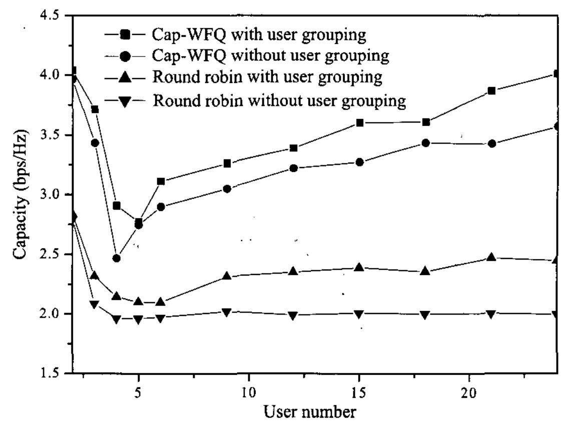

Fig. 4. Channel efficiency as a function of user number.

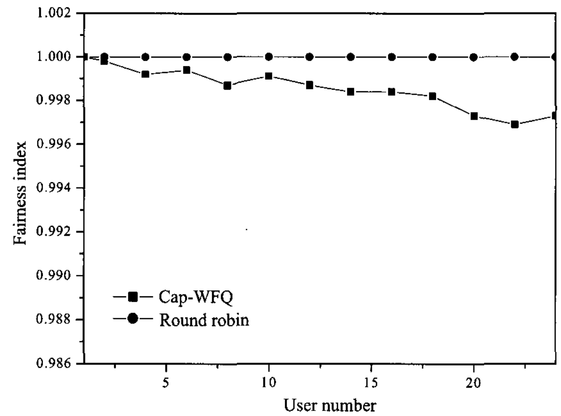

Fig. 5. Fairness index as a function of user number.

In Fig. 4, we evaluate the channel efficiency as a function of user number. In this simulation, the average SNR was set to be 10 dB and the group number was set to be $\frac{\text{user number}}{2}$. As the figure shows, when the user number increases from 2 to 4, multiuser interference increases, then capacity decreases. As the user number continues to increase, user grouping algorithm can eliminate some interference (the performance of round robin with user grouping over that without user grouping), however, multiuser diversity gain is more notable. The system capacity for Cap-WFQ algorithms increases with the user number.

We evaluated the fairness of Cap-WFQ by use of fairness index defined as

$$\delta_F = \frac{(\sum_{i=1}^{M} \bar{R}_i)^2}{M \sum_{i=1}^{M} \bar{R}_i^2} \tag{12}$$

where $\bar{R}_i$ is user i's ($i = 1, 2, \cdots, M$) throughput in time scale average. δ_F is less than or equal to 1 and larger δ_F indicates better fairness. We suppose in round robin scheduling all users can share the system throughput equally in long time average, thus δ_F is equal to 1 (where all $\bar{R}_i$s are equal). The fairness comparison of Cap-WFQ and round robin scheduling is shown in Fig. 5. In this simulation, we set all the users in the system having the same weights. We simulated 50 time slots and recorded throughput of each user. As the figure indicates, the fairness index of Cap-WFQ decreases slightly with the increment of user number while the minimal fairness index is no less than 0.997. This means the fairness guarantee of Cap-WFQ is excellent.

VI. CONCLUSIONS

In this paper, we propose a multi-dimensional radio resource scheduling scheme in MIMO-OFDM systems. OFDMA and SDMA are taken into consideration. CCI introduced by frequency reuse under SDMA is eliminated by controlling subcarrier sharing of spatial users by scheduling. By numerical examples, system throughput and fairness superiority of the proposed scheduling scheme are verified.

APPENDIX: SOLUTION TO OPTIMIZATION PROBLEM IN (8)

In the following analysis, we denote $n = NK$, $\beta = \frac{\alpha}{N_0}$ for simplicity.

A. Equivalent Problem

$$\begin{cases} \max_{\mathbf{X}} f(\mathbf{X}) = \prod_{k=1}^{n} (1 + \beta_k x_k) \\ \text{subject to:} \\ \sum_{k=1}^{n} x_k \le P_T \\ x_k \ge 0, \quad k = 1, 2, \cdots, n \end{cases} \tag{13}$$

where $\mathbf{X} = [x_1, x_2, \cdots, x_n]^T$, $P_T > 0$, $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n > 0$.

B. Extreme Value in Feasible Region

The gradient of $f(\mathbf{X})$ is

$$\nabla f(\mathbf{X}) = f(\mathbf{X}) \left[\frac{\beta_1}{1+\beta_1 x_1}, \frac{\beta_2}{1+\beta_2 x_2}, \cdots, \frac{\beta_n}{1+\beta_n x_n}\right]^T. \quad (14)$$

Because all the elements of $\nabla f(\mathbf{X})$ is always larger than zero in feasible region, there will be no extreme value in feasible region for $f(\mathbf{X})$. Therefore, the maximum value of $f(\mathbf{X})$ must be on the feasible region boundary.

C. Maximum Value on the Feasible Region Boundary

For any integer i between 1 and n, for any point $\mathbf{X}_1 = [x_{11}, x_{12}, \cdots, x_{1n}]^T$ on the feasible region boundary

$$\begin{cases} x_i = 0 \\ \sum_{k=1}^{n} x_k < P_T \end{cases} \quad (15)$$

there is a point $\mathbf{X}_2 = [x_{21}, x_{22}, \cdots, x_{2n}]^T$ in the feasible region, so as to

$$\begin{cases} x_{2k} = x_{1k} & k \neq i \\ x_{2k} = P_T - \sum_{j=1}^{n} x_{1j} & k = i. \end{cases} \quad (16)$$

Then, we get

$$f(\mathbf{X}_1) = \prod_{k=1,k\neq i}^{n} (1+\beta_k x_{1k}) < (1+\beta_i(P_T - \sum_{j=1}^{n} x_{1j})) \prod_{k=1,k\neq i}^{n} (1+\beta_k x_{1k}) = f(\mathbf{X}_2). \quad (17)$$

Therefore, the maximum value of $f(\mathbf{X})$ is not on this feasible region boundary.

From the analysis above, we conclude that the maximum value of $f(\mathbf{X})$ must be on the following feasible region boundary

$$\begin{cases} \sum_{k=1}^{n} x_k = P_T, \\ x_k \geq 0, \quad k = 1, 2, \cdots, n. \end{cases} \quad (18)$$

D. Optimization on the Feasible Region Boundary

Put the boundary equation $\sum_{k=1}^{n} x_k = P_T$ into $f(\mathbf{X})$, then the optimization problem in (16) turns into the following optimization problem

$$\begin{cases} \max_{\mathbf{X}_b} F(\mathbf{X}_b) = (1+\beta_n P_T - \beta_n \sum_{k=1}^{n-1} x_k) \prod_{k=1}^{n-1} (1+\beta_k x_k) \\ \text{subject to:} \\ \sum_{k=1}^{n-1} x_k \leq P_T \\ x_k \geq 0, \quad k = 1, 2, \cdots, n-1 \end{cases} \quad (19)$$

where $\mathbf{X}_b = [x_1, x_2, \cdots, x_{n-1}]^T$.

E. Stagnation Point of $F(\mathbf{X}_b)$

$$\begin{aligned} \frac{\partial F}{\partial x_i} &= (\beta_i(1+\beta_n P_T - \beta_n \sum_{k=1}^{n-1} x_k) - \beta_n(1+\beta_i x_i)) \\ &\quad \times \prod_{k=1,k\neq i}^{n-1} (1+\beta_k x_k) \\ &= (\beta_i + \beta_i\beta_n P_T - \beta_n - \beta_i\beta_n \sum_{k=1}^{n-1} x_k - \beta_i\beta_n x_i) \\ &\quad \times \prod_{k=1,k\neq i}^{n-1} (1+\beta_k x_k). \end{aligned} \quad (20)$$

Let $\nabla F(\mathbf{X}_b^*) = 0$, then we get the stagnation point equation from (20),

$$\mathbf{A}\mathbf{X}_b^* = \mathbf{B} \quad (21)$$

where

$$\mathbf{A} = \begin{bmatrix} 2\beta_1 & \beta_1 & \cdots & \beta_1 \\ \beta_2 & 2\beta_2 & \cdots & \beta_2 \\ \cdots & \cdots & \ddots & \cdots \\ \beta_{n-1} & \beta_{n-1} & \cdots & 2\beta_{n-1} \end{bmatrix} \quad (22)$$

$$= \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \beta_{n-1} \end{bmatrix} \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \cdots & \cdots & \ddots & \cdots \\ 1 & 1 & \cdots & 2 \end{bmatrix} \quad (23)$$

$$\begin{aligned} \mathbf{B} &= [b_1, b_2, \cdots, b_{n-1}]^T \\ &= \left[\beta_1(\frac{1}{\beta_n} + P_T) - 1, \beta_2(\frac{1}{\beta_n} + P_T) - 1, \right. \\ &\quad \left. \cdots, \beta_{n-1}(\frac{1}{\beta_n} + P_T) - 1\right]^T. \end{aligned} \quad (24)$$

Let

$$\mathbf{C} = \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \cdots & \cdots & \ddots & \cdots \\ 1 & 1 & \cdots & 2 \end{bmatrix} \quad (25)$$

then, we get

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & \frac{n-1}{n} & \cdots & -\frac{1}{n} \\ \cdots & \cdots & \ddots & \cdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & \frac{n-1}{n} \end{bmatrix}. \quad (26)$$

Therefore, we get

$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{n-1}{n\beta_1} & -\frac{1}{n\beta_2} & \cdots & -\frac{1}{n\beta_{n-1}} \\ -\frac{1}{n\beta_1} & \frac{n-1}{n\beta_2} & \cdots & -\frac{1}{n\beta_{n-1}} \\ \cdots & \cdots & \ddots & \cdots \\ -\frac{1}{n\beta_1} & -\frac{1}{n\beta_2} & \cdots & \frac{n-1}{n\beta_{n-1}} \end{bmatrix}. \quad (27)$$

Thus, the stagnation point of $F(\mathbf{X}_b)$ is

$$\mathbf{X}_b^* = [x_1^*, x_2^*, \cdots, x_{n-1}^*]^T = \mathbf{A}^{-1}\mathbf{B} \tag{28}$$

where

$$x_k^* = \frac{b_k}{\beta_k} - \frac{1}{n}\sum_{l=1}^{n-1}\frac{b_l}{\beta_l} = \frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_k} \qquad k = 1, 2, \cdots, n-1. \tag{29}$$

F. Remarks on the Stagnation Point of $F(\mathbf{X}_b)$

According to (29),

$$\begin{aligned}\sum_{k=1}^{n-1} x_k^* &= \frac{n-1}{n}P_T + \frac{n-1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \sum_{k=1}^{n-1}\frac{1}{\beta_k} \\ &= P_T - (\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_n}).\end{aligned} \tag{30}$$

Thus, when $\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} > \frac{1}{\beta_n}$, we get

$$\sum_{k=1}^{n-1} x_k^* < P_T. \tag{31}$$

That is

$$x_n^* = P_T - \sum_{k=1}^{n-1} x_k^* = \frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_n} > 0. \tag{32}$$

We notice that $\beta_n = \min(\beta_1, \beta_2, \cdots, \beta_n)$, therefore

$$x_k^* \geq \frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_n} > 0, \quad k = 1, 2, \cdots, n-1. \tag{33}$$

In summary, we get when the following inequation holds

$$P_T > n(\frac{1}{\beta_n} - \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l}) \tag{34}$$

the stagnation point of $\mathbf{X}_b^*$ is in the feasible region of the optimization problem (19).

G. Analysis of the Extremum of $F(\mathbf{X}_b)$

$$\frac{\partial^2 F}{\partial x_i \partial x_j} = \begin{cases} -2\beta_i\beta_n \prod_{k=1,k\neq i}^{n-1}(1+\beta_k x_k), & i = j \\ \Big(-\beta_i\beta_n(1+\beta_j x_j) + \beta_j(\beta_i + \beta_i\beta_n P_T - \beta_n \\ -\beta_i\beta_n \sum_{k=1}^{n-1} x_k - \beta_i\beta_n x_i)\Big) \prod_{k=1,k\neq i,j}^{n-1}(1+\beta_k x_k), & i \neq j. \end{cases} \tag{35}$$

On the stagnation point of $\mathbf{X}_b^*$, (35) becomes

$$\left.\frac{\partial^2 F}{\partial x_i \partial x_j}\right|_{\mathbf{X}_b = \mathbf{X}_b^*} = \begin{cases} -\frac{2\beta_i}{1+\beta_i x_i^*}\beta_n \prod_{k=1}^{n-1}(1+\beta_k x_k^*), & i = j \\ -\frac{\beta_i}{1+\beta_i x_i^*}\beta_n \prod_{k=1}^{n-1}(1+\beta_k x_k^*), & i \neq j. \end{cases} \tag{36}$$

We notice that

$$1 + \beta_i x_i^* = \beta_i(\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l}). \tag{37}$$

Insert (37) into (36), then we get the Hessian matrix on the point $\mathbf{X}_b^*$ for $F(\mathbf{X}_b)$

$$\mathbf{H} = -(\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l})^{n-2}\prod_{k=1}^{n}\beta_k\mathbf{C}. \tag{38}$$

It is easy to get the eigenvalues of matrix $\mathbf{C}$

$$\lambda_1 = 1, \lambda_2 = 1, \cdots, \lambda_{n-1} = 1, \lambda_n = n+1. \tag{39}$$

Because all the eigenvalues of $\mathbf{C}$ are positive, $\mathbf{C}$ is positive definite. $\mathbf{H}$ is negative definite. Hence $F(\mathbf{X}_b)$ reaches local maximum value on $\mathbf{X}_b^*$. Since $F(\mathbf{X}_b)$ has only extremum point, it reaches global maximum on $\mathbf{X}_b^*$,

$$\begin{aligned} F(\mathbf{X}_b^*) &= (1 - \beta_n(\frac{1}{\beta_n} - \frac{1}{n}P_T - \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l})) \\ &\quad \times \prod_{k=1}^{n-1}(1 + \beta_k(\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_k})) \\ &= (\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l})^n \prod_{k=1}^{n}\beta_k. \end{aligned} \tag{40}$$

Concluding the analysis above, the solution to optimization problem in (13) can be described as: If the condition

$$P_T > n(\frac{1}{\beta_n} - \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l}) \tag{41}$$

holds, when $\mathbf{X} = \mathbf{X}^* = [x_1^*, x_2^*, \cdots, x_n^*]^T$, where

$$x_k^* = \frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l} - \frac{1}{\beta_k}, \quad k = 1, 2, \cdots, n. \tag{42}$$

$f(\mathbf{X})$ gets its global maximum value in its feasible region

$$f(\mathbf{X}^*) = (\frac{1}{n}P_T + \frac{1}{n}\sum_{l=1}^{n}\frac{1}{\beta_l})^n \prod_{k=1}^{n}\beta_k. \tag{43}$$

H. Optimization on the Feasible Region Boundary in (19)

If condition in (41) does not hold, that is $\mathbf{X}_b^*$ is out of the feasible region or on the feasible region boundary, $F(\mathbf{X}_b)$ has no extreme value in feasible region. Therefore, the maximum value must be on the feasible region boundary. Then, there will be an integer m, $1 < m < n$ and satisfying

$$0 < m(\frac{1}{\beta_m} - \frac{1}{m}\sum_{l=1}^{m}\frac{1}{\beta_l}) < P_T \leq (m+1)(\frac{1}{\beta_{m+1}} - \frac{1}{m+1}\sum_{l=1}^{m+1}\frac{1}{\beta_l}) \tag{44}$$

such that the solution to optimization problem in (13) is on the feasible region boundary,

$$\begin{cases} x_k \geq 0, & k = 1, 2, \cdots, m \\ \sum_{k=1}^{m} x_k = P_T & \\ x_k = 0, & k = m+1, m+2, \cdots, n. \end{cases} \tag{45}$$

At this time, the optimization problem in (13) can be simplified as the following optimization problem

$$\begin{cases} \max_{\mathbf{X}_c} G(\mathbf{X}_c) = (1 + \beta_m P_T - \beta_m \sum_{k=1}^{n-1} x_k) \times \prod_{k=1}^{m-1}(1 + \beta_k x_k) \\ \text{subject to:} \\ \sum_{k=1}^{m-1} x_k \leq P_T \\ x_k \geq 0, \quad k = 1, 2, \cdots, m \end{cases} \tag{46}$$

where $\mathbf{X}_c = [x_1, x_2, \cdots, x_{m-1}]^T$. (45) is similar as (19). According to the analysis above, we get that when $\mathbf{X}_c = \mathbf{X}_c^* = [x_1^*, x_2^*, \cdots, x_{m-1}^*]^T$, where

$$x_k^* = \frac{1}{m}P_T + \frac{1}{m}\sum_{l=1}^{m}\frac{1}{\beta_l} - \frac{1}{\beta_k}, \quad k = 1, 2, \cdots, m-1. \tag{47}$$

$G(\mathbf{X}_c)$ gets its global maximum value in its feasible region

$$G(\mathbf{X}_c^*) = (\frac{1}{m}P_T + \frac{1}{m}\sum_{l=1}^{m}\frac{1}{\beta_l})^m \prod_{k=1}^{m}\beta_k. \tag{48}$$

Therefore, the the solution to optimization problem in (13) can be described as: When $\mathbf{X} = \mathbf{X}^* = [x_1^*, x_2^*, \cdots, x_n^*]^T$, where

$$x_k^* = \begin{cases} \frac{1}{m}P_T + \frac{1}{m}\sum_{l=1}^{m}\frac{1}{\beta_l} - \frac{1}{\beta_k} & k = 1, 2, \cdots, m \\ 0 & k = m+1, m+2, \cdots, n \end{cases} \tag{49}$$

$f(\mathbf{X})$ gets its global maximum value in its feasible region

$$f(\mathbf{X}^*) = (\frac{1}{m}P_T + \frac{1}{m}\sum_{l=1}^{m}\frac{1}{\beta_l})^m \prod_{k=1}^{m}\beta_k. \tag{50}$$

REFERENCES

[1] T. Keller and L. Hanzo, "Adaptive multicarrier modulation: A convenient framework for time-frequency processing in wireless communications," *Proc. IEEE*, vol. 88, no. 5, pp. 611–640, May 2000.

[2] J. Jang and K. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 171–178, Feb. 2003.

[3] G. J. Foschini and M. J. Gans, "On limits of wireless communications in fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, pp. 311–335, Mar. 1998.

[4] I. E. Telatar, "Capacity of multi-antenna Gaussian channel," *AT&T-Bell Labs. Int. Tech. Memorandum*, June 1995.

[5] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[6] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications–A key to gigabit wireless," *Proc. IEEE*, vol. 92, no. 2, pp. 198–218, Feb. 2004.

[7] I. Koffman and V. Roman, "Broadband wireless access solutions based on OFDM access in IEEE 802.16," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 96–103, Apr. 2002.

[8] Y. Xiao, "IEEE 802.11n: Enhancements for higher throughput in wireless LANs," *IEEE Wireless Commun.*, vol. 12, no. 6, pp. 82–91, Dec. 2005.

[9] M. Ergen, S. Coleri, and P. Varaiya, "QoS aware adaptive resource allocation techniques for fair scheduling in OFDMA based broadband wireless access systems," *IEEE Trans. Broadcasting*, vol. 49, no. 4, pp. 362–370, Dec. 2003.

[10] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE VTC 2000-spring*, Tokyo, Japan, vol. 2, May 2000, pp. 1085–1089.

[11] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1747–1758, Oct. 1999.

[12] Y. J. Zhang and K. B. Letaief, "An efficient resource-allocation scheme for spatial multiuser access in MIMO/OFDM systems," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 107–116, Jan. 2005.

[13] Q. H. Spencer and A. L. Swindlehurst, "Channel allocation in multi-user MIMO wireless communications systems," in *Proc. IEEE ICC 2004*, vol. 5, 2004, pp. 3035–3039.

[14] J. Cai, X. Shen, and J. W. Mark, "Downlink resource management for packet transmission in OFDM wireless communication systems," in *Proc. IEEE GLOBECOM 2003*, vol. 6, Dec. 2003, pp. 2999–3003.

[15] L. Li and Z. Niu, "A multi-dimensional radio resource scheduling scheme for MIMO-OFDM systems with channel dependent parallel weighted fair queueing (CDPWFQ)," in *Proc. IEEE PIMRC 2004*, vol. 4, Sept. 2004, pp. 2367–2371.

[16] J. C. R. Bennett and H. Zhang, "WF2Q: Worst-case fair weighted fair queueing," in *Proc. IEEE INFOCOM'96*, vol. 1, Mar. 1996, pp. 120–128.

[17] T. Yoo and A. Goldsmith, "On the optimality of multi-antenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 528–541, Mar. 2006.

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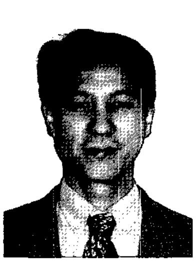

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