Performance Evaluation of Block Error of FS MC-CDMA System in Various Nakagami Fading Channels

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Abstract—In this paper, we discusses that the theoretical analysis is made for the performance of FS MC-CDMA by the aid of the Nakagami fading channels and the block error probabilities of the FS MC-CDMA in Nakagami fading channel are presented. The channel fading speed, slow or fast, is considered in evaluating block error probabilities. The effectiveness of diversity combining in improving block error performance is examined.

Index Terms—FS MC-CDMA, MC-CDMA, Nakagami fading, block error probability

I. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA), which efficiently combines CDMA with orthogonal frequency division multiplexing (OFDM), has gained considerable attention as a promising mulitple access technique for future mobile communication[1]-[3]. A direct-sequence spread-spectrum (DS-SS) code-division multiple-access (CDMA) scheme used for high-data-rate applications is usually know as broadband CDMA (B-CDMA)[1] due to its large bandwidth. However, the high bandwidth everywhere is a key requirement of the future communicating systems[2]. Broadband wireless mobile channels are typically time-varying and the received signals may experience both frequency-selective and time-selective fading[3],[4].

Bit error probabilities are often examined for a particular channel environment and a particular modulation format. In this case of Nakgmi fading, the bit error probabilities have been derived for FS MC-CDMA(fractionally spread multicarrier CDMA)[5]. In data communication applications, the expressions of block error probabilities are important in evaluating system performance.

In this paper, we analyzed FS MC-CDMA system considered in [5], when communicating over wireless channels exhibiting both frequency-selective and time-selective fading. Therefore we applies the block error probability to display the performance of FS MC-CDMA system in the slow and fast Nakagami fading channel environment.

II. FS MC-CDMA SYSTEM MODEL

A. Transmitter Model

The transmitter diagram of the kth user is shown in Fig. 1 for the FS MC-CDMA system. In this scheme the original binary data stream having a bit duration of T_b is S-P converted to U parallel sub-streams, which are expressed as $\{b_{k1}, b_{k2}, ..., b_{ku}\}$. The new bit duration or symbol duration after S-P conversion is given by $T_S = UT_b$. As shown in Fig. 1, after S-P conversion each of the substreams is spread using two time (T)-domain spreading codes, namely $\alpha_k(t)$ and. $c_k(t)$. More explicitly, the first T-domain spreading code is applied at the fraction level and it is expressed as $\alpha_k(t) = \sum_{i=-\infty}^{\infty} \alpha_{ki} P_{T_D}(t - iT_D)$ where $T_D = T_S$ N_1 represents the fraction's time duration, a_{ki} assumes the binary values of +1 or -1 with equal probability, while $P_{T_D}(t)$ represents the rectangular pulses of duration T_D [5].

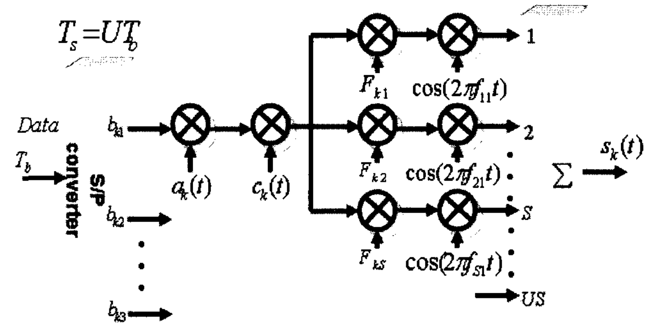

Fig. 1 Transmitter model of fractionally spread MC-CDMA.

The second T-domain spreading code $c_k(t)$ at the chip level and is defined as $c_k(t) = \sum_{i=-\infty}^{\infty} c_{ki} P_{T_D}(t - iT_D)$ where c_{ki} is again a random

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sequence with $c_{ki} \in \{+1, -1\}$ and $P_{T_C}(t)$ is the rectangular chip waveform defined over the time interval $[0, T_C]$.

Let the total number of subcarrier frequencies, namely US, where S is defined as the length of the F-domain spreading codes to be invoked additionally. As shown in Fig. 1, after T-domain spreading the u th substream, where $u = 1, 2, \cdots, U$, is further spread in the frequency(F)-domain using an S-chip F-domain spreading code $\{F_{k1}, F_{k2}, \ldots, F_{kS}\}$ associated with the US number of subcarrier frequencies of $\{f_{u1}, f_{u2}, \ldots, f_{uS}\}$. Finally, the US number of subcarrier-modulated substreams are superimposed on each other in order to form the transmitted signal, which can be expressed as

$$S_k(t) = \sqrt{\frac{2E_b}{T_S S}} \sum_{U=1}^{U} \sum_{S=1}^{S} b_{ku}(t) a_k(t) c_k(t) F_{ks} \times \cos(2\pi f_{su} t + \varphi_{su}^{(k)}) \tag{1}$$

where E_b represents the energy per bit, $b_{ku}(t)$ denotes the u th binary data's waveform after the S-P conversion, while $\phi_{us}^{(k)}$ represents a random phase due to carrier modulation. Assuming $N_2 = T_D / T_c$ being an integer, then the total T-domain spreading factor is

$$N = T_s / T_c = T_s / T_D \times T_D / T_c = N_1 / N_2 .$$

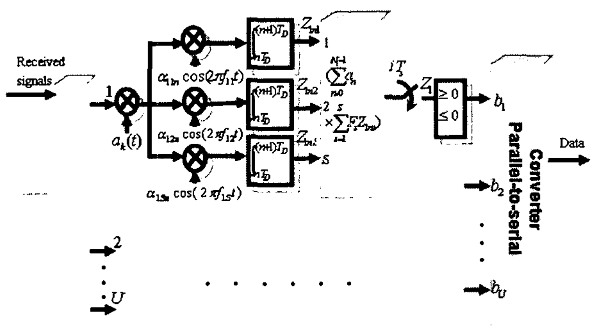

Fig.2 Receiver schematic diagram of fractionally spread MC-CDMA.

B. Receiver Model

Since we assume each subcarrier signal experiences flat fading. As shown in Fig. 1, the FS MC-CDMA transmitter usually employs S-P conversion and U data bits are transmitted in parallel within each symbol duration[5]. Hence, the symbol duration is $T_S = UT_b$. Based on the above assumptions, the asynchronous signal received by the base station can be expressed as

$$r(t) = \sqrt{\frac{2E_b}{T_S S}} \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{U=1}^{U} \sum_{S=1}^{S} b_{ku}(t - \tau_k) \alpha_{kn} P_{T_D}(t - nT_D - \tau_k) c_k(t - \tau_k) F_{ks} \times \cos(2\pi f_{su} t + \psi_{su}^{(k)} + n(t) \tag{2}$$

where $n(t)$ represents the AWGN noise having zero mean and a double-sided power spectrum density of $N_0 / 2$. Furthermore, in (2) $\alpha_{uns}^{(k)}$ is an amplitude fading parameter associated with the k th user, with the n th fraction as well as with the subcarrier indexed by the values of, u and s.

In Fig. 2 each subcarrier signal is first despread using the T-domain spreading code $c(t)$ of the reference user associated with each fraction. Then, the subcarrier signals conveying the same data bit are despread using the F-domain spreading code $\{F_1, F_2, \ldots, F_S\}$ and combined using a MRC scheme with the aid of the channel's fading envelope estimates $\{\alpha_{un1}, \alpha_{un2}, \ldots, \alpha_{unS}\}_{u=1}^{U}$. Finally, the decision variable Z_U, $u = 1, \ldots, U$ acquired for the u th binary bit. The process of generating the decision variable Z_U for the first symbol can be summarized using the following equations.

$$Z_u = \sum_{n=0}^{N_1 - 1} \alpha_n Z_{un} , \quad u = 1, \ldots, U \tag{3}$$

$$Z_{un} = \sum_{s=1}^{S} F_s Z_{uns} \tag{4}$$

$$Z_{uns} = \alpha_{uns} \times \int_{nT_D}^{(n+1)T_D} r(t) c(t) \cos(2\pi f_{us} t) dt \tag{5}$$

where we assumed that $\tau_1 = 0$ and $\psi_{uns} = 0$, representing perfect synchronization with the subcarrier signal of the fraction that is being considered.

III. BLOCK ERROR PROBABILITY

A. Nakagami Fading Model

A Nakagami distribution characterizes channels with different fading depth through a parameter called amount of fading AF. The AF of a signal is assumed to be 1/m. The signal envelope, α is a random variable with a Nakagami probability density function (pdf), i.e.,

$$p_a(\alpha) = \frac{2}{\Gamma(m)}\left(\frac{m}{2X}\right)\alpha^{2m-1}\exp\left(\frac{m}{2X}\alpha^2\right) \qquad (6)$$

where $\Gamma(\cdot)$ is the Gamma function, X is the mean signal power, and $m \geq 1/2$.

B. BER Model

In this section we summarize the BER expressions for the FS MC-CDMA system, when communicating over the AWGN, and over the frequency-selective Nakagami fading channels.

The BER of the FS MC-CDMA system communicating over AWGN channels can be expressed as

$$\begin{aligned} P_b &= Q(\sqrt{2 \cdot SINR}) \\ &= Q\left(\frac{K-1}{3N_1N_2S} + \left(\frac{3E_b}{N_0}\right)^{-1}\right)^{-1/2} \end{aligned} \qquad (7)$$

where Q(x) represents the Gaussion Q-function, which can be represented in the form of

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-\frac{t^2}{2})dt.$$

In the context of the frequency-selective fast Nakagami fading channels, the BER of the FS MC-CDMA systems can be expressed as

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{m\sin^2\theta}{\gamma_C + m\sin^2\theta}\right)^{mSN_1} d\theta \qquad (8)$$

where m is the Nakagami-m fading parameter, γ_c represents the average signal-to-noise ratio (SNR) received and can be expressed as

$$\gamma_c = \left[\frac{2(K-1)}{3N_2} + \left(\frac{\Omega E_b}{SN_0N_1}\right)^{-1}\right]^{-1} \qquad (9)$$

where $\Omega = E[(\alpha_{uns}^{(k)})^2]$. Explicitly, (8) shows that the diversity order achieved is SN_1.

In the context of communicating over frequency-selective slow Nakagami fading channels, the BER of the FS MC-CDMA systems can be expressed as

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{m\sin^2\theta}{\gamma_c + m\sin^2\theta}\right)^{mS} d\theta \qquad (10)$$

which shows the diversity order achieved over the frequency-selective slow Nakagami fading channels is S. Furthermore, the SNR in (10) is given by

$$\gamma_c = \left[\frac{2(K-1)}{3N_1N_2} + \left(\frac{\Omega E_b}{SN_0}\right)^{-1}\right]^{-1} \qquad (11)$$

It can be shown [6] that the limit (8) or (10) with respect to $m \to \infty$ will converge to (7), which quantifies the BER in the context of AWGN channels. This characteristic implies that when the channel quality improves and the fading envelope becomes near-constant, the FS MC-CDMA will, automatically leverage the diversity gain into spreading gain.

C. Block Error Probability Model

In deriving block error probabilities, we need to calculate the probability of more than M bit errors in a block of N bit, P(M, N). The block error probability P(M, N) of FS MC-CDMA with signal-to-noise γ_c in a Nakagami fading channel is

$$P(\gamma_c, M, N) = \sum_{i=M+1}^{N}\binom{N}{i}P_b^i(\gamma_c)[1-P_b(\gamma_c)]^{N-i} \qquad (12)$$

IV. PERFORMANCE ANALYSIS AND DISSCUSION

Fig. 3 and 4 show some numerical results for the block error probability performance of FS MC-CDMA system over slow and fast Nakagami fading. The values presented were with $m =$ 1/2, 1, 2, 10, 20, ∞; $M = 0$, $N = 31$, $N_1 = 4$, $N_2 = 31$, $S = 4$ and $K = 30$. We show the corresponding comparison of the block error probability performance of the FS MC-CDMA system, Note that the block error probability in fast fading channels increases less abruptly than in the slow fading channels. However, in Fig. 5 as the number of the users K increases, the block error probability of fast fading better than slow fading, which the reason for the performance tends was the diversity order, fast fading is $SN_1 = 16$ but slow fading channel is only $S = 4$.

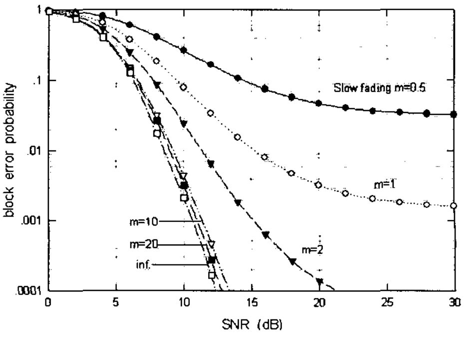

Fig. 3 Block error probability versus SNR performance of the FS MC-CDMA, when communicating over slow frequency-selective (M = 0) fading channels.

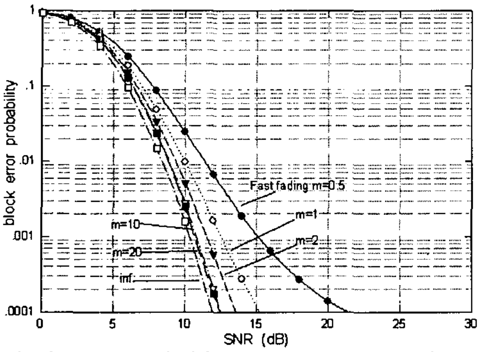

Fig. 4 Block error probability versus SNR perfomance of the FS MC-CDMA, when communicating over fast frequency-selective (M=0) fading channels.

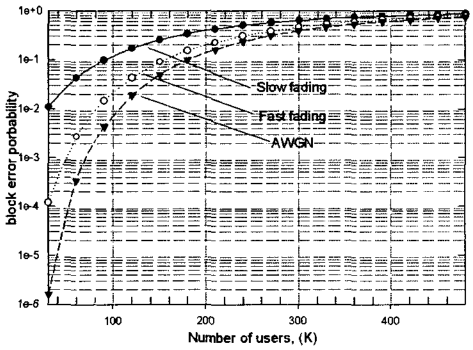

Fig. 5 Block error probability versus the number of users K for the FS MC-CDMA, when communicating over AWGN, as well as slow and fast frequency-selective Rayleigh (m = 1; M = 1) fading channels.

Finally, Fig 6 and 7 show the block error probability FS MC-CDMA system, with respect to the bit error (M = 0, 1, 2), when communicating over the frequency-selective slow and fast fading channels, assuming both Rayleigh (m = 1) and Rician (m = 2) fading models. The results of Fig 5 and 6, illustrate that when the value of M increases, the block error probability performance of FS MC-CDMA over both fast and slow fading channels will approach better. However, for most given parameter M, FS MC-CDMA system is capable of achieving a lower block error probability, in then fast fading case than in the slow fading case.

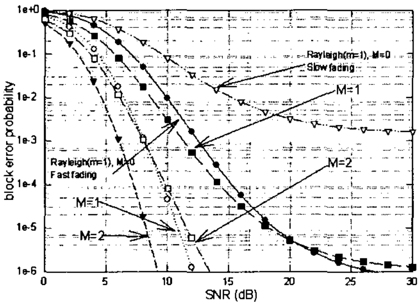

Fig. 6 Block error probability versus bit error (M) performance of the FS MC-CDMA, when communicating over slow and fast frequency-selective Rayleigh (m = 1) fading channels.

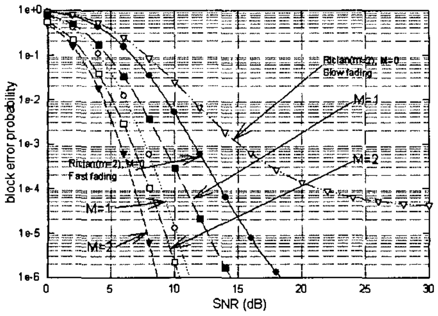

Fig. 7 Block error probability versus bit error(M) performance of the FS MC-CDMA, when communicating slow and fast frequency-selective Rician (m = 2) fading channels.

V. CONCLUSION

The block error probability analysis for FS MC-CDMA system over slow and fast Nakagami fading channels was presented in this paper. The analysis was applied to evaluate the block error probability performance over Nakagami fading channels. Furthermore, the block error probabilities of the FS MC-CDMA system under slow and fast Nakagami fading are derived in this paper and our numerical results show that in FS MC-CDMA the block error probability performance attained, the effect of fading amount on block error performance is observed. It is shown that the block error performance under the FS MC-CDMA is beneficial for employment over wireless channels exhibiting frequency-selective fading or time-selective fading.

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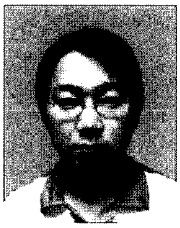

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