다중사용자 다이버시티가 적용된 MRT/MRC MIMO의 Capacity Bound의 Closed-form 표현

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Closed-form Expressions for Capacity Bounds of MRT/MRC MIMO with Multiuser Diversity

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요 약

본 논문에서는 독립적이며 주파수 비선택적인 정적 MIMO Rayleigh 페이딩 채널 가정 하에 다중사용자 다이버시티가 적용된 MRT/MRC MIMO의 capacity bound를 closed-form 형태의 수식으로 표현하였다. 분석 결과는 수치 검증 결과와 정확히 일치하며 MRT/MRC가 다중 사용자 다이버시티에 미치는 영향을 명확히 보여준다.

Key Words : Multiuser diversity, MRT/MRC MIMO, Channel capacity

ABSTRACT

Closed-form expressions for capacity bounds of multiuser diversity combined with maximum ratio transmission (MRT) and maximum ratio combining (MRC) at each link are presented under the assumption of independent and quasi-static flat multiple-input multiple-output (MIMO) Rayleigh fading channels. The analysis results precisely agree with the numerical verification results and clearly show the impact of MRT/MRC on multiuser diversity.

I. Introduction

Recently, increasing demands for wireless services has been driving the demand to increase system capacity. Multi-antenna techniques and radio resource management employing packet scheduling are the key techniques to increase system capacity in packet switched systems[1-2]. To date, the interaction between packet scheduling and antenna diversity techniques has been studied in [3-7]. Most of these studies have analyzed the combined performance of transmit diversity (TD) techniques in accordance with scheduling by using a system level simulation model[3-4] while the interactions between multiuser diversity and spatial diversity have been assessed using an analytical models[5-7]. In [5-6], however, the derived capacity expressions are only given in integral form, which still requires a numerical integration. On the other hand, the closed-form expressions are presented for capacity bounds of antenna diversity with multiuser diversity in a Nakagami fading channel in [7]. However, the closed-form expressions for capacity bounds are confined to the approximated expressions[7]. To the best of the authors' knowledge, closed-form expressions for capacity bounds

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of MRT/MRC combined with multiuser diversity in MIMO Rayleigh fading channels without approximation have yet to be proposed in the literature.

In this paper, we derive closed-form expressions without approximation for a upper and a lower bounds of the average channel capacity of MRT/MRC combined with a fair-access scheduling algorithm under the assumption of independent and quasi-static flat MIMO Rayleigh fading channels.

Ⅱ. MRT/MRC System Model

We consider a downlink of a single-cell wireless system employing n_T transmit antennas and n_R receive antennas with K mobile users. The channel is assumed to be a frequency flat block MIMO Rayleigh fading channel so that the channel coefficients are constant over the packet duration. The channel state is assumed to be known at both the receiver and the transmitter. The transmitter adapts to the channel variations using a constant power variable rate strategy.

If we consider a MIMO system using both MRT and MRC, the effective signal to noise ratio (SNR) of the kth user, γ_k is given by[8]

$$\gamma_k = \overline{\gamma}_k |\mathrm{w}_k^H \mathrm{H}_k \mathrm{v}_k|^2 = \overline{\gamma}_k \alpha_k, \tag{1}$$

where $\overline{\gamma}_k$ denotes the average SNR per receive antenna of the kth user and α_k denotes the instantaneous channel gain of the kth user. The $n_R \times n_T$ channel matrix H_k for the kth user consists of the complex channel coefficient for the path traveled from the transmit antenna j to the receive antenna i, $\{h_{i,j,k}\}_{i=1,\cdots,n_R, j=1,\cdots,n_T}$. Channel coefficients are assumed to be independent zero-mean and unit variance complex Gaussian random variables. An $n_T \times n_T$ optimal transmit weight vector v_k and $n_R \times n_R$ receive weight vector w_k for the kth user are jointly determined by the MRT/MRC scheme in order to maximize the effective SNR γ_k[8].

The resultant α_k obtained by the MRT/MRC scheme becomes the maximum eigenvalue of $\mathrm{H}_k\mathrm{H}_k^H$. Since $\mathrm{H}_k\mathrm{H}_k^H$ is a random matrix, its maximum eigenvalue $\lambda_{\max,k}$ is still a random variable, and the well-known bounds of $\lambda_{\max,k}$, i.e., α_k are given by[8]

$$\sum_{i=1}^{n_R}\sum_{j=1}^{n_T} \frac{|h_{i,j,k}|^2}{S} \leqq \alpha_k \leqq \sum_{i=1}^{n_R}\sum_{j=1}^{n_T} |h_{i,j,k}|^2, \tag{2}$$

where $S = \min(n_T, n_R)$. The upper bound of α_k corresponds to the sum of the eigenvalues of $\mathrm{H}_k\mathrm{H}_k^H$. Therefore, the equality between the upper bound and α_k holds if the eigenvalues of $\mathrm{H}_k\mathrm{H}_k^H$ are zeros except $\lambda_{\max,k}$, which results from fully correlated channels. The lower bound of α_k corresponds to the average of the nonzero eigenvalues of $\mathrm{H}_k\mathrm{H}_k^H$ and thus is achieved if all the nonzero eigenvalues are same.

The probability density functions (PDFs) for the lower and the upper bounds of α_k are given by the same expression each with different values of β_k, S and 1, respectively, as follows:

$$f_{\alpha_k}(x) = \frac{\beta_k^{n_T n_R}}{(n_T n_R - 1)!} x^{n_T n_R - 1} e^{-\beta_k x}. \tag{3}$$

Ⅲ. Capacity Bounds of Multiuser Diversity Combined with MRT/MRC

We consider a fair-access scheduler that decides to send a packet to the user k^* with the largest instantaneous SNR to mean SNR ratio at each time slot. The scheduling algorithm is expressed as:

$$k^* = \arg\max_{k \in \{1,2,\cdots,K\}} \frac{\gamma_k}{\overline{\gamma}_k}. \tag{4}$$

Therefore, the final decision variables for the fair-access scheduling become the instantaneous channel gain of the candidates, α_k, because $\alpha_k = \gamma_k / \overline{\gamma_k}$.

The fair-access scheduler does not consider the average SNR of the candidates for packet scheduling. Therefore, the combined instantaneous SNR of the selected users at each time slot $\tilde{\gamma}$ can be expressed as a multiplication process of two independent random variables of the combined average SNR of the scheduled users, $\bar{\bar{\gamma}}$, and the combined instantaneous channel gain of the scheduled users, $\tilde{\alpha}$, as follows[6]

$$\tilde{\gamma} = \bar{\bar{\gamma}} \cdot \tilde{\alpha} \tag{5}$$

On the assumption of identical small-scale fading statistics across candidates, the random variable of the combined average SNR of the scheduled users $\bar{\bar{\gamma}}$ has identical statistics with those of the average SNR of the candidates $\bar{\gamma}$ because all candidates have equal access time. Thus, let us assume the distribution of $\bar{\gamma}$, i.e., $\bar{\bar{\gamma}}$ is given by

$$f_{\bar{\bar{\gamma}}}(\gamma) = \frac{1}{K} \sum_{k=1}^{K} \delta\left(\gamma - \overline{\gamma_k}\right). \tag{6}$$

The PDF of the instantaneous channel gain combined by the fair-access scheduler $\tilde{\alpha}_k$ can be computed using order statistics[9] on the assumption of identical distributions of all α_k

$$f_{\tilde{\alpha}}(\gamma) = K f_{\alpha}(\gamma) \left(F_{\alpha}(\gamma)\right)^{K-1}, \tag{7}$$

where $F_{\alpha}(\gamma)$ denotes the cumulative distribution function (CDF) of α_k and the index k of α_k is omitted by the assumption of identical fading statistics for all mobile users.

The PDF of $\tilde{\gamma}$ is obtained by (3), (6), and (7) as follows

$$\begin{aligned} f_{\tilde{\gamma}}(\gamma) &= \int_{-\infty}^{\infty} \frac{1}{|x|} f_{\bar{\bar{\gamma}}}(x) f_{\tilde{\alpha}}\left(\frac{\gamma}{x}\right) dx\,. \\ &= \frac{1}{K} \sum_{k=1}^{K} \frac{1}{\overline{\gamma_k}} f_{\tilde{\alpha}}\left(\frac{\gamma}{\overline{\gamma_k}}\right) \end{aligned} \tag{8}$$

Thus, the average channel capacity is given by

$$C = \int_0^{\infty} f_{\tilde{\gamma}}(\gamma) \log_2(1+\gamma) d\gamma. \tag{9}$$

Theorem: The upper and the lower bounds of the average channel capacity of MRT/MRC combined with the fair-access packet scheduler are given by the following expression each with different values of β_k, S and 1, respectively,

$$\begin{aligned} C = \frac{1}{\ln 2} \sum_{k=1}^{K} \sum_{l=1}^{K} (-1)^{l+1} \frac{(K-1)! e^{l\beta_k/\overline{\gamma_k}}}{(K-l)!} \times \\ \left[\frac{\left(\frac{\beta_k}{\bar{\bar{\gamma}}_k}\right)^{n(l_i)} \prod_{m=0}^{n_T n_R - 1} (m!)^{-l_m}}{l_0! l_1! \cdots l_{n_T n_R - 1}!} \times \right. \\ \left[\sum_{l_i}^{l} \left[(-1)^{n(l_i)} E_1\left(\frac{l\beta_k}{\bar{\bar{\gamma}}_k}\right) + \right. \right. \\ \left. \left. \left. \sum_{p=1}^{n(l_i)} \left[\frac{(-1)^{n(l_i)-p} n(l_i)!}{p(n(l_i)-p)!} \times \left(\frac{l\beta_k}{\bar{\bar{\gamma}}_k}\right)^{-p} P_p\left(\frac{l\beta_k}{\bar{\bar{\gamma}}_k}\right) \right] \right] \right] \right], \end{aligned} \tag{10}$$

where $n(l_i) = l_1 + 2l_2 + \cdots + (n_T n_R - 1) l_{n_T n_R - 1}$ and l_i is a non-negative integer satisfying $\sum l_i = l$. $E_1(\cdot)$ denotes the exponential integral function of the first kind order defined as $E_1(x) \equiv \int_1^{\infty} t^{-1} e^{-xt} dt$ for $x \geqq 0$, and $P_n(\cdot)$ designates the Poisson distribution defined as $P_n(x) = \sum_{k=0}^{n-1} (x^k / k!) e^{-x}$ for all positive integers n.

Proof: Integrating the right-hand side of (9) by parts leads to

$$C = \frac{-1}{\ln 2} \int_0^{\infty} \frac{F_{\tilde{\gamma}}(\gamma) - 1}{1+\gamma} d\gamma. \tag{11}$$

The cumulative distribution function (CDF) of $\tilde{\gamma}$, $F_{\tilde{\gamma}}(\cdot)$ can be obtained from (8) and written as

$$F_{\tilde{\gamma}}(x) = \frac{1}{K} \sum_{k=1}^{K} \left(1 - e^{-\mu_k x} \sum_{m=0}^{M} \frac{(\mu_k x)^m}{m!}\right)^K, \tag{12}$$

where $\mu_k = \beta_k / \overline{\gamma_k}$ and $M = n_T n_R - 1$. Using a binomial expansion of $e^{-\mu_k x}$ and a multinomial expansion of $\left(\sum_{m=0}^{M}(\mu_k x)^m/m!\right)^l$, (12) can be rewritten as:

$$F_{\bar{\gamma}}(x)=1+\frac{1}{K}\sum_{k=1}^{K}\left(\sum_{l=1}^{K}\frac{(-1)^l e^{-l\mu_k x}K!}{l!(K-l)!}\times\left(\sum_{l_i}^{l}\frac{l!}{l_0!l_1!\cdots l_n!}\times\prod_{m=0}^{M}\left(\frac{1}{m!}\right)^{l_m}(\mu_k x)^{ml_m}\right)\right), \tag{13}$$

where the sum $\sum_{l_i}^{l}(\cdot)$ is taken over all non-negative integers l_0, l_1, ..., and l_M satisfying $l_0+l_1+\cdots+l_M=l$. Substituting (13) into (11), we obtain

$$C=\frac{1}{K\ln 2}\sum_{k=1}^{K}\sum_{l=1}^{K}\frac{(-1)^{l+1}K!}{l!(K-l)!}\times\sum\frac{l!\prod_{m=0}^{M}(m!)^{-l_m}}{l_0!l_1!\cdots l_m!}\times\int_0^{\infty}\frac{e^{-l\mu_k x}(\mu_l x)^{l_1+2l_2+\cdots+Ml_M}}{1+x}dx. \tag{14}$$

Operating the change of variable $t=1+x$ within the integral term and using a binomial expansion of $(t-1)^{n(l_i)}$, the integral term in (14) can be rewritten as:

$$\int_0^{\infty}\frac{e^{-l\mu_k x}(\mu_k x)^{n(l_i)}}{1+x}dx = e^{l\mu_k}\mu_k^{n(l_i)}\sum_{p=0}^{n(l_i)}\left(\frac{(n(l_i))!}{p!(n(l_1)-p)!}(-1)^{n(l_i)-p}\times\int_1^{\infty}t^{p-1}e^{-l\mu_k t}dt\right). \tag{15}$$

By isolating the first term, corresponding to $p=0$ in the summation over p, the integral in (15) can be written in a closed-form with the help of [10] as follows:

$$C=\frac{1}{\ln 2}\sum_{k=1}^{K}\sum_{l=1}^{K}\frac{(-1)^{l+1}(K-1)!e^{l\mu_k}}{(K-l)!}\times\left[\frac{(\mu_k)^{n(l_i)}\prod_{m=0}^{M}(m!)^{-l_m}}{l_0!l_1!\cdots l_M!}\left[\sum_{l_i}^{l}\left[(-1)^{n(l_i)}E_1(l\mu_k)+\sum_{p=1}^{n(l_i)}\frac{(-1)^{n(l_i)-p}n(l_i)!\Gamma(p,l\mu_k)}{p!(n(l_i)-p)!(l\mu_k)^p}\right]\right]\right], \tag{16}$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete gamma function defined as $\Gamma(\alpha,x)=\int_x^{\infty}t^{\alpha-1}e^{-t}dt$ and $\Gamma(\alpha,x)=(\alpha-1)!P_{\alpha}(x)$ for a positive integer α. Finally, replacing μ_k with $\beta_k/\bar{\gamma}_k$ leads to (10), which concludes the proof.

Ⅳ. Numerical Results

The capacity bounds for the fair-access packet scheduler combined with MRC/MRT are presented. For simplicity, it is assumed that the average SNR, $\bar{\gamma}_k$ is 0.0dB for all users. 10,000 samples of a fading channel are used for numerical verification. The average channel capacity per sector is used as a performance measure and is obtained by averaging a large number of capacity variables of the upper and the lower bounds, which are obtained from the instantaneous channel gains computed by (2) in every channel realization. The numerical verification results are compared with the analysis results computed by (10).

Fig. 1 compares the bounds of the average channel capacity between the analysis and simulation results when a $n_R\times n_T$ multi-antenna system is considered. For different values of K and the number of transmitting and receiving antennas, the analysis curves precisely correspond with the numerical verification curves, thus verifying our proposed closed-form expressions.

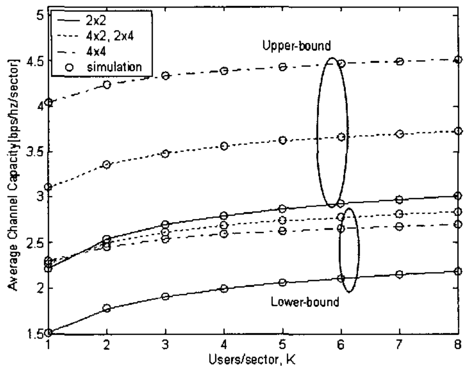

Fig. 1 Comparisons of the average capacity bounds between the analysis and numerical verifications.

The upper bound of the average channel capacity increases with the total number of transmit and receive antennas irrespective of K because the sum of the eigenvalues is proportional to the total number of available links between the transmit and receive antennas. On the other hand, the lower bound of the instantaneous channel gain of each link is directly proportional to the sum of the eigenvalues, and reversely proportional to the channel rank. This leads to an identical mean value but smaller variance in a 4×4 multi-antenna system relative to that of a 4×2 multi-antenna system in the lower bound of the instantaneous channel gain. The smaller variance of the instantaneous channel gain not only reduces the severity of destructive fades but also the probability of encountering very high constructive fading peaks, which has destructive impact on multiuser diversity. Thus, in the case of multiuser environments, a 4×2 multi-antenna system shows larger average channel capacity of the lower bound than a 4×4 multi-antenna system.

V. Conclusion

In this paper, we derive exact closed-form expressions for the upper and the lower bounds of the average channel capacity of MRT/MRC combined with a fair-access scheduling algorithm under the assumption of independent and quasi-static flat MIMO Rayleigh fading channels. It is shown that the analysis results precisely agree with the numerical verification results and the results clearly show the impact of MRT/MRC on multiuser diversity.

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