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ADAPTIVE BACKSTEPPING CONTROL FOR SATELLITE FORMATION FLYING WITH MASS UNCERTAINTY

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ABSTRACT

Satellite formation flying has become a critical issue in the aerospace engineering because it is considered as an enabling technology for many space missions. Thus, many nonlinear control theories have been developed for the tracking problem of satellite formation flying, which include full-nonlinear dynamics, external disturbances and parameter uncertainty. In this study, nonlinear adaptive control law is developed using an adaptive backstepping technique to solve the relative position tracking problem of the satellite formation flying in the presence of mass uncertainty and the bounded external disturbance. Simulation studies are included to demonstrate the proposed controller performance. The proposed controller is shown to guarantee the system stability against the external bounded disturbances in the presence of mass uncertainty.

Keywords: satellite formation flying, enabling technology, adaptive backstepping, mass uncertainty

1. INTRODUCTION

Satellite formation flying (SFF) is the placing of small satellites into nearby orbits to form a cluster for the achievement of same mission. It has become a critical issue in the aerospace engineering because it is considered as an enabling technology for many space missions and can provide potential benefits compared to a single large spacecraft. NASA (National Aeronautics and Space Administration) launched the Earth Observing-1 (EO-1) in November 2000 which was the first satellite demonstrating SFF technology and was scheduled to acquire a stereo image with Lansat-7 satellite. Recently, ST5 (Space Technology 5) was successfully launched on March 22, 2006 to test and validate new innovative technologies for future science missions, which consists of three micro-satellites. ESA (European Space Agency) is also implementing several space missions using SFF technology such as LISA Pathfinder, SWARM, PROBA-3, MAX and so on. In addition, many universities and institutes plan or are developing many missions to demonstrate some technologies of SFF and distributed systems.

Many SFF control laws have been designed on the base of the simplified relative dynamic equations such as Clohessy-Wiltshire equations (Clohessy & Wiltshire 1960) which are known as Hill's

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equations . These equations cannot capture the J_2 perturbation effect because they are derived under the assumption that the reference satellite moves in the circular orbit, the Earth is spherically symmetric, and the target satellite is very close to the reference satellite. Control designs based on Hill's equations require high fuel consumption and can imperil the formation flying mission with long duration and large separation between satellites since Hill's equations disregard the perturbation and nonlinear terms on the relative dynamics. So many nonlinear control laws have been presented for the tracking problem of SFF, which include full-nonlinear dynamics, external disturbances and parameter uncertainty.

Queiroz et al. (2000) developed a nonlinear adaptive control law for the relative position tracking of multiple satellites. Gurfil et al. (2003) proposed a nonlinear adaptive neural control methodology for deep-space SFF. Pongvthithum et al. (2005) developed the tracking control law using the universal adaptive control scheme. Most nonlinear control approaches for SFF are based on full state feedback controllers which require both position and velocity sensors of satellites. However, Wong et al. (2002) designed an adaptive output feedback tracking control in the absence of velocity measurements. In addition, sliding mode controller was developed to track the desired trajectories with the extended Kalman filter for estimating the state vector based on measurements of relative distance between two satellites (Lim et al. 2003).

Some unknown parameters can be appeared in the relative dynamics of SFF, which may be mass, thruster magnitude error and misalignment. Adaptive backstepping technique has been considered as a powerful tool for stabilizing nonlinear systems with parameter uncertainties both for tracking and regulation problems. This technique was introduced by Kanellakopoulos et al. (1991) for feedback linearizable systems and has been extended to a wide class of nonlinear systems by many researchers. However, adaptive control based on disturbance-free models can cause the system to become unstable in the presence of small bounded external disturbances. This problem leads to the development of robust adaptive control which adds a leakage term such as the switching σ-modification and the e_1-modification into the parameter update law or uses projection operator (Ioannou & Sun 1996).

Mass uncertainty has been handled in the control problem of SFF using an adaptive scheme because the mass of satellites could be changed due to fuel consumption (Pongvthithum et al. 2005). Even though the satellite mass vary slowly in time for the orbit maneuver, it can be considered as a constant because the variation of mass is tiny in each configuration or reconfiguration step. In this study, nonlinear control law is developed for the tracking problem of SFF with mass uncertainty in the presence of the bounded external disturbance. The controller is designed using the adaptive backstepping technique and Lyapunov-based design. And the switching σ-modification (Ikhouane & Krstic 1998) is added to the parameter update law to avoid the parameter drift. No previous paper uses the same controller design approach as in this paper for SFF control problem. In addition, this approach does not need the requirement for the disturbances to be periodic compared to Wong et al. (2001). The proposed controller was shown to guarantee the system stability against the external bounded disturbances in the presence of mass uncertainty. Even though the tracking error converges into zero, the estimated parameter could not approach to the true value because Lyapunov function is globally uniformly ultimately bounded but not asymptotically stable.

2. RELATIVE DYNAMICS

A rotating local-vertical-local-horizontal (LVLH) frame is used to visualize the relative motion with respect to the reference satellite. The x-axis points in the radial direction, the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points in the along-track direction as shown in Figure 1.

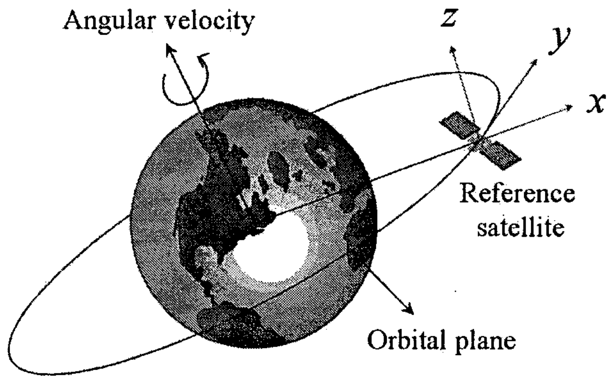

Figure 1. LVLH coordinate system for the relative motion.

The relative motion dynamics can be derived using the Lagrangian mechanics based on the LVLH frame (Wong et al. 2002). This derivation utilizes the fact that the total energy(sum of potential and kinetic energy) of the satellite is conserved under the gravitational field. The relative dynamics for an eccentric reference satellite is given by

$$\begin{aligned}
&\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x + \frac{\mu(R+x)}{[(R+x)^2+y^2+z^2]^{3/2}} - \frac{\mu}{R^2} = \frac{u_{lx}}{m_l} + \frac{1}{m}(D_x + u_x)\\
&\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y + \frac{\mu y}{[(R+x)^2+y^2+z^2]^{3/2}} = \frac{u_{ly}}{m_l} + \frac{1}{m}(D_y + u_y)\\
&\ddot{z} + \frac{\mu z}{[(R+x)^2+y^2+z^2]^{3/2}} = \frac{u_{lz}}{m_l} + \frac{1}{m}(D_z + u_z)
\end{aligned} \tag{1}$$

where $\mathbf{x} = [x, y, z] \in \mathrm{R}^3$ is a state vector denoting the position of the follower satellite, $\mathbf{D} = [D_x, D_y, D_z] \in \mathrm{R}^3$ denotes external disturbances including J_2, solar radiation pressure and so on, $\mathbf{u} = [u_x, u_y, u_z] \in \mathrm{R}^3$ and $\mathbf{u}_l = [u_{lx}, u_{ly}, u_{lz}] \in \mathrm{R}^3$ describe the control inputs of the follower satellite and the reference satellite, respectively. In addition, m represents the mass of the follower satellite, and m_l does one of the reference satellite. R presents the radius of the reference satellite, and μ is the gravitational constant. θ refers to the latitude angle of the reference satellite, which means the circular orbit for the case of $\dot{\theta}$ = constant, i.e. $\ddot{\theta} = 0$.

In general, Clohessy-Wiltshire equation based on the LVLH frame is utilized to describe the relative motion and control strategies between neighboring satellites, which is usually called as Hill's equations. So, Eq. (1) can be transformed into the Hill's equations under some assumptions. However, non-linear relative dynamics is used for designing adaptive backstepping controller in this study. Even though spacecraft mass is slow-varying in the configuration or reconfiguration of SFF, it is assumed that it is an unknown and constant parameter during orbit maneuvering. In addition, we assume that the reference satellite moves in the circular orbit and the external disturbances are unknown and bounded. If the reference satellite is considered as a virtual reference with no control inputs, the first term of the right equation in Eq. (1) can be canceled. So the above nonlinear dynamic equation can be rewritten to the simple form of Eq. (2) for deriving the control law.

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2$$

$$\dot{\mathbf{x}}_2 = f(\mathbf{x}_1, \mathbf{x}_2) + \frac{1}{m}(\mathbf{D} + \mathbf{u}) \tag{2}$$

where

$$f(\mathbf{x}_1, \mathbf{x}_2) = \begin{bmatrix} 2\dot{\theta}\dot{y} + \dot{\theta}^2 x - \frac{\mu(R+x)}{[(R+x)^2+y^2+z^2]^{3/2}} + \frac{\mu}{R^2} \\ -2\dot{\theta}\dot{x} + \dot{\theta}^2 y - \frac{\mu y}{[(R+x)^2+y^2+z^2]^{3/2}} \\ -\frac{\mu z}{[(R+x)^2+y^2+z^2]^{3/2}} \end{bmatrix}$$

where $\mathbf{x}_1 = [x, y, z] \in \mathrm{R}^3$ and $\dot{\mathbf{x}}_1 = [\dot{x}, \dot{y}, \dot{z}] \in \mathrm{R}^3$ are a position vector and a velocity vector of the follower satellite, respectively. It is worth noting that there is one unknown parameter of satellite mass and external disturbances in the nonlinear relative dynamics of Eq. (2). In the feedback linearization control, the precise model is required and some useful nonlinearities can often be cancelled. However, the adaptive backstepping scheme offers a choice of design tools to accommodate uncertainties and nonlinearities, and has the flexibility to avoid cancellations of useful nonlinearities. So it is regarded as good candidates for design of stabilizing controllers for uncertain nonlinear systems (Manosa et al. 2005). In this study, the adaptive backstepping scheme is used for a class of strict-feedback nonlinear systems for the tracking problem.

3. ADAPTIVE BACKSTEPPING CONTROL DESIGN

In this section, an adaptive backstepping control law is designed for the tracking problem of SFF using nonlinear relative dynamics of Eq. (2). We pose two constraints on the nonlinear relative dynamics that the external disturbance is bounded, i.e. $|\mathbf{D}| \leq \bar{\mathrm{D}}$ and the unknown mass of the follower satellite is bounded, i.e. $0 < \mathrm{M}_1 \leq m \leq \mathrm{M}_2$. Note that the control $\mathbf{u}(t)$ is multiplied by an unknown mass in Eq. (2). Thus we need to construct an estimator $\hat{m}(t)$ of the real mass m.

For the tracking problem of SFF, we introduce the new auxiliary variables.

$$\begin{aligned} \mathbf{z}_1 &= \mathbf{x}_1 - \mathbf{x}_d \\ \mathbf{z}_2 &= \mathbf{x}_2 - \dot{\mathbf{x}}_d - \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_1 &= -c_1 \mathbf{z}_1 \\ \tilde{m} &= m - \hat{m} \end{aligned} \tag{3}$$

$\mathbf{z}_1 \in \mathrm{R}^3$ means a tracking error, and $\mathbf{x}_d \in \mathrm{R}^3$ represents a desired trajectory. $\boldsymbol{\alpha}_1 \in \mathrm{R}^3$ is a stabilizing function which appears in the backstepping control design, in which c_1 is a positive design parameters.

In addition $\tilde{m} \in \mathrm{R}$ is the estimated mass error of the follower satellite, and $\hat{m} \in \mathrm{R}$ will be updated through the parameter update law.

Consider the Lyapunov function candidate

$$V = \frac{a}{2}\left(\frac{\mathbf{z}_1^{\mathrm{T}}\mathbf{z}_1}{a^2}\right) + \frac{b}{2}\left(\frac{\mathbf{z}_2^{\mathrm{T}}\mathbf{z}_2}{b^2}\right) + \frac{m}{2\gamma}\left(\frac{\tilde{m}}{m}\right)^2 \tag{4}$$

where a, b and γ are positive design parameters such as c_1. The derivative of Lyapunov function is given by

$$\begin{aligned} \dot{V} &= \frac{1}{a}\mathbf{z}_1^{\mathrm{T}}\dot{\mathbf{z}}_1 + \frac{1}{b}\mathbf{z}_2^{\mathrm{T}}\dot{\mathbf{z}}_2 + \frac{1}{m\gamma}\tilde{m}\dot{\tilde{m}} \\ &= -\frac{c_1}{a}||\mathbf{z}_1||^2 + \frac{1}{m\gamma}\left(\frac{\gamma}{a}\hat{m} + \frac{\gamma}{a}\tilde{m}\right)\mathbf{z}_1^{\mathrm{T}}\mathbf{z}_2 \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{b}\mathbf{z}_2^{\mathrm{T}}\left\{f+\frac{1}{m}(\mathbf{D}+\mathbf{u})-\ddot{\mathbf{x}}_d+c_1\dot{\mathbf{x}}_1-c_1\dot{\mathbf{x}}_d\right\}-\frac{1}{m\gamma}\tilde{m}\dot{\hat{m}} \\
= \ &-\frac{c_1}{a}||\mathbf{z}_1||^2+\frac{1}{bm}\mathbf{z}_2^{\mathrm{T}}\left\{\frac{b}{a}\hat{m}\mathbf{z}_1+\hat{m}f+\mathbf{D}+\mathbf{u}-\hat{m}\ddot{\mathbf{x}}_d+\hat{m}c_1\dot{\mathbf{x}}_1-\hat{m}c_1\dot{\mathbf{x}}_d\right\} \\
&+\frac{1}{m\gamma}\tilde{m}\left(\frac{\gamma}{a}\mathbf{z}_1^{\mathrm{T}}\mathbf{z}_2+\frac{\gamma}{b}\mathbf{z}_2^{\mathrm{T}}f-\frac{\gamma}{b}\mathbf{z}_2^{\mathrm{T}}\ddot{\mathbf{x}}_d+\frac{\gamma}{b}c_1\mathbf{z}_2^{\mathrm{T}}\dot{\mathbf{x}}_1-\frac{\gamma}{b}c_1\mathbf{z}_2^{\mathrm{T}}\dot{\mathbf{x}}_d-\dot{\hat{m}}\right) \qquad (5)
\end{aligned}
$$

Thus, we choose the control law and update law so as to make the derivative of Lyapunov function non-positive. In specialty, a switching σ-modification (Ioannou & Sun 1996) can be added into the parameter update law so as to avoid the parameter drift. But, a switching σ-modification is utilized based on the assumption that the unknown parameter is bounded. For the case of unknown satellite mass, the modified switching σ-modification is added to the parameter update law because satellite mass is assumed to be positively bounded, i.e. $0 < \mathrm{M}_1 \leq m \leq \mathrm{M}_2$.

$$
\dot{\hat{m}}=\gamma\mathbf{z}_2^{\mathrm{T}}\left(\frac{1}{a}\mathbf{z}_1+\frac{1}{b}f-\frac{1}{b}\ddot{\mathbf{x}}_d+\frac{c_1}{b}\dot{\mathbf{x}}_1-\frac{c_1}{b}\dot{\mathbf{x}}_d\right)-\gamma\sigma_\eta(\hat{\eta})\hat{\eta} \qquad (6)
$$

$$
\mathbf{u}=-c_2\mathbf{z}_2-\frac{b}{a}\hat{m}\mathbf{z}_1-\hat{m}f-\bar{\mathrm{D}}Sgn(\mathbf{z}_2)+\hat{m}\ddot{\mathbf{x}}_d-\hat{m}c_1\dot{\mathbf{x}}_1+\hat{m}c_1\dot{\mathbf{x}}_d \qquad (7)
$$

In the parameter update law of Eq. (6), the modified switching σ-modification is defined as

$$
\sigma_\eta(\hat{\eta})=\begin{cases} 0 & \text{if } ||\hat{\eta}|| \leq \frac{\mathrm{M}_2-\mathrm{M}_1}{2} \\ \bar{\sigma}_m\left(\frac{2||\hat{\eta}||}{\mathrm{M}_2-\mathrm{M}_1}-1\right) & \text{if } \frac{\mathrm{M}_2-\mathrm{M}_1}{2} \leq ||\hat{\eta}|| \leq \mathrm{M}_2-\mathrm{M}_1 \\ \bar{\sigma}_m & \text{if } ||\hat{\eta}|| \geq \mathrm{M}_2-\mathrm{M}_1 \end{cases} \qquad (8)
$$

where $\hat{\eta}=\hat{m}-\frac{\mathrm{M}_1+\mathrm{M}_2}{2}$ and $\eta=m-\frac{\mathrm{M}_1+\mathrm{M}_2}{2}$
$\bar{\sigma}_m$ of Eq. (8) is a positive constant as a design parameter. The $Sgn(\psi)$ function in the control law, Eq. (7) is defined using the signum function as

$$
Sgn(\psi)=[\mathrm{sgn}(\psi_1),\mathrm{sgn}(\psi_2),...,\mathrm{sgn}(\psi_m)]^{\mathrm{T}}, \quad \forall\psi=[\psi_1,\psi_2,...,\psi_m]^{\mathrm{T}} \qquad (9)
$$

Now we will investigate the stability analysis of an adaptive backstepping controller for the the closed-loop system. The useful property of the modified switching σ-modification is introduced to prove the closed-loop system stability, which can be derived from the appendix A of Manosa et al. (2005).

$$
\sigma_\eta(\hat{\eta})\tilde{\eta}\hat{\eta} \leq -\frac{3}{4}\bar{\sigma}_m\tilde{\eta}^2+\frac{39}{4}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right) \qquad (10)
$$

By substituting Eq. (6) and (7) into (5), and using the property of Eq. (10), the derivative of the candidate Lyapunov function is rewritten by

$$
\begin{aligned}
\dot{V} \ = \ &-\frac{c_1}{a}||\mathbf{z}_1||^2-\frac{c_2}{bm}||\mathbf{z}_2||^2+\frac{1}{bm}\mathbf{z}_2^{\mathrm{T}}[\mathbf{D}-\bar{\mathrm{D}}\cdot\mathrm{sgn}(\mathbf{z}_2)]+\frac{1}{m}\sigma_m(\hat{\eta})\tilde{\eta}\hat{\eta} \\
&\leq -\frac{c_1}{a}||\mathbf{z}_1||^2-\frac{c_2}{bm}||\mathbf{z}_2||^2+\frac{1}{bm}\mathbf{z}_2^{\mathrm{T}}[\mathbf{D}-\bar{\mathrm{D}}\cdot\mathrm{sgn}(\mathbf{z}_2)]-\frac{3}{4m}\bar{\sigma}_m\tilde{m}^2 \\
&+\frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right)^2 \\
&\leq -\frac{c_1}{a}||\mathbf{z}_1||^2-\frac{c_2}{bm}||\mathbf{z}_2||^2-\frac{3}{4m}\bar{\sigma}_m\tilde{m}^2+\frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
&\leq -\min\left(2c_1, \frac{2c_2}{m}, \frac{3}{2}\gamma\bar{\sigma}_m\right)\left[\frac{a}{2}\left(\frac{\mathbf{z}_1^T\mathbf{z}_1}{a^2}\right)+\frac{b}{2}\left(\frac{\mathbf{z}_2^T\mathbf{z}_2}{b^2}\right)+\frac{m}{2\gamma}\left(\frac{\tilde{m}}{m}\right)^2\right] \\
&\quad +\frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right)^2 \\
&= -c_0V+d_0
\end{aligned}
\tag{11}
$$

where

$$c_0 = \min\left(2c_1, \frac{2c_2}{m}, \frac{3}{2}\gamma\bar{\sigma}_m\right)$$

$$d_0 = \frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M2}}{2}\right)^2$$

We can integrate the differential inequality of the Lyapunov function derivative because c_0 and d_0 are constants. So

$$V(t) \leq V(0)\exp[-c_0t]+\frac{d_0}{c_0}(1-\exp[-c_0t]) \leq V(0)\exp[-c_0t]+\frac{d_0}{c_0} \tag{12}$$

From Eq. (12), we conclude that $V(t)$ is globally uniformly ultimately bounded. This conclusion leads that $\mathbf{z}_1, \mathbf{z}_2$ and $\tilde{m}$ are bounded because the Lyapunov function candidate consists of them. So, we can derive from this result that the state variables $(\mathbf{x}_1, \mathbf{x}_2)$ and the parameter estimate $(\hat{m})$ are also bounded, and so the control input **u**(t) is bounded.

The system performance can be improved by selecting the appropriate design parameters. It can be proven by investigating the properties of the asymptotic tracking error which is defined as Eq. (13).

$$||\mathbf{z}_1||^2_{\mathrm{r\cdot m\cdot s},[\mathrm{t}_0,\infty]} = \lim_{\mathrm{T}\to\infty}\frac{1}{\mathrm{T}}\int_{\mathrm{t}_0}^{\mathrm{t}_0+\mathrm{T}}||\mathbf{z}_1||^2\,\mathrm{dt} \tag{13}$$

From the third equation of Eq. (11), we can rewrite the derivative of the Lyapunov function as Eq. (14), and derive the inequality of $||\mathbf{z}_1||^2$ as Eq. (15) by simple manipulation of Eq. (14).

$$
\begin{aligned}
\dot{V} &\leq -\frac{c_1}{a}||\mathbf{z}_1||^2-\frac{c_2}{bm}||\mathbf{z}_2||^2-\frac{3}{4m}\bar{\sigma}_m\tilde{m}^2+\frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right)^2 \\
&\leq -\frac{c_1}{a}||\mathbf{z}_1||^2+\frac{39}{4m}\bar{\sigma}_m\left(\frac{\mathrm{M}_1+\mathrm{M}_2}{2}\right)^2 \\
&= -\frac{c_1}{a}||\mathbf{z}_1||^2+d_0
\end{aligned}
\tag{14}
$$

$$||\mathbf{z}_1||^2 \leq -\left(\frac{a}{c_1}\right)\dot{V}+\left(\frac{a}{c_1}\right)d_0 \tag{15}$$

As shown in Eq. (12), the Lyapunov function is globally uniformly ultimately bounded. So, we can get the Eq. (16) by using the property of the Lyapunov function.

$$\lim_{\mathrm{T}\to\infty}\left\{\frac{V(t_0+\mathrm{T})-V(t_0)}{\mathrm{T}}\right\}=0 \tag{16}$$

Table 1. Parameters for the numerical simulation.

	Parameter	Value
Follower satellite	Semi-major axis [m]	7.2E+6
	Initial position [m]	$[100\ \ -100\ \ 100]^{\mathrm{T}}$
	Initial velocity [m/s]	$[0\ \ 0\ \ 0]^{\mathrm{T}}$
	Mass [kg]	150
	Initial estimated mass [kg]	155
Design parameters	c_1, c_2	5.0E-1, 5.0E-1
	a, b	1.0E-1, 1.0E-3
	M_1, M_2	140, 160
	$\bar{\sigma}_m$	1
	Disturbance limit ($\bar{D}$) [N]	[2.0E-4 2.0E-4 2.0E-4]

where t_0 and T represent the initial time and time variable, respectively. By substituting Eq. (15) and Eq. (16) into Eq. (13), we can obtain the asymptotic tracking error equation as Eq. (17).

$$\begin{aligned} ||\mathbf{z}_1||^2_{\mathrm{r\cdot m\cdot s},[\mathrm{t_0},\infty]} &\leq \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_{\mathrm{t_0}}^{\mathrm{t_0+T}} \left[-\left(\frac{a}{c_1}\right)\dot{V} + \left(\frac{a}{c_1}\right) d_0 \right] \mathrm{dt} \\ &= \lim_{\mathrm{T}\to\infty} \left\{ \left(\frac{a}{c_1}\right) \frac{V(t_0) - V(t_0 + \mathrm{T})}{\mathrm{T}} + \left(\frac{a}{c_1}\right) d_0 \right\} \\ &= \left(\frac{a}{c_1}\right) d_0 \end{aligned} \tag{17}$$

Eq. (17) implies that the asymptotic tracking error is within any boundary which consists of the design parameters, and it can be improved by selecting the design parameters.

4. NUMERICAL SIMULATION AND RESULTS

Robust adaptive backstepping control law and parameter update law were designed for satellite formation flying with external disturbance and mass uncertainty in the previous section. In this section, robust adaptive backstepping controller is simulated to demonstrate the performance of the proposed controller for the tracking problem of SFF. The control law drives the follower satellite to track the desired trajectory against the external bounded disturbances in the presence of mass uncertainty. All parameters used in this simulation are shown in Table 1.

The desired trajectory is described by Eq. (18) for the tracking problem, which converges to the zero due to the exponential term. In addition, the disturbance (**D**) is also necessary for the simulation, which is given by Eq. (19) and shown in Figure 2. The absolute value of each disturbance component should be not larger than the disturbance limit ($\bar{\mathrm{D}}$) in Table 1.

$$\mathbf{x}_d(t) = 100 \times \begin{bmatrix} \cos(\dot{\theta} \times t)\exp(-5.0E - 7 \times t^2) \\ -\cos(\dot{\theta} \times t)\exp(-5.0E - 7 \times t^2) \\ \cos(\dot{\theta} \times t)\exp(-5.0E - 7 \times t^2) \end{bmatrix} \tag{18}$$

$$\mathbf{D}(t) = \sin(2\pi \times \dot{\theta} \times t) \times \begin{bmatrix} 1.9106E - 4 \\ -1.9106E - 4 \\ -1.1517E - 4 \end{bmatrix} \tag{19}$$

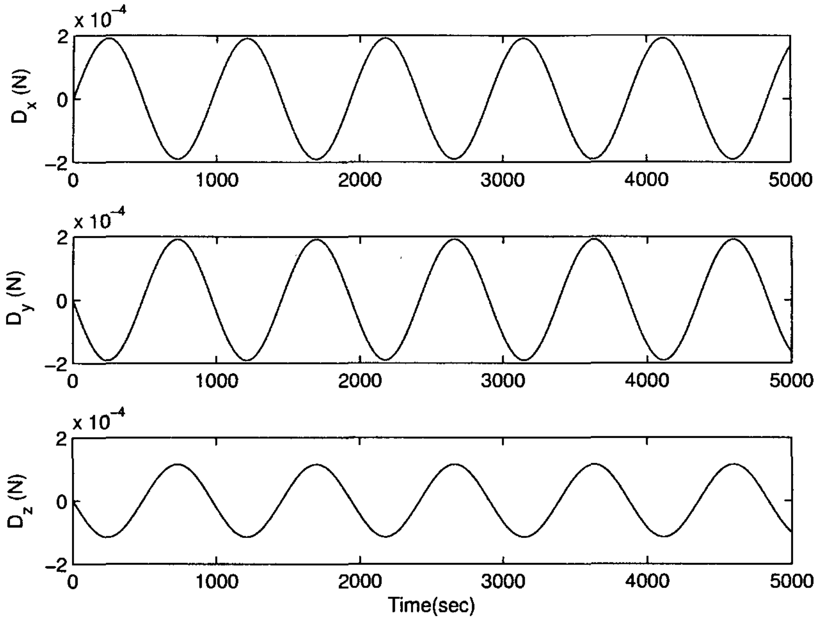

Figure 2. External Disturbance on the follower satellite.

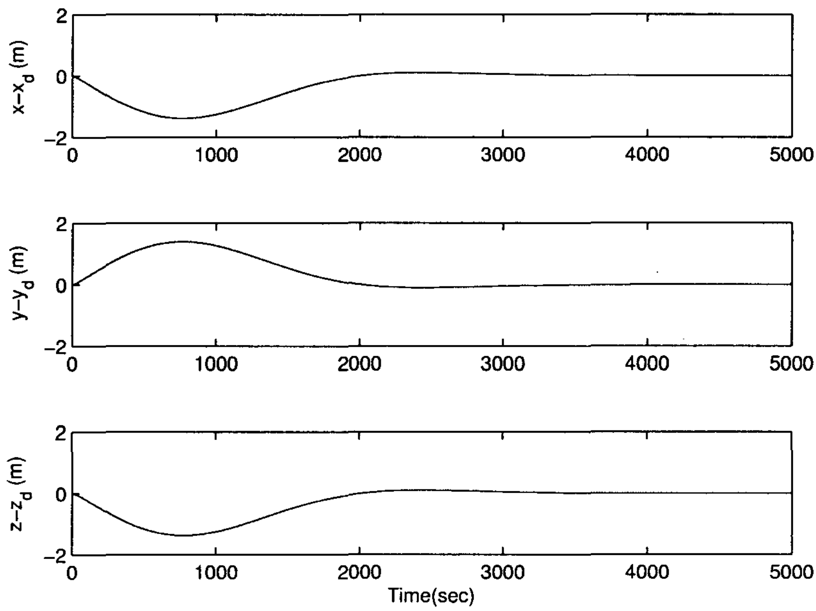

Figure 3. Tracking error of the follower satellite.

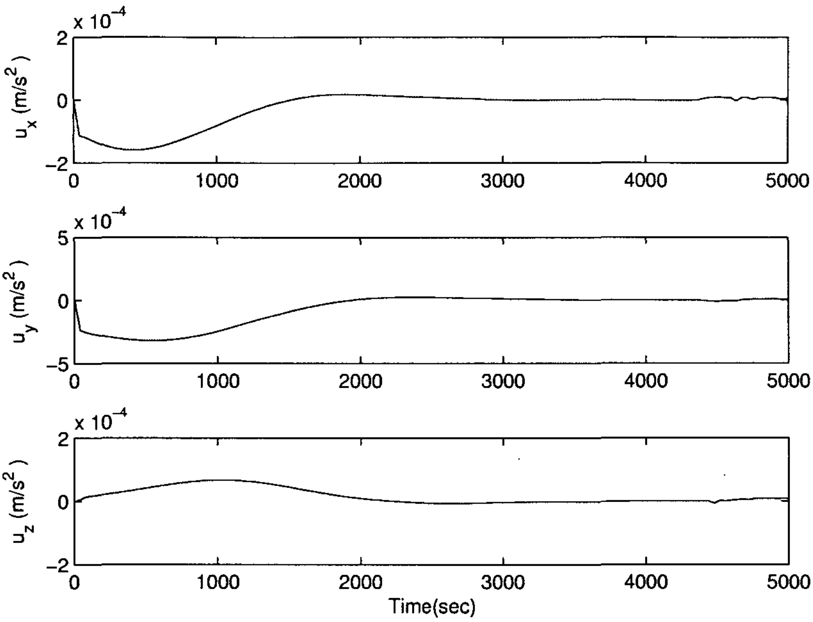

Figure 4. Control input of the follower satellite.

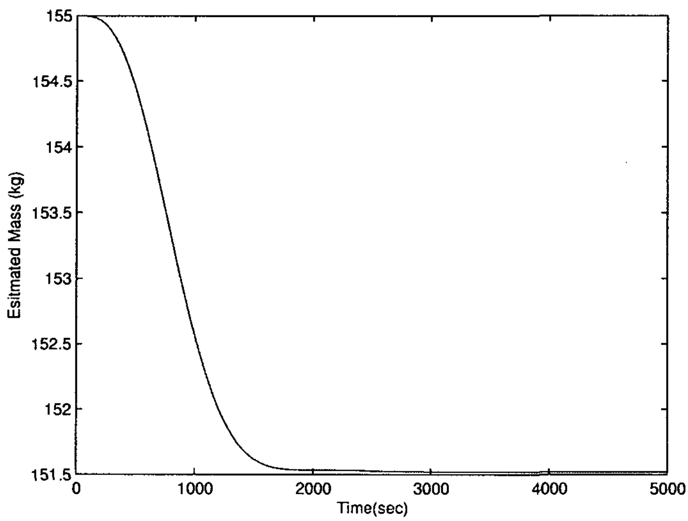

Figure 5. Estimated mass from the update law.

Figure 3, 4 and 5 show the tracking error, control input and the estimated mass of the follower satellite, respectively. Figure 3 shows that the follower satellite tracks the desired trajectory well under the external disturbance by the adaptive backstepping controller. The control inputs are kept relatively small during the whole simulation time, which can be generated by small electric thrusters. However, the estimated mass doesn't converge into the real value because the Lyapunov function is globally uniformly ultimately bounded. It means that Lyapunov function does not approach into the zero due to the characteristics of Eq. (12).

5. CONCLUSIONS

Nonlinear control law was developed based on the adaptive backstepping technique for the tracking problem of satellite formation flying which has mass uncertainty and the bounded external disturbance. The modified switching σ-modification was added into the parameter update law to avoid the parameter drift. The closed-loop system was proven to be stable through the control law and update law using Lyapunov stability theorem. The proposed controller was shown to guarantee the system stability against the external bounded disturbances in the presence of mass uncertainty. The tracking error converges into zero, but the estimated mass could not approach to the true value because the Lyapunov function candidate is globally uniformly ultimately bounded but not asymptotically stable.

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