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ON WEIGHTED WEYL SPECTRUM, II

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Abstract. In this paper, we show that if T is a hyponormal operator on a non-separable Hilbert space $\mathcal{H}$, then $\mathrm{Re}\,\omega_\alpha^0(T) \subset \omega_\alpha^0(\mathrm{Re}\,T)$, where $\omega_\alpha^0(T)$ is the weighted Weyl spectrum of weight α with $\aleph_0 \leq \alpha \leq h := \dim \mathcal{H}$. We also give some conditions under which the product of two α-Weyl operators is α-Weyl and its converse implication holds, too. Finally, we show that the weighted Weyl spectrum of a hyponormal operator satisfies the spectral mapping theorem for analytic functions under certain conditions.

1. Introduction

If T is a normal operator on a separable Hilbert space and if $\sigma(T)$ denotes the spectrum of T, then $\mathrm{Re}\,\sigma(T)$, the real part of $\sigma(T)$, always equals $\sigma(\mathrm{Re}\,T)$, the spectrum of real part of T. S. K. Berberian in [2] showed that the equality of these two subsets of real numbers remains true even for some non-normal operators. Arora and Arora [1] further generalized the work for a non-separable Hilbert space $\mathcal{H}$ and showed that for hyponormal operators, $\sigma_\alpha(\mathrm{Re}\,T) = \mathrm{Re}\,\sigma_\alpha(T)$, where $\omega_\alpha^0(T)$ is the weighted Weyl spectrum of weight α with $\aleph_0 \leq \alpha \leq h := \dim \mathcal{H}$. Motivated by these results, we show in Section 2 that if T is hyponormal, then $\mathrm{Re}\,\omega_\alpha^0(T) \subset \omega_\alpha^0(\mathrm{Re}\,T)$. Section 3 focusses on the product of α-Weyl operators. The aim of Section 4 is to discuss the spectral mapping theorem for weighted Weyl spectrum. We show that the weighted Weyl spectrum of a hyponormal operator satisfies spectral mapping theorem for analytic functions under certain conditions.

Throughout the paper, $\mathcal{H}$ is a fixed (complex) non separable Hilbert space of dimension $h > \aleph_0$ and $\mathcal{L}(\mathcal{H})$ denotes the algebra of all bounded linear operators on $\mathcal{H}$. For each cardinal α, $\aleph_0 \leq \alpha \leq h$, ϑ_α denotes the two sided ideal in $\mathcal{L}(\mathcal{H})$ of all bounded linear operators of rank less

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than α and $\mathcal{T}_\alpha$ denotes the norm closure of ϑ_α in $\mathcal{L}(\mathcal{H})$. We write $\sigma_\alpha(T)$, $\pi_\alpha(T)$, and $\omega_\alpha^0(T)$ for the weighted spectrum, the weighted approximate spectrum, and the weighted Weyl spectrum of T of weight α, respectively. We denote $R(T)$, $N(T)$, and $\nu(T)$ for the range, the null space, and the nullity of T, respectively. For the definitions of α-Fredholm operators, α-compact operators etc., we refer to [6].

2. Operator satisfying $\mathrm{Re}\,\omega_\alpha^0(T) \subset \omega_\alpha^0(\mathrm{Re}\,T)$

If T is a normal, closed range and α-compact operator, then $\omega_\alpha^0(T) = \{0\}$ [5]. Since $\mathcal{T}_\alpha$ is self adjoint ideal [6], $\mathrm{Re}\,\omega_\alpha^0(T) = \{0\} = \omega_\alpha^0(\mathrm{Re}\,T)$. To proceed further, we begin with the following:

LEMMA 2.1. *Let T be an α-Fredholm operator. Then there is an $\eta > 0$ such that for any S in $\mathcal{L}(\mathcal{H})$ satisfying $\|S\| < \eta$, $T + S$ is also α-Fredholm and $i(T + S) = i(T)$.*

Proof. Since T is α-Fredholm, there exist operators T_1 and T_2 in $\mathcal{L}(\mathcal{H})$ and K_1 and K_2 in $\mathcal{T}_\alpha$ such that

$$TT_1 = I + K_1 \quad \text{and} \quad T_2T = I + K_2\,.$$

Then, $(T + S)T_1 = I + K_1 + ST_1$ and $T_2(T + S) = I + K_2 + T_2S$. Put $\eta = \min(\|T_1\|^{-1}, \|T_2\|^{-1})$. Then

$$\|ST_1\| \le \|S\|\,\|T_1\| < 1 \quad \text{and} \quad \|T_2S\| \le \|T_2\|\,\|S\| < 1 \quad \text{if} \quad \|S\| < \eta\,.$$

Hence the operators $I + ST_1$ and $I + T_2S$ are invertible. Thus

$$(I + T_2S)^{-1}T_2(T + S) = I + (I + T_2S)^{-1}K_2$$

and

$$(T + S)T_1(I + ST_1)^{-1} = I + K_1(I + ST_1)^{-1}\,.$$

Since $\mathcal{T}_\alpha$ is a two sided ideal, the above equations show that $(T + S)$ is α-Fredholm. Also, since the index 'i' [3] is a group homomorphism,

$$i[(I + T_2S)^{-1}] + i[T_2] + i[(T + S)] = 0\,.$$

Since $i[(I+T_2S)^{-1}] = 0$, this implies that $i(T+S) = -i(T_2) = i(T)$. □

LEMMA 2.2. *For an operator T in $\mathcal{L}(\mathcal{H})$, $\omega_\alpha^0(T) - \sigma_\alpha(T)$ is an open set.*

Proof. We have

$$\begin{aligned} & \omega_\alpha^0(T) - \sigma_\alpha(T) \\ = & \{\lambda \in \mathbb{C} \,|\, (T-\lambda I) \text{ is } \alpha\text{-Fredholm with } i(T-\lambda I) \neq 0\} \\ = & \theta_\alpha(T) \quad \text{(say)}. \end{aligned}$$

If $\theta_\alpha(T)$ is empty, it is trivially open. So, suppose some λ_0 is in $\theta_\alpha(T)$. Then $(T-\lambda_0 I)$ is α-Fredholm and $i(T-\lambda_0 I)$ is nonzero. Hence, by Lemma 2.1, there is some $\epsilon > 0$ such that for any bounded linear operator S satisfying $\|S\| < \varepsilon$, $(T-\lambda_0 I+S)$ is α-Fredholm and $i(T-\lambda_0 I+S)$ is nonzero. In particular, if $|\mu-\lambda_0| < \epsilon$, then $\|(\mu-\lambda_0)I\| < \epsilon$ and hence

$$i((T-\lambda_0 I)-(\mu-\lambda_0)I) = i(T-\mu I) \neq 0$$

and $(T-\mu I)$ is α-Fredholm. This implies that μ is in $\theta_\alpha(T)$ or $\theta_\alpha(T)$ is open. □

LEMMA 2.3. *$\omega_\alpha^0(T)$ is closed for any operator T in $\mathcal{L}(\mathcal{H})$.*

Proof. Let (λ_n) be a sequence in $\omega_\alpha^0(T)$ such that (λ_n) converges to λ_0 in $\mathbb{C}$. Since λ_n is in $\omega_\alpha^0(T)$ for each n, $T-\lambda_n I$ is not α-Weyl for each n. Also, since $\omega_\alpha^0(T) = \sigma_\alpha(T) \cup \theta_\alpha(T)$, where $\theta_\alpha(T) = \omega_\alpha^0(T) - \sigma_\alpha(T)$, and the union is disjoint, either $\sigma_\alpha(T)$ contains only finitely many λ_n or $\sigma_\alpha(T)$ contains infinitely many λ_n. In the first case, without loss of generality, it can be assumed that λ_n is not in $\sigma_\alpha(T)$ for each n. Hence, $T-\lambda_n I$ is α-Fredholm with nonzero index for each n. Since λ_n converges to λ_0, by Lemma 2.1, $T-\lambda_0 I$ is also α-Fredholm with nonzero index and hence, λ_0 is in $\theta_\alpha(T)$, which, in turn, implies that λ_0 is in $\omega_\alpha^0(T)$. If $\sigma_\alpha(T)$ contains infinitely many λ_n, then there exists a subsequence (λ_{n_k}) of (λ_n) such that each λ_{n_k} is in $\sigma_\alpha(T)$ and (λ_{n_k}) converges to λ_0. Since $\sigma_\alpha(T)$ is compact, λ_0 is in $\sigma_\alpha(T)$, which leads to the conclusion. □

THEOREM 2.4. *Let T be an operator in $\mathcal{L}(\mathcal{H})$. Then $\partial\omega_\alpha^0(T) \subset \sigma_\alpha(T)$, i.e., the boundary of $\omega_\alpha^0(T)$ is contained in $\sigma_\alpha(T)$.*

Proof. Let λ_0 be an arbitrary element of $\partial\omega_\alpha^0(T)$. Then there exists a sequence (λ_n) in $\omega_\alpha^0(T)$ and also, a sequence (μ_n) in $\omega_\alpha^0(T)'$, the complement of $\omega_\alpha^0(T)$ such that both (λ_n) and (μ_n) converge to λ_0. Now, the following two cases arise:

Case 1. If infinitely many λ_n are in $\sigma_\alpha(T)$, then there exists a subsequence (λ_{n_k}) of (λ_n) such that each λ_{n_k} is in $\sigma_\alpha(T)$ and (λ_{n_k}) converges to λ_0. As $\sigma_\alpha(T)$ is compact, λ_0 is in $\sigma_\alpha(T)$ and we are done.

Case 2. If finitely many λ_n belong to $\sigma_\alpha(T)$ only, then without loss of generality, we can say that λ_n is not in $\sigma_\alpha(T)$ for each n. Thus, λ_n is in $\omega_\alpha^0(T) - \sigma_\alpha(T)$ for each n. Hence, each $T - \lambda_n I$ is an α-Fredholm operator with index nonzero. Also, λ_n converges to λ_0. Therefore, $T - \lambda_0 I$ is an α-Fredholm operator with index nonzero [Lemma 2.1].

Again, μ_n is not in $\omega_\alpha^0(T)$ for each n. Therefore, $T - \mu_n I$ is α-Fredholm with index zero for each n. Let λ_0 be not in $\sigma_\alpha(T)$. Then, $T - \lambda_0 I$ is an α-Fredholm operator and since $T - \mu_n I$ converges to $T - \lambda_0 I$, index of $T - \lambda_0 I$ is also zero, which provides a contradiction. Hence, λ_0 is in $\sigma_\alpha(T)$ and $\partial\omega_\alpha^0(T) \subset \sigma_\alpha(T)$. □

THEOREM 2.5. *If T is a hyponormal operator in $\mathcal{L}(\mathcal{H})$, then* $\operatorname{Re}\omega_\alpha^0(T) \subset \omega_\alpha^0(\operatorname{Re}T)$.

Proof. Let a be in $\operatorname{Re}\omega_\alpha^0(T)$. Consider the vertical line $\operatorname{Re}\lambda = a$. This line meets the boundary of $\omega_\alpha^0(T)$ at a point $\mu = a + ib$. Now, $\partial\omega_\alpha^0(T) \subset \sigma_\alpha(T)$ (Theorem 2.4). Thus a is in $\operatorname{Re}\sigma_\alpha(T)$. But since $\operatorname{Re}\sigma_\alpha(T) = \sigma_\alpha(\operatorname{Re}T)$ for a hyponormal operator T [1], a belongs to $\sigma_\alpha(\operatorname{Re}T)$, which implies that $\operatorname{Re}\omega_\alpha^0(T) \subset \omega_\alpha^0(\operatorname{Re}T)$. □

3. The product of α-Weyl operators

In this section, we present few results giving the conditions under which the product of two α-Weyl operators is α-Weyl and the converse implication holds. We now begin with the section as the following lemma.

LEMMA 3.1. *Let $S \in \mathcal{L}(\mathcal{H})$ and $T \in \mathcal{L}(\mathcal{H})$ be α-Fredholm. Then ST is α-Fredholm.*

Proof. Let S and T be α-Fredholm. This implies that both are invertible modulo $\mathcal{T}_\alpha$. Hence, there exist operators S_1, T_1 and S_2, T_2 in $\mathcal{L}(\mathcal{H})$, K_1, K_1' and K_2, K_2' in $\mathcal{T}_\alpha$ such that

$$S_1S = I - K_1;\ SS_2 = I - K_1' \quad \text{and} \quad T_1T = I - K_2;\ TT_2 = I - K_2'.$$

And so we now have that

$$(T_1S_1)(ST) = T_1(I - K_1)(T) = (T_1T - T_1K_1T) = I - K_1 - T_1K_1T$$

is in $I + \mathcal{T}_\alpha$. Similarly, $(ST)(T_2S_2)$ is in $I + \mathcal{T}_\alpha$. Thus, ST is α-Fredholm. □

For the converse, we have :

LEMMA 3.2. *Let $S \in \mathcal{L}(\mathcal{H})$ and $T \in \mathcal{L}(\mathcal{H})$. If ST is α-Fredholm and $ST = TS$, then S and T both are α-Fredholm.*

Proof. Let ST be α-Fredholm. This implies that ST is invertible modulo $\mathcal{T}_\alpha$ and in particular, is left invertible modulo $\mathcal{T}_\alpha$. Hence, ST is bounded from below on some closed subspace $\mathbf{M}$ of codimension less than α [6], which means that there is an $\epsilon > 0$, such that $\|(ST)x\| \geq \epsilon\|x\|$, for every x in $\mathbf{M}$. Now,

$$\epsilon\|x\| \leq \|(ST)x\| = \|(TS)x\| \leq \|T\| \, \|Sx\|$$

for every x in $\mathbf{M}$. Thus, there exists an ϵ_1 $(= \epsilon/\|T\| > 0)$ such that $\epsilon_1\|x\| \leq \|Sx\|$ for every x in $\mathbf{M}$. Thus, S is bounded from below on a closed subspace $\mathbf{M}$ of codimension less than α. Since $ST = TS$, similarly we can get T to be bounded from below on some closed subspace of codimension less than α. Thus, S and T both are left invertible modulo $\mathcal{T}_\alpha$ [6]. Also, this gives that S^* and T^* are left invertible modulo $\mathcal{T}_\alpha$, which implies that S and T are right invertible modulo $\mathcal{T}_\alpha$. Thus S and T are α-Fredholm. □

Recall that an operator is said to be α-Weyl if it is α-Fredholm and is in $\ker i$ [5]. We have:

LEMMA 3.3. *If S and T are commuting closed range operators with $\nu(S) = \nu(S^*)$ and $\nu(T) = \nu(T^*)$, then ST is α-Weyl if and only if S and T are α-Weyl.*

Proof. By Lemma 3.1 and Lemma 3.2, we have ST is α-Fredholm if and only if S and T are α-Fredholm. Since $\nu(S) = \nu(S^*)$ and $\nu(T) = \nu(T^*)$, both S and T are in $\ker i$ [5], which implies that S and T are α-Weyl. Conversely, since S and T are α-Weyl and $i(S) + i(T) = i(ST)$, $i(ST) = 0$, we have that (ST) is α-Weyl. □

LEMMA 3.4. *If S and T are commuting closed range α-Weyl operators and ST is α-Weyl, then S is α-Weyl if and only if T is α-Weyl.*

Proof. If S and T are α-Weyl, then by Lemma 3.3, ST is α-Weyl. If ST is α-Fredholm, then S and T are α-Fredholm by Lemma 3.2. Since $i(ST) = i(S)+i(T)$, $i(S) = 0$ if and only if $i(T) = 0$ whenever $i(ST) = 0$. Thus, ST is α-Weyl, which implies that S is α-Weyl if and only if T is α-Weyl. □

4. Spectral mapping theorem

Our aim in this section is to discuss the spectral mapping theorem for weighted Weyl spectrum. We show that the weighted Weyl spectrum of a hyponormal operator satisfies the spectral mapping theorem for analytic functions under certain conditions. The motivation for the following result comes from [7]. The main result is as follows:

THEOREM 4.1. *If T is a hyponormal operator with $\nu(T-\lambda I) = \nu(T^*-\bar{\lambda}I)$ and $\overline{R(T-\lambda I)}$ being the smallest α-closed subset of $R(T-\lambda I)$ for all λ in $\sigma(T)$, and if f is a function analytic on a neighborhood of $\sigma(T)$, then*

$$f(\omega_\alpha^0(T)) = \omega_\alpha^0(f(T)).$$

Before beginning the proof, first we recall upper and lower semi-continuity of a mapping Γ on $\mathcal{L}(\mathcal{H})$ with values as compact subsets of $\mathbb{C}$ [9] and show that the mapping $T \mapsto \omega_\alpha^0(T)$ is upper semi-continuous.

DEFINITION 4.2 ([9]). Let (G_n) be a sequence of compact subsets of $\mathbb{C}$. The limit inferior, $\liminf G_n$, is the set of all λ in $\mathbb{C}$ such that every neighborhood of λ has a non-empty intersection with all but finitely many G_n. The limit superior, $\limsup G_n$, is the set of all λ in $\mathbb{C}$ such that every neighborhood of λ intersects infinitely many G_n. If $\liminf G_n = \limsup G_n$ then $\lim G_n$ is said to exist and is equal to this common limit.

DEFINITION 4.3 ([9]). A mapping Γ defined on $\mathcal{L}(\mathcal{H})$ where values are compact subsets of $\mathbb{C}$ is said to be *upper semi-continuous* at T if $T_n \to T$ then $\limsup \Gamma(T_n) \subset \Gamma(T)$, Γ is *lower semi-continuous* at T if $\Gamma(T) \subset \liminf \Gamma(T_n)$. If Γ is both upper and lower semi-continuous at T, then it is said to be *continuous* at T and in this case, $\lim \Gamma(T_n) = \Gamma(T)$.

The following is an analogous result to Theorem 1 of [9].

THEOREM 4.4. *The mapping $T \mapsto \omega_\alpha^0(T)$ is upper semi-continuous at T.*

Proof. Let λ be not in $\omega_\alpha^0(T)$. Then $T-\lambda I$ is α-Weyl, i.e., $T-\lambda I$ is α-Fredholm and $i(T-\lambda I) = 0$. By Lemma 2.1 of Section 2, there exists an $\eta > 0$ such that if S is in $\mathcal{L}(\mathcal{H})$ and $\|\lambda I - T - S\| < \eta$, then S is α-Weyl. Let (T_n) be a sequence of operators in $\mathcal{L}(\mathcal{H})$ such that $T_n \to T$. Then, corresponding to $\eta > 0$, there exists a positive integer N such that

$$\|(\lambda I - T) - (\lambda I - T_n)\| < \frac{\eta}{2} \quad \text{for all } n \geq N.$$

Let U_λ be an open $(\eta/2)$-neighborhood of λ. Then, for μ in U_λ and $n \geq N$, $\|(\lambda I - T) - (\lambda I - T_n)\| < \eta$ so that $(\mu I - T_n)$ are α-Weyl for all $n \geq N$. This implies that λ is not in $\limsup \omega_\alpha^0(T_n)$. Thus,

$$\limsup \omega_\alpha^0(T_n) \subset \omega_\alpha^0(T)\,.$$

□

THEOREM 4.5. *Let $T_n \to T$. Then it holds that if* $\lim \sigma_\alpha(T_n) = \sigma_\alpha(T)$, *then* $\lim \omega_\alpha^0(T_n) = \omega_\alpha^0(T)$.

Proof. In the light of Theorem 4.4, it suffices to show that $\omega_\alpha^0(T) \subset \liminf \omega_\alpha^0(T_n)$. Suppose λ is not in $\liminf \omega_\alpha^0(T_n)$. Then there is a neighborhood U_λ of λ that does not intersect infinitely many $\omega_\alpha^0(T_n)$. Since $\sigma_\alpha(T_n) \subset \omega_\alpha^0(T_n)$, U_λ does not intersect infinitely many $\sigma_\alpha(T_n)$, i.e., λ is not in $\lim \sigma_\alpha(T_n) = \sigma_\alpha(T)$. Hence $T - \lambda I$ is α-Fredholm. By Lemma 2.1, there exists an $\eta > 0$ such that if S is in $\mathcal{L}(\mathcal{H})$ and $\|(T - \lambda I) - S\| < \eta$, then S is α-Fredholm and $i(S) = i(T - \lambda I)$. Let V_λ be $(\eta/2)$-open neighborhood of λ. Then, for μ in V_λ and $n \geq N$, $\|(T_n - \mu I) - (T - \lambda I)\| < \eta$ so that, for all $n \geq N$, $(T_n - \mu I)$ are α-Fredholm and $i(T_n - \mu I) = i(T - \lambda I)$. But $i(T_n - \mu I) = 0$ for all $n \geq N$. Hence, $i(T - \lambda I) = 0$, so λ is not in $\omega_\alpha^0(T)$. □

COROLLARY 4.6. *Let $T_n \to T$. Then it holds that if $T_nT = TT_n$ for each n, then* $\lim \omega_\alpha^0(T_n) = \omega_\alpha^0(T)$.

Proof. Since $T_n \to T$ in $\mathcal{L}(\mathcal{H})$, $\widehat{T}_n \to \widehat{T}$ in $\mathcal{L}(\mathcal{H})/\mathcal{I}_\alpha$. Also, $T_nT = TT_n$ for each n. This implies $\widehat{T}_n\widehat{T} = \widehat{T}\widehat{T}_n$, for each n. Hence, by Theorem 4 [8], $\lim \sigma(\widehat{T}_n) = \sigma(\widehat{T})$. Thus, by Theorem 4.5, $\lim \omega_\alpha^0(T_n) = \omega_\alpha^0(T)$. □

Proof of Theorem 4.1. Suppose p is any polynomial. Let

$$p(T) - \lambda I = a_0(T - \mu_1 I) \cdots (T - \mu_n I)\,.$$

If T is hyponormal, then $(T - \mu_i I)$ $(i = 1, \ldots, n)$ are commuting hyponormal operators, and hence it follows from Section 3 that

$$\begin{aligned} \lambda \notin \omega_\alpha^0(p(T)) &\iff a_0(T - \mu_1 I) \cdots (T - \mu_n I) \text{ is } \alpha\text{-Weyl} \\ &\iff (T - \mu_i I) \text{ is } \alpha\text{-Weyl for all } i = 1, \ldots, n \\ &\iff \mu_i \notin \omega_\alpha^0(T) \text{ for all } i = 1, \ldots, n \\ &\iff \lambda \notin p(\omega_\alpha^0(T)), \end{aligned}$$

which implies that

$$\omega_\alpha^0(p(T)) = p(\omega_\alpha^0(T))\,. \tag{1}$$

Now, suppose that r is any rational function with no poles in $\sigma(T)$. Write $r = p/q$ where p and q are polynomials and q has no zeros in $\sigma(T)$. Then,

$$r(T) - \lambda I = (p/q)(T) - \lambda I = (p - \lambda q)(T)(q(T))^{-1}.$$

By (1), $(p - \lambda q)(T)$ is α-Weyl if and only if $(p - \lambda q)$ has no zeros in $\omega_\alpha^0(T)$. Thus,

$$\begin{aligned}
\lambda \notin \omega_\alpha^0(r(T)) &\iff (p - \lambda q)(T) \text{ is } \alpha\text{-Weyl} \\
&\iff (p - \lambda q) \text{ has no zeros in } \omega_\alpha^0(T) \\
&\iff (p - \lambda q)(x)(q(x))^{-1} \neq 0 \text{ for any } x \text{ in } \omega_\alpha^0(T) \\
&\iff \lambda \notin r(\omega_\alpha^0(T)),
\end{aligned}$$

which implies that $\omega_\alpha^0(r(T)) = r(\omega_\alpha^0(T))$. If f is an analytic function on a neighborhood of $\sigma(T)$, then by Runge's theorem [4], there is a sequence $\{r_n\}$ of rational functions with no poles in $\sigma(T)$ such that $\{r_n\}$ converges to f uniformly on $\sigma(T)$. Since $\{r_n(T)\}$ commutes with $f(T)$, the result follows from Corollary 4.6. □

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