Delay-dependent Stabilization for Systems with Multiple Unknown Time-varying Delays

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Abstract: This paper deals with the delay-dependent and rate-independent stabilization of systems with multiple unknown time-varying delays and time-varying structured uncertainties. All the linear matrix inequalities based conditions are derived by employing free-weighting matrices to express the relationships between the terms in the Leibniz-Newton formula. The criteria do not require any tuning parameters. Numerical examples demonstrate the validity of the method.

Keywords: Delay-dependent and rate-independent, linear matrix inequality, multiple delays, stabilization, unknown time-varying delays.

1. INTRODUCTION

Stability and stabilization criteria for a delay system are an important issue in control theory that has been attracting a great deal of attention over the past few decades. To reduce the conservatism of existing criteria, considerable effort has been expended on finding delay-dependent ones [1-16]. Most of these criteria are based on four model transformations of the original system [1]. Specifically, the descriptor model transformation method combined with the inequalities proposed in [2,3] is very efficient [1,4-6,8]. However, as pointed out by [9-11], it uses fixed weighting matrices to express the relationships between the terms in the Leibniz-Newton formula, which can lead to conservatism. In [9-11], He *et al.* presented a free-weighting-matrix approach, which not only solved the problem with fixed weighting matrices, but also avoided any model transformation.

In many practical time-delay systems, either the delays are time-varying or only the bounds on the delays are known. Typical time-delay systems with multiple time-varying delays include a turbojet engine, a microwave oscillator, the inferred grinding model, and models of population dynamics [12]. Most delay-dependent stability and stabilization criteria, which are for slow delays, i.e., the upper bounds on the derivatives of the delays are less than 1, are not applicable to unknown, time-varying delays. The Razumikhin approach is the main method for dealing with systems with rapidly varying or unknown delays [12-14]. Recently, Fridman and Shaked [1,4], Han [5] and Jing *et al.* [6] applied a Lyapunov-Krasovskii functional approach and a descriptor system approach to the problem of fast time-varying delays or unknown delays and derived delay-dependent and rate-independent criteria. In fact, the free-weighting-matrix approach [10,11] can also be employed to study this topic.

On the other hand, even though linear matrix inequalities (LMIs) are known to be an efficient method for solving standard convex optimization problems numerically, surprisingly few LMI-based delay-dependent stabilization criteria have been reported. A first-order model transformation can yield LMI-based conditions, but the resulting system is not equivalent to the original one [17,18]. Although the use of Park's or Moon *et al.*'s inequalities can improve the results obtained with such a transformation, the stabilization conditions are no longer LMIs. In [3], Moon *et al.* converted the nonlinear matrix inequalities to a nonlinear minimization problem subject to LMI constraints and presented an algorithm for the design of a delay-dependent state-feedback controller to stabilize the system. Lee *et al.* [8] extended this method and combined it with the descriptor model transformation to study the H_∞

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control problem, but the results were suboptimal.

In this paper, all the LMI-based stabilization conditions for systems with multiple unknown time-varying delays are derived from the delay-dependent and rate-independent stability criterion extended from [11]. And they are extended to systems with time-varying structured uncertainties. To the best of our knowledge, these are the first LMI-based delay-dependent and rate-independent stabilization results using Lyapunov-Krasovskii functional approach for systems with unknown time-varying delays. Two numerical examples are used to demonstrate that the criteria presented in this paper are effective.

2. PRELIMINARIES

Consider a linear system, Σ, with time-varying structured uncertainties and multiple unknown time-varying delays:

$$\Sigma : \begin{cases} \dot{x}(t) = \sum_{i=0}^{m} \left(A_i + \Delta A_i(t)\right) x(t - d_i(t)) \\ \qquad\qquad + \left(B + \Delta B(t)\right) u(t), \quad t > 0, \\ x(t) = \phi(t), \qquad\qquad t \in [-h,\ 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector and $u(t) \in \mathbf{R}^p$ is the control input. $A_i \in \mathbf{R}^{n\times n}$ $(i = 0,\ 1, \cdots, m)$ and $B \in \mathbf{R}^{n\times p}$ are constant matrices with appropriate dimensions. The time-varying structured uncertainties, $\Delta A_i(t) \in \mathbf{R}^{n\times n}$ $(i = 0,\ 1, \cdots, m)$ and $\Delta B(t) \in \mathbf{R}^{n\times p}$, are assumed to be of the form

$$\begin{aligned} &\left[\Delta A_0(t) \ \ \Delta A_1(t) \ \ \cdots \ \ \Delta A_m(t) \ \ \Delta B(t)\right] \\ &= DF(t)\left[E_0 \ \ E_1 \ \ \cdots \ \ E_m \ \ E_b\right], \end{aligned} \tag{2}$$

where $D \in \mathbf{R}^{n\times l}$, $E_i \in \mathbf{R}^{k\times n} (i = 0, 1, \cdots, m)$, and $E_b \in \mathbf{R}^{k\times p}$ are constant matrices; and $F(t) \in \mathbf{R}^{l\times k}$ is an unknown real, and possibly time-varying with Lebesgue-measurable elements satisfying

$$F^T(t)F(t) \le I, \quad \forall t \in [0, \infty). \tag{3}$$

The time delays, $d_0(t) = 0$ and $d_i(t)\,(i = 1, \cdots, m)$, are time-varying continuous functions that are unknown but bounded and satisfy

$$0 \le d_i(t) \le h_i, \quad i = 1, \cdots, m, \tag{4}$$

where h_i $(i = 1, \cdots, m)$ are constants and we define $h = \max_{i=1,\cdots,m} \{h_i\}$. The initial condition, $\phi(t)$, is a continuous vector-valued initial function of $t \in [-h,\ 0]$. We are interested in designing a memoryless state-feedback controller

$$u(t) = Kx(t), \tag{5}$$

where $K \in \mathbf{R}^{p\times n}$ is a constant gain matrix, to stabilize Σ.

The nominal system of Σ, Σ_0, is

$$\Sigma_0 : \begin{cases} \dot{x}(t) = \sum_{i=0}^{m} A_i x(t - d_i(t)) + Bu(t), & t > 0, \\ x(t) = \phi(t), & t \in [-h,\ 0], \end{cases} \tag{6}$$

will be discussed first.

The following lemma is needed to deal with a system with time-varying structured uncertainties.

Lemma 1 [19,20]**:** Given matrices $Q = Q^T$, H, E and $R = R^T$ with appropriate dimensions,

$$Q + HF(t)E + E^T F^T(t) H^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \le R$, if and only if there exists an $\varepsilon > 0$ such that

$$Q + \varepsilon^{-1} HH^T + \varepsilon E^T R E < 0.$$

3. STABILITY AND STABILIZATION

First, the nominal system, Σ_0, is discussed. The following Lemma is extended from [11]. It solves the problem of designing a stabilization controller for systems with multiple unknown time-varying delays.

Lemma 2: Given scalars $h_i > 0\,(i = 1, \cdots, m)$, the nominal system Σ_0 with $u(t) = 0$ and multiple unknown time-varying delays, $d_i(t) > 0\,(i = 1, \cdots, m)$, satisfying (4) is asymptotically stable if there exist $P = P^T > 0$, $Z_i = Z_i^T > 0$ $(i = 1, \cdots, m)$, and any appropriately dimensioned matrices $N_{ij}\,(i = 0, 1, \cdots, m; j = 1, \cdots, m)$, such that the following matrix inequality holds:

$$\Phi = \begin{bmatrix} H^T A + A^T H & A_h & N_h \\ A_h^T & -Z_{-h} & 0 \\ N_h^T & 0 & -Z_h \end{bmatrix} < 0, \tag{7}$$

where

$$H = \begin{bmatrix} P_0 & N_1 & N_2 & \cdots & N_m \end{bmatrix}^T,$$

$$A = \begin{bmatrix} A_0 & A_1 & A_2 & \cdots & A_m \\ I & -I & 0 & \cdots & 0 \\ I & 0 & -I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & 0 & 0 & \cdots & -I \end{bmatrix},$$

$$A_h = [h_1 A_{0m} \quad h_2 A_{0m} \quad \cdots \quad h_m A_{0m}],$$
$$N_h = [h_1 N_1 \quad h_2 N_2 \quad \cdots \quad h_m N_m],$$
$$Z_h = \mathrm{diag}\{-h_1 Z_1 \quad -h_2 Z_2 \quad \cdots \quad -h_m Z_m\},$$
$$Z_{-h} = \mathrm{diag}\{-h_1 Z_1^{-1} \quad -h_2 Z_2^{-1} \quad \cdots \quad -h_m Z_m^{-1}\},$$
$$N_j = [N_{0j}^T \quad N_{1j}^T \quad \cdots \quad N_{mj}^T]^T,$$
$$A_{0m} = [A_0 \quad A_1 \quad \cdots \quad A_m]^T,$$
$$P_0 = [P_0 \quad 0 \quad \cdots \quad 0]^T.$$

Proof: Choose a candidate Lyapunov-Krasovskii functional to be

$$V(x_t) = x^T(t)Px(t) + \sum_{i=1}^{m} \int_{-h_i}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_i \dot{x}(s) ds d\theta, \quad (8)$$

where $P = P^T > 0$ and $Z_i = Z_i^T > 0 \, (i = 1, \cdots, m)$ are to be determined. According to the Leibniz-Newton formula, for $j = 1, \cdots, m$, and for any matrices N_{ij} $(i = 1, \cdots, m)$, the following equation holds:

$$2\left[\sum_{i=0}^{m} x^T(t - d_i(t)) N_{ij}\right] \times \left[x(t) - x(t - d_j(t)) - \int_{t-d_j(t)}^{t} \dot{x}(s) ds\right] = 0. \quad (9)$$

On the other hand, for any symmetric semi-positive definite matrices with appropriate dimensions

$$X^{(j)} = \begin{bmatrix} X_{00}^{(j)} & X_{01}^{(j)} & \cdots & X_{0m}^{(j)} \\ \left[X_{01}^{(j)}\right]^T & X_{11}^{(j)} & \cdots & X_{1m}^{(j)} \\ \vdots & \vdots & \ddots & \vdots \\ \left[X_{0m}^{(j)}\right]^T & \left[X_{1m}^{(j)}\right]^T & \cdots & X_{mm}^{(j)} \end{bmatrix} \geq 0, \; j = 1, \cdots m,$$

the following holds:

$$\varsigma_1^T(t)\left(h_j X^{(j)}\right)\varsigma_1(t) - \int_{t-d_j(t)}^{t} \varsigma_1^T(t)\left(h_j X^{(j)}\right)\varsigma_1(t) ds \geq 0, \quad (10)$$

where

$$\varsigma_1^T(t) = [x^T(t) \; x^T(t - d_1(t)) \; x^T(t - d_2(t)) \cdots x^T(t - d_m(t))].$$

Calculating the derivative of $V(x_t)$ along the solutions of Σ_0 and adding (9) and (10) for $j=1, \cdots, m$, to it yields

$$\dot{V}(x_t) = \varsigma_1^T(t) \Xi \varsigma_1(t) - \sum_{j=1}^{m} \int_{t-d_j(t)}^{t} \varsigma_2^T(t,s) \Psi \varsigma_2(t,s) ds, \quad (11)$$

where

$$\varsigma_2^T(t,s) = \left[\varsigma_1^T(t) \quad \dot{x}^T(s)\right],$$

$$\Xi = HA + A^T H^T + A_{0m}\left[\sum_{j=1}^{m} h_j Z_j\right] A_{0m}^T + \sum_{j=1}^{m} h_j X^{(j)},$$

$$\Psi_j = \begin{bmatrix} X^{(j)} & N_j \\ N_j^T & Z_j \end{bmatrix}.$$

If $\Xi < 0$ and $\Psi_j \geq 0 \, (j = 1, \cdots, m)$, then $\dot{V}(x_t) < -\varepsilon \|x(t)\|^2$ for a sufficiently small positive number $\varepsilon > 0$, which ensures the asymptotic stability of Σ_0 [21]. Set

$$X^{(j)} = N_j Z_j^{-1} N_j^T, \; j = 1, \cdots, m, \quad (12)$$

which implies that $\Psi_j \geq 0 \; (j = 1, \cdots, m)$. Replacing $X^{(j)}$ in Ξ with (12) and applying the Schur complement ([22]) show that $\Phi < 0$ implies $\Xi < 0$. So, Σ_0 is asymptotically stable if (7) holds. □

Remark 1: The conditions in (7) are not LMIs due to the term Z_{-h}. However, they can be expressed as LMIs by the same method used in [11]. The form of (7) is aimed at using LMIs to find a state-feedback controller that stabilizes the system.

Based on Lemma 2, we can derive a control law, (5), to stabilize the nominal system Σ_0 as follow:

Theorem 1: Given scalars $h_i > 0 \; (i = 1, \cdots, m)$, the control law (5) stabilizes the nominal system Σ_0 with multiple unknown time-varying delays, $d_i(t)$ $i=1, \cdots, m$, satisfying (4) if there exist $L = L^T > 0$, R_j $R_j^T > 0 \; (j = 1, \cdots, m)$, and any appropriately dimensioned matrices S_{ij} $(i = 0, 1 \cdots, m; \; j = 1, \cdots, m)$ and U such that the following LMI holds:

$$\Pi = \begin{bmatrix} \Pi_{00} & \Pi_{01} & \cdots & \Pi_{0m} & h_1 G_0 & h_2 G_0 & \cdots & h_m G_0 \\ \Pi_{01}^T & \Pi_{11} + h_1 R_1 & \cdots & \Pi_{1m} & h_1 G_1 & h_2 G_1 & \cdots & h_m G_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_{0m}^T & \Pi_{1m}^T & \vdots & \Pi_{mm} + h_m R_m & h_1 G_m & h_2 G_m & \cdots & h_m G_m \\ h_1 G_0^T & h_1 G_1^T & \cdots & h_1 G_m^T & -h_1 R_1 & 0 & \cdots & 0 \\ h_2 G_0^T & h_2 G_1^T & \cdots & h_2 G_m^T & 0 & -h_2 R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_m G_0^T & h_m G_1^T & \cdots & h_m G_m^T & 0 & 0 & \cdots & -h_m R_m \end{bmatrix} < 0, \quad (13)$$

where

$$\Pi_{00} = A_0L + LA_0^T + BU + U^TB^T + \sum_{j=1}^{m}(A_jS_{0j} + S_{0j}^TA_j^T),$$

$$\Pi_{0k} = \sum_{j=1}^{m} A_jS_{kj} + L - S_{0k}, \quad k = 1,\cdots,m,$$

$$\Pi_{ik} = -S_{ki} - S_{ik}^T, \quad i = 1,\cdots,m;\ i \le k \le m,$$

$$G_0 = LA_0^T + U^TB^T + \sum_{j=1}^{m} S_{0j}^TA_j^T,$$

$$G_i = \sum_{j=1}^{m} S_{ij}^TA_j^T, \quad i = 1,\cdots,m.$$

Moreover, a stabilizing controller is given by $u(t) = UL^{-1}x(t)$.

Proof: (7) implies that $N_{ij}(i=1,\cdots,m)$ are negative-definite. Thus $N_{ij}(i=1,\cdots,m)$ are nonsingular. So, H is nonsingular and

$$\bar{H} := H^{-1} = \begin{bmatrix} L & 0 & \cdots & 0 \\ S_{01} & S_{11} & \cdots & S_{m1} \\ \vdots & \vdots & \ddots & \vdots \\ S_{0m} & S_{1m} & \cdots & S_{m1} \end{bmatrix}. \quad (14)$$

Pre- and post-multiply Φ in (7) by diag$\{\bar{H}^T, I_m, I_m\}$ and diag$\{\bar{H}, I_m, I_m\}$, respectively, where I_m is an $mn \times mn$ identity matrix. Then, (7) can be written to

$$\begin{bmatrix} A\bar{H} + \bar{H}^TA^T & \bar{H}^TA_h & \bar{H}^TN_h \\ A_h^T\bar{H} & -Z_{-h} & 0 \\ N_h^T\bar{H} & 0 & -Z_h \end{bmatrix} < 0, \quad (15)$$

where A_0 in A and A_h is replaced with A_0+BK. Introducing the following changes to the variables

$$R_j := Z_j^{-1}(j = 1,\cdots,m), \quad U := KL$$

yields

$$A\bar{H} + \bar{H}^TA^T = \begin{bmatrix} \Pi_{00} & \Pi_{01} & \cdots & \Pi_{0m} \\ \Pi_{01}^T & \Pi_{11} & \cdots & \Pi_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ \Pi_{0m}^T & \Pi_{1m}^T & \cdots & \Pi_{mm} \end{bmatrix}, \quad (16)$$

and

$$\bar{H}^TA_h = \begin{bmatrix} h_1G_0 & h_2G_0 & \cdots & h_mG_0 \\ h_1G_1 & h_2G_1 & \cdots & h_mG_1 \\ \vdots & \vdots & \ddots & \vdots \\ h_1G_m & h_2G_m & \cdots & h_mG_m \end{bmatrix}. \quad (17)$$

Since

$$I = \bar{H}^T \cdot H^T = \bar{H}^T \cdot \begin{bmatrix} P_0 & N_1 & N_2 & \cdots & N_m \end{bmatrix}, \quad (18)$$

then

$$\bar{H}^T \cdot N_h = \bar{H}^T \cdot \begin{bmatrix} h_1N_1 & h_2N_2 & \cdots & h_mN_m \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ h_1I & 0 & \cdots & 0 \\ 0 & h_2I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_mI \end{bmatrix}.$$

Taking the Schur complement of $\bar{H}^TN_h$ and Z_h, we obtain (13). □

Remark 2: For given scalars $h_i > 0\,(i = 1,\cdots,m)$ condition (13) is an LMI. Since they do not depend on the derivative of the delays, $\dot{d}_j(t)$ $(j = 1,\cdots,m)$, but depend on the upper bounds on $d_j(t)$ $(j = 1,\cdots,m)$, these criteria are delay-dependent and rate-independent.

Theorem 1 is now extended to a system with time-varying structured uncertainties.

Theorem 2: Given scalars $h_i > 0$ $(i = 1,\cdots,m)$, the control law (5) stabilizes system Σ with time-varying structured uncertainties and multiple unknown time-varying delays, $d_i(t)$ $(i = 1,2,\cdots,m)$ satisfying (4) if there exist $L = L^T > 0$, $R_j = R_j^T > 0$ $(j = 1,\cdots,m)$ and any appropriately dimensioned matrices S_{ij} $(i = 0,1,\cdots,m; j = 1,\cdots,m)$, U, and a scalar $\lambda > 0$ such that the following LMI holds:

$$\begin{bmatrix} \bar{\Pi}_{00} & \Pi_{01} & \cdots & \Pi_{0m} & h_1\bar{G}_0 & h_2\bar{G}_0 & \cdots & h_m\bar{G}_0 & E^T \\ \Pi_{01}^T & \Pi_{11} & \cdots & \Pi_{1m} & h_1G_1 & h_2G_1 & \cdots & h_mG_1 & E_{a1}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \Pi_{0m}^T & \Pi_{1m}^T & \cdots & \Pi_{mm} & h_1G_m & h_2G_m & \cdots & h_mG_m & E_{am}^T \\ h_1\bar{G}_0^T & h_1G_1^T & \cdots & h_1G_m^T & \Theta_{11} & \Theta_{12} & \cdots & \Theta_{1m} & 0 \\ h_2\bar{G}_0^T & h_2G_1^T & \cdots & h_2G_m^T & \Theta_{12}^T & \Theta_{22} & \cdots & \Theta_{2m} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h_m\bar{G}_0^T & h_mG_1^T & \cdots & h_mG_m^T & \Theta_{1m}^T & \Theta_{2m}^T & \cdots & \Theta_{mm} & 0 \\ E & E_{a1} & \cdots & E_{am} & 0 & 0 & \cdots & 0 & -\lambda I \end{bmatrix} < 0, \quad (19)$$

where

$$\bar{\Pi}_{00} = \Pi_{00} + \lambda DD^T,$$

$$\bar{G}_0 = G_0 + \lambda DD^T,$$

$$\Theta_{ik} = \lambda h_ih_kDD^T, i = 1,\cdots,m; i < k \le m,$$

$$\Theta_{ii} = -h_i R_i + \lambda h_i^2 DD^T, i = 1,\cdots,m,$$

$$E_{ai} = \sum_{j=1}^{m} E_j S_{ij}, i = 0,1,\cdots,m,$$

$$E = E_0 L + E_{a0} + E_b U.$$

Moreover, a stabilizing controller is given by $u(t) = UL^{-1}x(t)$.

Proof: Replacing $A_j (j = 0,1,\cdots,m)$ and B in (13) with $A_j + DF(t)E_j (j = 0,1,\cdots,m)$ and $B + DF(t)E_b$ respectively, we find that (13) for Σ is equivalent to the following condition:

$$\Pi + \Gamma_d^T F(t)\Gamma_e + \Gamma_e^T F^T(t)\Gamma_d < 0,$$

where

$$\Gamma_e = \begin{bmatrix} E & E_{a1} & E_{a2} & \cdots & E_{am} & 0 & \cdots & 0 \end{bmatrix},$$

$$\Gamma_d = \begin{bmatrix} D^T & 0 & \cdots & 0 & h_1 D^T & h_2 D^T & \cdots & h_m D^T \end{bmatrix}.$$

By Lemma 1, a sufficient condition guaranteeing (13) for Σ is that there exists a positive number $\lambda > 0$ such that

$$\Pi + \lambda \Gamma_d^T \Gamma_d + \lambda^{-1}\Gamma_e^T \Gamma_e < 0. \tag{20}$$

Applying the Schur complement shows that (20) is equivalent to (19). □

Remark 3: There is an error in [4]. In that paper, the delay-dependent and rate-independent conditions were given as LMIs in Theorem 2 for $\bar{\varepsilon}_i = I$ $(i = 1,2)$. In (28a) in the theorem, blocks of the fourth, fifth, sixth, and seventh rows and columns were deleted. That is, both $\bar{S}_1$ and $\bar{S}_2$ must be zero. However, (28a) derived from (17), in which $L = E_0$, $L_1 = E_1$, $L_2 = E_2$ by pre- and post- multiplying (17) by Δ^T and Δ ($\Delta = \text{diag}\{Q, I, \bar{S}_1, \bar{S}_2, I\}$ but not by $\text{diag}\{Q, I\}$ and $\bar{S}_1 = S_1^{-1}, \bar{S}_2 = S_2^{-1}$). Since $\bar{S}_1$ and $\bar{S}_2$ must be zero, this treatment is equivalent to making all the blocks of the third and fourth rows and columns in (17) zero and then deleting them, which is clearly not correct. On the other hand, if either L_1 or L_2 in (17) is non-zero, the corresponding bounded-real-lemma representation of (17) cannot be extended to make it delay-dependent and rate-independent. In fact, it is clearly not correct that E_1 and E_2 are not contained in (28) if there are uncertainties in A_1 and A_2. So, the delay-dependent and rate-independent conditions in [4] are not valid for a delay including an uncertainty. Note that all the equation numbers mentioned above are those in [4].

4. EXAMPLES

In this section, two examples are used to demonstrate the effectiveness of the proposed method.

Example 1: Consider the uncertain system Σ with $m = 1$ and

$$A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$D = I,\ E_0 = 0.2I,\ E_1 = \alpha I,\ E_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where α was 0.2 in [3] and 0 in [4].

In [3], Moon *et al.* considered this system with a constant delay, and the upper bound on h that stabilized the system was found to be 0.45. For a system containing a fast time-varying delay, the upper bound on h for which the system is stabilized by a state-feedback controller was found to be 0.496 in [4] and 0.489 in [1] for $\alpha = 0$. We obtained a value of 0.496 for $\alpha = 0$ by solving LMI (19) in Theorem 2. Although this theorem yields the same upper bound on h as [4], the conditions in the theorem are LMIs, and no adjustment of parameters is needed. So, from a computational viewpoint, the method in this paper is superior to existing ones. In addition, when $\alpha = 0.2$, the upper bound on h for which the system is stabilized by a state-feedback controller was found to be 0.451. However, the method of [4] cannot handle this case.

Example 2: Consider the uncertain system Σ with $m = 2$ and

$$A_0 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 0.6 & -0.4 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ D = I,\ E_0 = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.16 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.04 \end{bmatrix}, E_2 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0 \end{bmatrix}, E_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

This example is similar to Example 2 in [12]. For $h = \max_{i=1,2}\{h_i\}$, the system can be stabilized if $h \le 0.1945$ in [12]. However, solving LMI (19), it was found that the system can be stabilized by (5) if $h \le 1.3303$ and $K = \begin{bmatrix} -475.1 & -1140 \end{bmatrix}$. In addition, the upper bounds on h_2 are listed in Table 1 for various h_1. Note that the method of [4] cannot handle this case.

Table 1. Upper bounder on h_2 for various h_1.

h_1	0.1	0.5	1.0
h_2	1.94	1.85	1.68
K	[−0.720 −1.866]	[−1.665 −2.323]	[−50.79 −64.70]
h_1	1.2	1.3303	1.4
h_2	1.52	1.3303	1.13
K	[−10.28 −19.62]	[−475.1 −1140]	[−30.40 −84.13]
h_1	1.5	1.6	--
h_2	0.73	0.22	--
K	[−146.6 −246.4]	[−174.9 −113.2]	--

5. CONCLUSION

In this paper, free-weighting matrices were employed to express the relationships between the terms in the Leibniz-Newton formula; and LMI-based delay-dependent and rate-independent stabilization conditions were presented for systems with unknown time-varying delays and time-varying structured uncertainties. The advantages of these conditions are that they are entirely LMI-based and no tuning parameters are needed. Numerical examples demonstrated the validity of the conditions.

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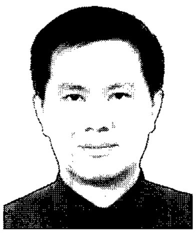

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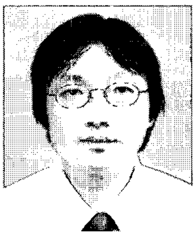

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