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ON THE HEREDITARILY HYPERCYCLIC OPERATORS

Bahman Yousefi and Ali Farrokhinia

Abstract. Let X be a separable Banach space. We give sufficient conditions under which $T : X \to X$ is hereditarily hypercyclic. Also, we prove that hereditarily hypercyclicity with respect to a special sequence implies the hereditarily hypercyclicity with respect to the entire sequence.

1. Introduction

Suppose that X is a topological vector space and $\{T_n : n \in \mathbb{N}\}$ is a sequence of continuous linear mappings on X. We say a vector $x \in X$ is hypercyclic for $\{T_n : n \in \mathbb{N}\}$ if the collection of the images $\{T_n x : n \in \mathbb{N}\}$ is dense in X. If such a vector exists, we call the original sequence of operators hypercyclic. In the special case, we say that a continuous linear mapping T on X is hypercyclic if the sequence of powers $\{T^n : n \in \mathbb{N}\}$ is hypercyclic. By Theorem 1.2 in [16] hypercyclicity implies transitivity, i.e., for each pair U and V of nonempty open subsets of X, there exists a positive integer n such that $T_n(U) \cap V \neq \phi$.

It is interesting to know that what type of continuous linear mappings can actually be hypercyclic. The first example of a hypercyclic operator on a Hilbert space was constructed by Rolewicz in 1969 [26]. He showed that if B is the backward shift on $\ell^2(\mathbb{N})$, then λB is hypercyclic if and only if $|\lambda| > 1$.

A nice criterion, namely the Hypercyclicity Criterion, was developed independently by Kitai [22], Gethner and Shapiro [15]. This criterion has been used to show that hypercyclic operators arise within the special classes of composition operators [10], weighted shifts [27], adjoints of multiplication operators [11], and adjoints of subnormal and hyponormal operators [9].

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Hypercyclicity is closely related to the well-known concept of transitivity from topological dynamics. By Theorem 1.2 in [16] every hypercyclic operator is transitive, but the converse need not be true, see [7] or [3, Example] for simple examples. In many spaces, the two concepts coincide, of [28, 1.10] or [19, Theorem 3].

The formulation of the Hypercyclicity Criterion in the following theorem was given by J. Bes in his PhD thesis [5].

THEOREM 1.1. (The Hypercyclicity Criterion). *Suppose X is a separable Banach space and T is a continuous linear mapping on X. If there exist two dense subsets Y and Z in X and a sequence $\{n_k\}$ such that:*

1. *$T^{n_k}y \to 0$ for every $y \in Y$, and*
2. *There exist functions $S_{n_k} : Z \to X$ such that for every $z \in Z$, $S_{n_k}z \to 0$, and $T^{n_k}S_{n_k}z \to z$,*

then T is hypercyclic.

In the above criterion it is said often that T satisfies the Hypercyclicity Criterion for the sequence $\{n_k\}$ ([12]).

In recent years, several variants of the criterion have been considered, see [19, Remark 3], [24, Theorem 1.1] and [14, Theorem 3.2]. All of them, however, were shown to be equivalent to the Hypercyclicity Criterion as stated above, see Peris [24, Theorem 2.3], Feldman [14, comment after Theorem 3.2] and Bermudez, Bonilla and Peris [1, Section 2]. For additional interesting discussions we refer to Grivaux [17].

The question about the necessity conditions for Hypercyclicity Criterion was raised by Herrero in [21] and by Bes and Peris in [6]. The question has so far evaded all attempts at being resolved. This has motivated the search for equivalent but less technical forms of the Hypercyclicity Criterion. The following result was obtained independently by Bernal-Gonzalez and Grosse-Erdmann, Leon-Saavedra, and by the authors (see [4, Remark 3.5], [23] and [39]).

THEOREM 1.2. *For any operator $T \in B(X)$, the following are equivalent:*

(i) T satisfies the hypothesis of the Hypercyclicity Criterion.

(ii) For each pair U, V of non-void open subsets of X, and each neighborhood W of zero, $T^n(U) \cap W \neq \phi$ and $T^n(W) \cap V \neq \phi$ for some integer n.

Also Bonet, Martinez-Gimenez and Peris, and the authors independently have shown that the Hypercyclicity Criterion for any operator T on a Hilbert space H, is equivalent to the hypercyclicity of some corresponding linear mapping on the operator algebra of Hilbert-Schmidt operators (see [8] and [38]). In [25], Peris and Saldivia have given equivalent conditions for a continuous linear operator on a separable $\mathcal{F}$-space satisfy the Hypercyclicity Criterion. Also, they proved that for any hypercyclic operator there exists a sequence $\{n_k\}$ such that T and $\{T^{n_k}\}$ do not share the same hypercyclic vectors.

DEFINITION 1.3. Let $T \in B(X)$ and $\{n_k\}$ be an increasing sequence of non-negative integers. We say that T is hereditarily hypercyclic with respect to $\{n_k\}$ provided for all subsequences $\{n_{k_j}\}$ of $\{n_k\}$, the sequence $\{T^{n_{k_j}}\}_{j\geq 1}$ is hypercyclic. Also, an operator T will be called hereditarily hypercyclic if it is hereditarily hypercyclic with respect to some sequence $\{n_k\}$.

It follows that the condition of being hereditarily hypercyclic characterizes the operators satisfying the criterion ([6]).

In this present paper for an operator acting on a separable Banach space we will investigate the hereditarily hypercyclicity with respect to a sequence.

2. Main results

In what follows, X will denote a separable Banach space. Also, $\{n_k\}_{k\geq 1} \subseteq \mathbb{N}$ will always refer to an increasing sequence of positive integers such that $n_k \to \infty$. By the entire sequence we will refer to the sequence $\{n\}_n = \mathbb{N}$.

LEMMA 2.1. *Suppose that $T \in B(X), \mathcal{F}$ is a countable dense subset of X and $\{n_k\}_{k\geq 2} \subseteq \mathbb{N}$, and $n_1 = 0$. Assume that if $\varepsilon > 0$ and vectors g, h are in $\mathcal{F}$, then for any $N \in \mathbb{N}$ there would exist an integer $k > N$ arbitrarily large and a vector f in $\mathcal{F}$ such that*
(i) $||f|| < \varepsilon$,
(ii) $||T^{n_k} f - g|| < \varepsilon$,
(iii) $||T^{n_k} h|| < \varepsilon$.
Then $\{T^{n_k}\}_k$ is hypercyclic. Moreover, T is hereditarily hypercyclic with respect to $\{n_k\}_{k\geq 1}$.

Proof. Assume that $\mathcal{F} = \{g_n : n \in \mathbb{N}\}$. Let $n_{k_1} = n_1 = 0$ and $q_1 = g_1$. By the induction, assume that for $2 \leq j \leq m$, the numbers $n_{k_j} \geq n_{k_{j-1}}$ and the vectors q_j in $\mathcal{F}$ have been chosen. Now choose $k > k_m$ and f by applying the hypothesis to $\varepsilon = ||T||^{-n_{k_m}} 2^{-m-1}$ and the vectors $g = g_{m+1}$, $h = \sum_{i=1}^{m} q_i$. Let the k and f so obtained be denoted by k_{m+1}

and q_{m+1} respectively. Then

$$||q_{m+1}|| \leq ||T||^{-n_{k_m}} 2^{-m-1},$$

$$||T^{n_{k_{m+1}}} q_{m+1} - g_{m+1}|| < ||T||^{-n_{k_m}} 2^{-m-1},$$

and

$$||T^{n_{k_{m+1}}} (\sum_{j=1}^{m} q_j)|| < ||T||^{-n_{k_m}} 2^{-m-1}.$$

Put $q = \sum\limits_{j=1}^{\infty} q_j$. We are now ready to verify that q is a hypercyclic vector for $\{T^{n_k}\}_k$. It follows that

$$||T^{n_{k_m}} q - g_m|| \leq ||T^{n_{k_m}} (\sum_{j=1}^{m-1} q_j)|| + ||T^{n_{k_m}} q_m - g_m|| + \sum_{j=m+1}^{\infty} ||T^{n_{k_m}}|| \, ||q_j||.$$

Note that by the hypothesis (i) and (ii) of the theorem, $||T|| > 1$. Thus

$$\begin{aligned} \sum_{j=m+1}^{\infty} ||T^{n_{k_m}}|| \, ||q_j|| &\leq ||T||^{n_{k_m}} (||T||^{-n_{k_m}} 2^{-m-1} \\ &\quad + ||T||^{-n_{k_{m+1}}} 2^{-m-2} + \cdots) \\ &\leq 2^{-m-1} \sum_{i \geq 0} 2^{-i} \\ &= 2^{-m}, \end{aligned}$$

and so

$$\begin{aligned} ||T^{n_{k_m}} q - g_m|| &\leq ||T||^{-n_{k_{m-1}}} 2^{-m} + ||T||^{-n_{k_{m-1}}} 2^{-m} + 2^{-m} \\ &\leq 2^{-m+2}. \end{aligned}$$

Thus $\{T^{n_k}\}_k$ is hypercyclic. Now let $\{n_{k_j}\}_{j \geq 1}$ be an arbitrary subsequence of $\{n_k\}$. By the assumptions if $\varepsilon > 0$, and $g, h \in \mathcal{F}$, then there exist j large enough and $f \in \mathcal{F}$ such that

$(i)'$ $||f|| < \varepsilon$,
$(ii)'$ $||T^{n_{k_j}} f - g|| < \varepsilon$,
$(iii)'$ $||T^{n_{k_j}} h|| < \varepsilon$.

By using conditions $(i)', (ii)'$ and $(iii)'$, with a similar method used by conditions $(i), (ii)$ and (iii), we can see that $\{T^{n_{k_j}}\}_{j \geq 1}$ is hypercyclic. So T is indeed hereditarily hypercyclic with respect to $\{n_k\}$. □

THEOREM 2.2. *An operator $T \in B(X)$ is hereditarily hypercyclic with respect to $\{n_k\}_{k \geq 1}$ if and only if given any two open sets U, V*

there is some positive integer N such that

$$T^{n_k}(U) \cap V \neq \phi$$

for any $k > N$.

Proof. Let T be hereditarily hypercyclic with respect to $\{n_k\}$. Suppose that there exist some open sets U, V such that

$$T^{n_{k_i}}(U) \cap V = \phi$$

for some subsequence $\{n_{k_i}\}_{i\geq 1}$ of $\{n_k\}_{k\geq 1}$. Since T is hereditarily hypercyclic with respect to $\{n_k\}$, thus $\{T^{n_{k_i}}\}_{i\geq 1}$ is hypercyclic and so we get a contradiction. Conversely suppose that $\{n_{k_i}\}_i$ is an arbitrary subsequence of $\{n_k\}$, and U, V are open sets in X satisfying

$$T^{n_k}(U) \cap V \neq \phi$$

for any $k > N$. So there exists i large enough such that $n_{k_i} > N$ and

$$T^{n_{k_i}}(U) \cap V \neq \phi.$$

This implies that $\{T^{n_{k_i}}\}_{i\geq 1}$ is hypercyclic and so T is indeed hereditarily hypercyclic with respect to $\{n_k\}$. □

THEOREM 2.3. *If an operator $T \in B(X)$ is hereditarily hypercyclic with respect to a sequence $\{n_k\}_{k\geq 1}$ with $\sup\limits_k(n_{k+1} - n_k) < \infty$, then T is hereditarily hypercyclic with respect to the entire sequence.*

Proof. Let T be hereditarily hypercyclic with respect to a sequence $\{n_k\}_k$ satisfying $\sup\limits_k(n_{k+1} - n_k) < \infty$. Also, let U and V be two nonempty open sets in X. We will show that there exists an integer N such that $T^k(U) \cap V \neq \phi$ for all $k > N$, which by Theorem 2.2 implies that T is indeed hereditarily hypercyclic with respect to the entire sequence. For this let $M = \sup\{n_{k+1} - n_k : k \in \mathbb{N}\}$. For each integer $0 \leq i \leq M$, put $U_i = U$ and $V_i = T^{-i}V$. Since T is hereditarily hypercyclic with respect to $\{n_k\}$, by Theorem 2.2, for all integer $0 \leq i \leq M$ there exists $N_i \in \mathbb{N}$ such that $T^{n_k}(U_i) \cap V_i \neq \phi$ for all $k > N_i$. Let $N = \max\{N_i : i = 0, 1, \ldots, M\}$, then $N = n_{m_0}$ for some integer $0 \leq m_0 \leq M$. Now we can show that $T^n(U) \cap V \neq \phi$ for all $n \geq N$. In fact if $n \geq N$, then there exists $k \geq m_0$ and $0 \leq j \leq M$ such that $n = n_k + j$. Note that $T^{n_k}(U_j) \cap V_j \neq \phi$ for $k \geq m_0$. Hence,

$$T^{n_k}(U) \cap T^{-j}(V) = T^{n_k}(U_j) \cap V_j \neq \phi$$

and so

$$T^n(U) \cap V = T^{n_k+j}(U) \cap V \neq \phi$$

for all $n \geq N$. So T is hereditarily hypercyclic with respect to the entire sequence and the proof is complete. $\square$

Note that the converse of Theorem 2.3 is automatically true.

COROLLARY 2.4. *Suppose that $T \in B(X)$ is hereditarily hypercyclic with respect to a sequence $\{n_k\}_k$ with $\sup\limits_k (n_{k+1} - n_k) < \infty$. Then given two nonempty open sets U and V in X, there exists an integer N such that $T^k(U) \cap V \neq \phi$ for all $k > N$.*

Remember that we say T satisfy the Hypercyclicity Criterion for a sequence $\{m_k\}_k$ if in the Hypercyclicity Criterion (Theorem 1.1) $\{n_k\}_k = \{m_k\}_k$.

COROLLARY 2.5. *Let T satisfies the Hypercyclicity Criterion for a sequence $\{n_k\}$ satisfying $\sup(n_{k+1} - n_k) < \infty$. Then T is hereditarily hypercyclic with respect to the entire sequence.*

Proof. By Theorem 2.3 in [6], if T satisfies the Hypercyclicity Criterion for a sequence $\{n_k\}$, then T is hereditarily hypercyclic with respect to $\{n_k\}$. Now by Theorem 2.3, the proof is complete. $\square$

REMARKS 2.6. (*i*) By Remark 3.3 in [2], if T satisfies the Hypercyclicity Criterion for a sequence $\{n_k\}$ such that $\sup(n_{k+1} - n_k) < \infty$, then T satisfies the Hypercyclicity Criterion for the entire sequence.

(*ii*) Theorem 1.1 in [12] follows immediately from Theorem 2.2 and Corollary 2.5 since clearly if $T \in B(X)$ satisfies the Hypercyclicity Criterion for a sequence $\{n_k\}$ satisfying $\sup(n_{k+1} - n_k) < \infty$, then given two nonempty open sets U and V in X, there exists an integer N such that $T^k(U) \cap V \neq \phi$ for all $k > N$. Note that Grivaux gave examples which shows that the converse of Theorem 1.1 in [12] is not true (see [17]).

Recall that if $\{\beta(n)\}_{n=-\infty}^{\infty}$ is a sequence of positive numbers with $\beta(0) = 1$ and $1 \leq p < \infty$, then the space of formal Laurent series consists of the sequences $f = \{\hat{f}(n)\}_{n=-\infty}^{\infty}$ such that $||f||^p = ||f||_\beta^p = \sum_{n=-\infty}^{\infty} |\hat{f}(n)|^p \beta(n)^p < \infty$. The notation $f(z) = \sum\limits_{n=-\infty}^{\infty} \hat{f}(n) z^n$ shall be used whether or not the series converges for any value of z. These are called formal Laurent series. Note that when n ranges on $\mathbb{N} \cup \{0\}$, they are called formal power series and are denoted by $H^p(\beta)$. Let $L^p(\beta)$ denotes the space of such formal Laurent series. These are reflexive Banach spaces with the norm $||.||_\beta$. Let $\hat{f}_k(n) = \delta_k(n)$. So $f_k(z) = z^k$ and the $\{f_k\}_{k \in \mathbb{Z}}$ is a basis for $L^p(\beta)$ such that $||f_k|| = \beta(k)$. Clearly

M_z, the operator of multiplication by z on $L^p(\beta)$, shifts the basis $\{f_k\}_k$. The operator M_z is bounded if and only if $\{\beta(k+1)/\beta(k)\}_k$ is bounded. The operator B on $L^p(\beta)$ is defined by $Bf_j = f_{j-1}$ for all $j \in \mathbb{Z}$. Clearly B is bounded if and only if the sequence $\{\beta(k)/\beta(k+1)\}_k$ is bounded. Sources on formal series include [13, 29–37].

In [27] H. Salas has extended Rolewicz's result ([26]) by completely characterizing the hypercyclic weighted shifts on l^2. In [20] K. G. Grosse-Erdmann has obtained a characterization of hypercyclic weighted shifts on an arbitrary $\mathcal{F}$-sequence space in which the canonical unit vectors form a Schauder basis and if the basis is unconditional it was given a characterization of those hypercyclic weighted shifts that are even chaotic. Here, we want to characterize the equivalent conditions for hereditarily hypercyclicity of the operator B (with respect to the entire sequence) acting on the Banach spaces $L^p(\beta)$.

In the following we assume that B is bounded on $L^p(\beta)$.

THEOREM 2.7. *The operator B is hereditarily hypercyclic with respect to the entire sequence if and only if $\lim_{|n|\to\infty} \beta(n) = 0$.*

Proof. Let $\lim_{|n|\to\infty} \beta(n) = 0$ and $m \in \mathbb{N}$. Consider the nonzero vectors g, h in the span $\{f_j : |j| \le m\}$. Then $g = \sum_{|j|\le m} \hat{g}(j) f_j$ and $h = \sum_{|j|\le m} \hat{h}(j) f_j$. Define $Sf_j = f_{j+1}$ for all $j \in \mathbb{Z}$. If $\alpha_0 = \min\{\beta(k) : |k| \le m\}$, then we have

$$\begin{aligned} ||B^n h||^p &= ||\sum_{|j|\le m} \hat{h}(j) f_{j-n}||^p \\ &= \sum_{|j|\le m} |\hat{h}(j)|^p \beta(j)^p (\frac{\beta(j-n)}{\beta(j)})^p \\ &\le ||h||^p \sup_{|j|\le m} (\frac{\beta(j-n)}{\beta(j)})^p. \end{aligned}$$

Similarly

$$\begin{aligned} ||S^n g||^p &= ||\sum_{|j|\le m} \hat{g}(j) f_{j+n}||^p \\ &\le ||g||^p \sup_{|j|\le m} (\frac{\beta(j+n)}{\beta(j)})^p. \end{aligned}$$

Since $\lim_{|n|\to\infty} \beta(n) = 0$, there exists a positive integer N large enough such that $\beta(k) < \alpha_0\varepsilon$ and $\beta(-k) < \alpha_0\varepsilon$ for all $k > N$. Now for $n > m + N$ we have $n + j > N$ and $n - j > N$ for all $|j| \leq m$, and so we get $||B^n h|| \leq ||h||\varepsilon$ and $||S^n g|| \leq ||g||\varepsilon$. By setting $f = S^n g$ we can see that the conditions in the hypothesis of Lemma 2.1 are satisfied. Thus B is hereditarily hypercyclic with respect to the entire sequence.

Conversely, suppose that B is hereditarily hypercyclic with respect to the entire sequence. Let $\varepsilon > 0$, then there exists $\alpha > 0$ such that $\frac{\alpha}{1-\alpha} < \varepsilon$. We will show that the relation

$$(*) \qquad \lim_{|n|\to\infty} \beta(n) = 0$$

holds. For this let $\{n_k\}$ be an arbitrary subsequence in $\mathbb{N}$ such that $n_k \to +\infty$. Since B is hereditarily hypercyclic with respect to the full sequence, the sequence $\{B^{n_k}\}_k$ is hypercyclic. By a theorem proved by Grosse-Erdmann [18, Theorem 1.2.2, p. 11], the set

$$\{(x, B^{n_k}x) : x \in L^p(\beta), \quad k \in \mathbb{N}\}$$

is dense in $L^p(\beta) \times L^p(\beta)$. So there exist a vector $x = \sum_j \hat{x}(j) f_j$ in $L^p(\beta)$ and an integer $k \geq 1$ such that $||x - f_0|| < \alpha$ and $||B^{n_k}x - f_0|| < \alpha$. Note that

$$||x - f_0||^p = |\hat{x}(0) - 1|^p + \sum_{|j|\geq 1} |\hat{x}(j)|^p \beta(j)^p < \alpha^p$$

and

$$||B^{n_k}x - f_0||^p = |\hat{x}(n_k) - 1|^p + \sum_{|j|\geq 1} |\hat{x}(j + n_k)|^p \beta(j)^p < \alpha^p.$$

Thus we get

$$(1) \qquad |\hat{x}(0)| > 1 - \alpha,$$

$$(2) \qquad |\hat{x}(j)|\beta(j) < \alpha \ ; \quad |j| \geq 1,$$

$$(3) \qquad |\hat{x}(n_k) - 1| < \alpha,$$

$$(4) \qquad |\hat{x}(j + n_k)|\beta(j) < \alpha \ ; \quad |j| \geq 1.$$

By relations (1) and (4) we obtain

$$(5) \qquad \beta(-n_k) < \frac{\alpha}{1 - \alpha} < \varepsilon$$

and by relations (2) and (3) we obtain

$$\beta(n_k) < \frac{\alpha}{1-\alpha} < \varepsilon. \tag{6}$$

Now to prove the relation $(*)$, we will argue by contradiction. Therefore, assume that $\limsup_{n\to+\infty} \beta(n) > 0$ or $\limsup_{n\to+\infty} \beta(-n) > 0$. If $\limsup_{n\to+\infty} \beta(n) > 0$, then there exist $\alpha_1 > 0$ and a sequence of integers $n_k \to \infty$ such that $\beta(n_k) > \alpha_1 > 0$ for all $k \in \mathbb{N}$. By taking $0 < \varepsilon < \alpha_1$, we get a contradiction by the relation (6). Also, if $\limsup_{n\to+\infty} \beta(-n) > 0$, similarly we can get a contradiction by the relation (5). Thus indeed $\lim_{|n|\to\infty} \beta(n) = 0$ and so the proof is complete. $\square$

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Bahman Yousefi
Department of Mathematics
College of Science
Shiraz University
Shiraz 71454, Iran
E-mail: byousefi@hafez.shirazu.ac.ir

Ali Farrokhinia
Department of Mathematics
Tarbiat Modares University
Tehran, Iran
E-mail: afarokha@modares.ac.ir