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Fuzzy strongly (r, s)-semiopen sets

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Abstract

In this paper, we introduce the concepts of fuzzy strongly (r, s)-semiopen sets and fuzzy strongly (r, s)-semicontinuous mappings on the intuitionistic fuzzy topological space in Šostak's sense and then we investigate some of their characteristic properties.

Key words : fuzzy strongly (r, s)-semiopen sets, fuzzy strongly (r, s)-semicontinuous mappings

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12]. Chang [2] defined fuzzy topological spaces. These spaces and their generalizations are later studied by several authors, one of which, developed by Šostak [11], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [10].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy strongly (r, s)-semiopen sets and fuzzy strongly (r, s)-semicontinuous mappings on the intuitionistic fuzzy topological spaces in Šostak's sense and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

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Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. *[1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then*

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.

(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\gamma_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space*.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. *[5] Let X be a nonempty set. An intuitionistic fuzzy topology in Šostak's sense (SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:*

(1) $\mathcal{T}_1(0_\sim) = \mathcal{T}_1(1_\sim) = 1$ and $\mathcal{T}_2(0_\sim) = \mathcal{T}_2(1_\sim) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The pair $(X, \mathcal{T})$ is said to be an intuitionistic fuzzy topological space in Šostak's sense (SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a gradation of openness of A and $\mathcal{T}_2(A)$ a gradation of nonopenness of A.

Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim \\ (r,s) & \text{if } A \in T - \{0_\sim, 1_\sim\} \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Definition 2.3. *[8] Let A be an intuitionistic fuzzy set of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be*

(1) fuzzy (r, s)-open if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,

(2) fuzzy (r, s)-closed if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4. *[8] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy (r, s)-closure is defined by*

$$cl(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy } (r, s)\text{-closed}\}$$

and the fuzzy (r, s)-interior is defined by

$$int(A, r, s) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is fuzzy } (r, s)\text{-open}\}.$$

Definition 2.5. *[8] Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be*

(1) a fuzzy (r, s)-continuous mapping if $f^{-1}(B)$ is a fuzzy (r, s)-open set of X for each fuzzy (r, s)-open set B of Y,

(2) a fuzzy (r, s)-open mapping if $f(A)$ is a fuzzy (r, s)-open set of Y for each fuzzy (r, s)-open set A of X,

(3) a fuzzy (r, s)-closed mapping if $f(A)$ is a fuzzy (r, s)-closed set of Y for each fuzzy (r, s)-closed set A of X.

Lemma 2.6. [8] For an intuitionistic fuzzy set A of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$,

(1) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

(2) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.

Definition 2.7. *[8, 9] Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be*

(1) fuzzy (r, s)-semiopen if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq cl(B, r, s)$,

(2) fuzzy (r, s)-semiclosed if there is a fuzzy (r, s)-closed set B in X such that $int(B, r, s) \subseteq A \subseteq B$,

(3) fuzzy (r, s)-preopen if $A \subseteq int(cl(A, r, s), r, s)$,

(4) fuzzy (r, s)-preclosed if $cl(int(A, r, s), r, s) \subseteq A$.

3. Fuzzy strongly (r, s)-semiopen sets

Definition 3.1. *Let A be an intuitionistic fuzzy set of a SOIFT $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be*

(1) a fuzzy strongly (r, s)-semiopen set if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq int(cl(B, r, s), r, s)$,

(2) a fuzzy strongly (r, s)-semiclosed set if there is a fuzzy (r, s)-closed set B in X such that $cl(int(B, r, s), r, s) \subseteq A \subseteq B$.

Theorem 3.2. Let A be an intuitionistic fuzzy set of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy strongly (r, s)-semiopen set.

(2) A^c is a fuzzy strongly (r, s)-semiclosed set.

(3) $A \subseteq \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s)$.

(4) $A^c \supseteq \text{cl}(\text{int}(\text{cl}(A^c, r, s), r, s), r, s)$.

(5) A is both a fuzzy (r, s)-semiopen set and a fuzzy (r, s)-preopen set.

(6) A^c is both a fuzzy (r, s)-semiclosed set and a fuzzy (r, s)-preclosed set.

Proof. (1) $\Leftrightarrow$ (2), (3) $\Leftrightarrow$ (4) and (5) $\Leftrightarrow$ (6) follow from Lemma 2.6.

(1) $\Rightarrow$ (3) Let A be a fuzzy strongly (r, s)-semiopen set of X. Then there is a fuzzy (r, s)-open set B of X such that $B \subseteq A \subseteq \text{int}(\text{cl}(B, r, s), r, s)$. Since B is fuzzy (r, s)-open and $B \subseteq A$, we have $B = \text{int}(B, r, s) \subseteq \text{int}(A, r, s)$. Thus

$$A \subseteq \text{int}(\text{cl}(B, r, s), r, s) \subseteq \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s).$$

(3) $\Rightarrow$ (1) Let $A \subseteq \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s)$ and take $B = \text{int}(A, r, s)$. Then B is a fuzzy (r, s)-open set of X. Also

$$\begin{aligned} B = \text{int}(A, r, s) \subseteq A &\subseteq \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s) \\ &= \text{int}(\text{cl}(B, r, s), r, s). \end{aligned}$$

Hence A is a fuzzy strongly (r, s)-semiopen set of X.

(1) $\Rightarrow$ (5) It is obvious.

(5) $\Rightarrow$ (3) Let A be a fuzzy (r, s)-semiopen and fuzzy (r, s)-preopen set of X. Then $A \subseteq \text{cl}(\text{int}(A, r, s), r, s)$ and $A \subseteq \text{int}(\text{cl}(A, r, s), r, s)$. Therefore

$$\begin{aligned} A &\subseteq \text{int}(\text{cl}(A, r, s), r, s) \\ &\subseteq \text{int}(\text{cl}(\text{cl}(\text{int}(A, r, s), r, s), r, s), r, s) \\ &= \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s). \end{aligned}$$

This completes the proof.

Remark 3.3. It is clear that every fuzzy (r, s)-open set is a fuzzy strongly (r, s)-semiopen set and every fuzzy strongly (r, s)-semiopen set is not only a fuzzy (r, s)-semiopen set but also a fuzzy (r, s)-preopen set. However, the following examples show that all of the converses need not be true.

Example 3.4. Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.1, 0.6), \quad A_1(y) = (0.5, 0.2);$$

and

$$A_2(x) = (0.2, 0.3), \quad A_2(y) = (0.7, 0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (r, s) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a SoIFT on X. The intuitionistic fuzzy set A_2 is a fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semiopen set which is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-open set.

Example 3.5. Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.2, 0.6), \quad A_1(y) = (0.1, 0.7);$$

and

$$A_2(x) = (0.4, 0.3), \quad A_2(y) = (0.5, 0.2).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (r,s) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a SoIFT on X. The intuitionistic fuzzy set A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set which is not a fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semiopen set.

Example 3.6. Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.5, 0.1), \quad A_1(y) = (0.4, 0.5);$$

and

$$A_2(x) = (0.6, 0.2), \quad A_2(y) = (0.3, 0.6).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (r,s) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a SoIFT on X. The intuitionistic fuzzy set A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set which is not a fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semiopen set.

Theorem 3.7. (1) Any union of fuzzy strongly (r, s)-semiopen sets is a fuzzy strongly (r, s)-semiopen set.

(2) Any intersection of fuzzy strongly (r, s)-semiclosed sets is a fuzzy strongly (r, s)-semiclosed set.

Proof. (1) Let $\{A_i\}$ be a collection of fuzzy strongly (r, s)-semiopen sets. Then for each i, there is a fuzzy (r, s)-open set B_i such that $B_i \subseteq A_i \subseteq \text{int}(\text{cl}(B_i, r, s), r, s)$. Since B_i is fuzzy (r, s)-open for each i, we have $\bigcup B_i$ is a fuzzy (r, s)-open set. Also

$$\begin{aligned} \bigcup B_i \subseteq \bigcup A_i &\subseteq \bigcup \text{int}(\text{cl}(B_i, r, s), r, s) \\ &\subseteq \text{int}(\text{cl}(\bigcup B_i, r, s), r, s). \end{aligned}$$

Then $\bigcup A_i$ is a fuzzy strongly (r, s)-semiopen set.

(2) It follows from (1) using Theorem 3.2.

Definition 3.8. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be*

(1) a fuzzy strongly (r, s)-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiopen set of X for each fuzzy (r, s)-open set B of Y,

(2) a fuzzy strongly (r, s)-semiopen mapping if $f(A)$ is a fuzzy strongly (r, s)-semiopen set of Y for each fuzzy (r, s)-open set A of X,

(3) a fuzzy strongly (r, s)-semiclosed mapping if $f(A)$ is a fuzzy strongly (r, s)-semiclosed set of Y for each fuzzy (r, s)-closed set A of X.

In general, it need not be true that f and g are fuzzy strongly (r, s)-semicontinuous (strongly (r, s)-semiopen and strongly (r, s)-semiclosed) mappings then so is $g \circ f$. But we have the following theorem.

Theorem 3.9. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$, $(Y, \mathcal{U}_1, \mathcal{U}_2)$ and $(Z, \mathcal{V}_1, \mathcal{V}_2)$ be SoIFTSs and let $f : X \to Y$ and $g : Y \to Z$ be mappings and $(r, s) \in I \otimes I$. Then the following statements are true:

(1) If f is a fuzzy strongly (r, s)-semicontinuous mapping and g is a fuzzy (r, s)-continuous mapping, then $g \circ f$ is a fuzzy strongly (r, s)-semicontinuous mapping.

(2) If f is a fuzzy (r, s)-open mapping and g is a fuzzy strongly (r, s)-semiopen mapping, then $g \circ f$ is a fuzzy strongly (r, s)-semiopen mapping.

(3) If f is a fuzzy (r, s)-closed mapping and g is a fuzzy strongly (r, s)-semiclosed mapping, then $g \circ f$ is a fuzzy strongly (r, s)-semiclosed mapping.

Proof. Straightforward.

Theorem 3.10. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy strongly (r, s)-semicontinuous mapping.

(2) $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiclosed set of X for each fuzzy (r, s)-closed set B of Y.

(3) $\text{cl}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s)r, s) \subseteq f^{-1}(\text{cl}(B, r, s))$ for each intuitionistic fuzzy set B of Y.

(4) $f(\text{cl}(\text{int}(\text{cl}(A, r, s), r, s), r, s)) \subseteq \text{cl}(f(A), r, s)$ for each intuitionistic fuzzy set A of X.

Proof. (1) $\Leftrightarrow$ (2) It is obvious.

(2) $\Rightarrow$ (3) Let B be any intuitionistic fuzzy set of Y. Then $\text{cl}(B, r, s)$ is a fuzzy (r, s)-closed set of Y. By (2), $f^{-1}(\text{cl}(B, r, s))$ is a fuzzy strongly (r, s)-semiclosed set of X. Thus

$$\begin{aligned} & f^{-1}(\text{cl}(B, r, s)) \\ & \quad \supseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(\text{cl}(B, r, s)), r, s), r, s), r, s) \\ & \quad \supseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s), r, s). \end{aligned}$$

(3) $\Rightarrow$ (4) Let A be any intuitionistic fuzzy set of X. Then $f(A)$ is an intuitionistic fuzzy set of Y. By (3),

$$\begin{aligned} f^{-1}(\text{cl}(f(A), r, s)) & \supseteq \text{cl}(\text{int}(\text{cl}(f^{-1}f(A), r, s), r, s), r, s) \\ & \supseteq \text{cl}(\text{int}(\text{cl}(A, r, s), r, s), r, s). \end{aligned}$$

Hence

$$\begin{aligned} \text{cl}(f(A), r, s) & \supseteq ff^{-1}(\text{cl}(f(A), r, s)) \\ & \supseteq f(\text{cl}(\text{int}(\text{cl}(A, r, s), r, s), r, s)) \end{aligned}$$

(4) $\Rightarrow$ (2) Let B be any fuzzy (r, s)-closed set of Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set of X. By (4),

$$\begin{aligned} & f(\text{cl}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s), r, s)) \\ & \quad \subseteq \text{cl}(ff^{-1}(B), r, s) \subseteq \text{cl}(B, r, s) = B \end{aligned}$$

and hence

$$\begin{aligned} & \text{cl}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s), r.s) \\ & \quad \subseteq f^{-1}f(\text{cl}(\text{int}(\text{cl}(f^{-1}(B), r, s), r, s), r, s)) \\ & \quad \subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiclosed set of X.

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