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Effect of Epoxy Cracking on Initial Quench Behavior about High Field Superconducting Magnet

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Abstract-- **The study to be presented related on initial behavior of quench concerned with many considerations, such as epoxy impregnated coil, critical current density related on strain and temperature, winding effect and behavior of internal superconducting wire. Especially, the deformation behavior of coils under magnetic field and thermal contractions at cryogenic temperatures to be dealt with the analytical method related on Fracture Mechanics. From the results, we know that the strain by self weight contribute to epoxy cracking at the edge of deformed coils and the deformation behavior relate on epoxy cracking must be dealt with biaxial loading problem. Then, the epoxy crack on rθ-plane under biaxial loading have been propagated with inclined crack angle and joined superconducting wire. Also, we can explain transfer of epoxy crack propagation energy from epoxy resin to superconducting wire.**

1. INTRODUCTION

The high field superconducting magnets used in applications such as biology, chemistry, high energy physics, medicine, and material science are susceptible to premature quench due to increasing strain energy [1-4]. These magnets are fabricated with closely compacted winding and impregnated with epoxy resin. When these magnets are energized after thermal contraction, the premature quench is caused by epoxy failures due to increasing magnetic force and by transfer of crack propagation energy due to epoxy cracking.

Especially, the axes of magnets for Fourier Transform Ion Cyclotron Resonance (FT-ICR) Mass Spectrometer and Magnetic Resonance Imaging (MRI) are oriented horizontally [5]. Thus, the quench due to friction on the end flanges by self weight and uncontrolled levitation is expected additionally [6]. Therefore, not only increasing hoop stress on midplane but also increasing shear stress on edge must be considered. When the winding boundaries are restrained by the coil form, transverse shear stresses which appear as principal tensile stresses lead to epoxy fracture [1-4, 7].

In the past, several researches related on heat generation in coil and fracture induced premature quench were tried to explain initial quench behavior [1-4, 7-12]. Experimental results had observed quench behavior [3-4, 8]. and analytical results had observed internal behavior in coils by using ANSYS [7, 9-11]. The theoretical approach based on fracture mechanics had introduced fracture toughness as uniaxial loading problem [2]. But these results had mentioned theoretical explanation indistinctly.

In this paper, we have dealt with biaxial loading problem in order to explain epoxy cracking phenomena obviously and to verify that increasing strain energy due to epoxy cracking contributes to initial quench behavior.

2. THEORETICAL APPROACH OF EPOXY CRACKING

As mentioned above, we present an analytical solution about epoxy cracking of the winding composite under biaxial loading. The crack propagation problem of epoxy impregnated coil in infinitesimal zone can be simplified deform behavior problem in orthotropic material on 2D plane by using maximum strain theory [13-18].

2.1. Fundamental equations in anisotropic solids

The plate of homogeneous rectilinearly anisotropic material whose principal axes of material symmetry coincides with the x and y directions are considered. From stress(σ_{ij})-strain(ε_{ij}) relationship, the governing equations for the two-dimensional case can be written in terms of compliance coefficients as

$$\begin{aligned}\varepsilon_{xx} &= a_{11}\sigma_{xx} + a_{12}\sigma_{yy} + a_{16}\tau_{xy}\\ \varepsilon_{yy} &= a_{12}\sigma_{xx} + a_{22}\sigma_{yy} + a_{26}\tau_{xy}\\ \gamma_{xy} &= a_{16}\sigma_{xx} + a_{26}\sigma_{yy} + a_{66}\tau_{xy}\end{aligned} \tag{1}$$

where a_{ij} (i,j=1,2,6) are the compliance coefficients.

The equation of stress equilibrium and strain compatibility can be represented in terms of Airy's stress function, $U(x,y)$ as [13-15].

$$\begin{aligned}a_{22}\frac{\partial^4 U}{\partial x^4} - 2a_{26}\frac{\partial^4 U}{\partial x^3 \partial y} + (2a_{12} + a_{66})\frac{\partial^4 U}{\partial x^2 \partial y^2}\\ - 2a_{16}\frac{\partial^4 U}{\partial x \partial y^3} + a_{11}\frac{\partial^4 U}{\partial y^4} = 0\end{aligned} \tag{2}$$

Assuming $U(x,y)=e^{x+sy}$, the general expression of (2) in plane elasticity problem can be written in real part terms of complex variables as [13-14].

$$U(x,y) = 2\,\mathrm{Re}[U_1(z_1) + U_2(z_2)] \tag{3}$$

where $U_1(z_1)$ and $U_2(z_2)$ are Airy's stress function of complex variables $z_1=x+s_1y$ and $z_2=x+s_2y$, and s_1 and s_2 are complex roots of the following characteristic equation and functions of material properties

$$a_{11}s^4 - 2a_{16}s^3 + (2a_{12} + a_{66})s^2 - 2a_{26}s + a_{22} = 0 \tag{4}$$

In orthotropic solids of elastic symmetry ($a_{16}=a_{26}=0$), the equation (4) can be simplified as

$$a_{11}s^4 + (2a_{12} + a_{66})s^2 + a_{22} = 0 \tag{5}$$

Then, if $s_1 \neq s_2$, the complex roots is given as

$$s_1 = \sqrt{\frac{\alpha_0 - \beta_0}{2}} + i\sqrt{\frac{\alpha_0 + \beta_0}{2}}$$
$$s_2 = -\sqrt{\frac{\alpha_0 - \beta_0}{2}} + i\sqrt{\frac{\alpha_0 + \beta_0}{2}} \tag{6}$$

If $\alpha_0 > \beta_0$, α_0 and β_0 are represented as functions of material properties

$$\alpha_0 = \sqrt{\frac{a_{22}}{a_{11}}} = \sqrt{\frac{E_{11}}{E_{22}}}$$
$$\beta_0 = \frac{1}{a_{11}}\left(\frac{a_{66}}{2} + a_{12}\right) = \frac{E_{11}}{2G_{12}} - \nu_{12} \tag{7}$$

where E_{11} and E_{22} are elastic modulus, G_{12} is shear modulus, and ν_{12} is Poission's ratio of superconducting coil.

2.2. Stress on crack tip

From (3), evaluation of the complex potential functions near the crack tip can be expressed by using stress and displacement components

$$\sigma_{xx} = 2\,\mathrm{Re}\left[s_1^2\frac{d^2U_1(z_1)}{dz_1^2} + s_2^2\frac{d^2U_2(z_2)}{dz_2^2}\right]$$
$$\sigma_{yy} = 2\,\mathrm{Re}\left[\frac{d^2U_1(z_1)}{dz_1^2} + \frac{d^2U_2(z_2)}{dz_2^2}\right] \tag{8}$$
$$\tau_{xy} = -2\,\mathrm{Re}\left[s_1\frac{d^2U_1(z_1)}{dz_1^2} + s_2\frac{d^2U_2(z_2)}{dz_2^2}\right]$$

As shown in Fig. 1, the inclined crack angle ϕ may be considered by use of polar coordinate ζ_j originating at the crack tip.

$$z_i - a = \zeta_i = r(\cos\phi + s_i\sin\phi) \tag{9}$$

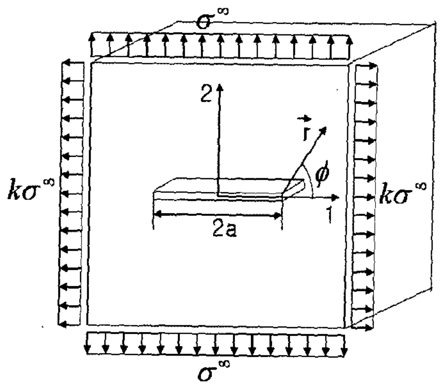

Fig. 1. The inclined crack geometry under biaxial loading.

Thus, related to fracture toughness $K_I = \sigma^\infty\sqrt{\pi a}$, the crack tip stresses including the nonsingular term can be expressed as

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{s_1s_2\sqrt{r}}{(s_1 - s_2)}\left(\frac{s_2}{\sqrt{\zeta_2}} - \frac{s_1}{\sqrt{\zeta_1}}\right)\right] + \sigma^\infty\,\mathrm{Re}[s_1s_2 + k]$$
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{\sqrt{r}}{(s_1 - s_2)}\left(\frac{s_1}{\sqrt{\zeta_2}} - \frac{s_2}{\sqrt{\zeta_1}}\right)\right] \tag{10}$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{s_1s_2\sqrt{r}}{(s_1 - s_2)}\left(\frac{1}{\sqrt{\zeta_2}} - \frac{1}{\sqrt{\zeta_1}}\right)\right]$$

The maximum circumferential tensile stress on crack tip can be calculated from

$$\sigma_{\phi\phi} = \sigma_{xx}\sin^2\phi + \sigma_{yy}\cos^2\phi - 2\tau_{xy}\sin\phi\cos\phi \tag{11}$$

Substituting the stress components of (10) into (11), the dimensionless maximum stress on the crack tip including nonsingular term can be expressed as

$$\frac{\sigma_{\phi\phi}}{\sigma^\infty} = \mathrm{Re}\left[\frac{s_1(\cos\phi + s_2\sin\phi)^{\frac{3}{2}} - s_2(\cos\phi + s_1\sin\phi)^{\frac{3}{2}}}{(s_1 - s_2)}\right] \times\sqrt{\frac{a}{2r}} + \mathrm{Re}[s_1s_2 + k]\sin^2\phi \tag{12}$$

where σ^∞ are minor stress related on radial direction, k are ratio between main stress related on tangential direction and minor stress, a/r is ratio between half crack length and crack propagation length.

3. EQUIVALENT ELASTIC MODULUS OF NB_3SN COIL

The superconducting coil is composed of superconducting wire and epoxy resin and is considered as orthotropic composite [19-20]. From relation between force and strain, equivalent elastic modulus is estimated by using elastic modulus, area, thickness of composite components.

The tangential elastic modulus related on longitudinal deformation of superconducting wire can be expressed by using superconducting wire area A_{sc} and epoxy area A_{epoxy}

$$E_{\tan} = \frac{E_{SC} A_{SC} + E_{epoxy} A_{epoxy}}{A_{SC} + A_{epoxy}} \tag{13}$$

where E_{sc} and E_{epoxy} are elastic modulus of superconducting wire and epoxy resin respectively.

The radial and axial elastic modulus can be written by using wire thickness t_{sc} and epoxy thickness t_{epoxy}

$$E_{rad}, E_{axial} = \frac{E_{SC} E_{epoxy} (t_{SC} + t_{epoxy})}{t_{epoxy} E_{SC} + t_{SC} E_{epoxy}} \tag{14}$$

From (13) and (14), we estimated elastic modulus of Nb_3Sn superconducting coil as shown in Table 1 because Nb_3Sn wire was known as strain sensitive material.

TABLE I
ESTIMATED EQUIVALENT ELASTIC MODULUS OF NB_3SN COILS.

Superconducting Wire Type	Tangential Elastic Modulus [GPa]	Axial Elastic Modulus [GPa]	Radial Elastic Modulus [GPa]
Round Wire	125.75	22.66	22.66
Rectangular Wire (Aspect Ratio 2:1)	142.24	55.70	33.50
Rectangular Wire (Aspect Ratio 1.5:1)	139.88	45.62	33.50

Elastic modulus of epoxy resin : 8 GPa
Elastic modulus of Nb_3Sn wire : 165 GPa

4. RESULTS AND DISCUSSION

In the epoxy resin, the crack begins to propagate when strain energy density approaches or exceeds its critical value. When new free surfaces are formed and the relative opening displacement between two points on the opposite sides of free crack surfaces must be greater than that which is allowed in continuum mechanics, we called crack propagation. But we can not adopt continuum hypothesis and many researchers are going to solve this problem [13-18]. The circumferential stress criteria and the circumferential strain criteria are similar in form. Both are given in single-valued ratio. Therefore, we have progressed qualitative analysis by using the dimensionless circumferential stress as shown in Fig. 2.

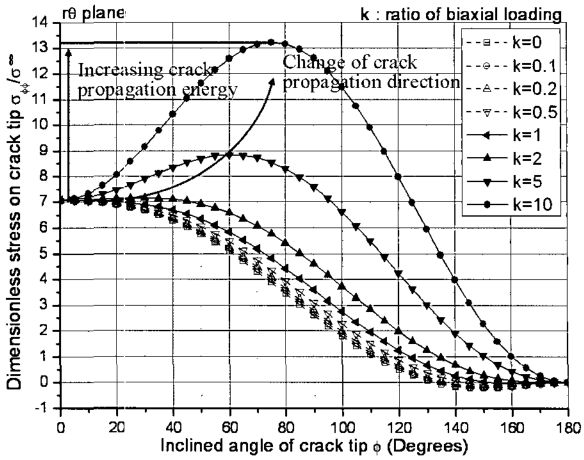

Fig. 2. Variation of dimensionless stress on crack tip with k for round wire coil.

In the present study, r/a was taken as 0.01 [14, 17-18]. and β_0 was fixed as 1 [14] in order to obtain information about effects of Young's modulus in superconducting coil.

As shown in Fig. 2, the maximum dimensionless stresses on crack tip can be seen to occur for nonzero value of inclined angle ϕ and to be changed crack propagation direction because the initial angle of crack growth corresponds to the direction where the stress on crack tip has a maximum value, which increases as stress ratio k increases. Also, the tangential stress σ_θ is greater than the radial stress σ_r and the crack propagations on $r\theta$ plane by biaxial loading have joined longitudinal deformation of superconducting wires. Because the crack propagated direction on $r\theta$ plane by uniaxial loading have paralleled longitudinal deformation of superconducting wires, we could not explain transfer of epoxy crack propagation energy in the previous study. Therefore, we can guess that stresses by epoxy cracking are contributed to increasing shear stress on superconducting wire and to generating quench in coil.

When stress ratio k is 10, the dimensionless stress by biaxial loading is estimated 1.86 times greater than the stress by uniaxial loading. Thus, the frictional force on edge of coil by self weight may be contributed to increasing circumferential stress on crack tip and deformation at the tip of inclined crack with frictional surfaces [16].

In the case of rectangular wire, the maximum dimensionless stress on crack tip is generated on $r\theta$ plane. Also, the value of maximum stress is depended on size of elastic modulus ratio in superconducting coil.

As shown in Fig. 3, for observing the size effect of elastic modulus ratio, we have compared maximum dimensionless stress on crack tip with various wire type. In those results, we can find that the maximum dimensionless stress on crack tip are generated at round wire coil and rectangular wire coils have an advantage in epoxy cracking. Also, we found information about effects of Young's modulus in superconducting coil.

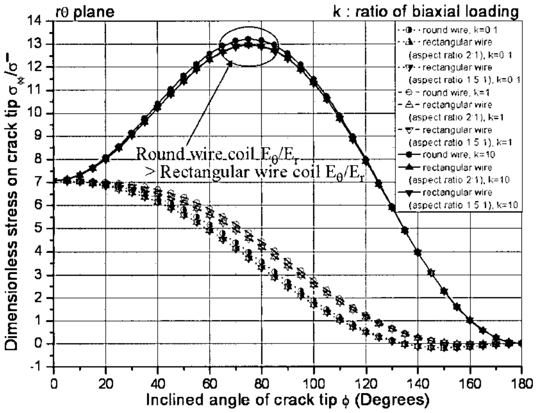

Fig. 3. Comparison of dimensionless stress on crack tip with k for various wire type.

When the magnitude of elastic modulus ratio is 1, the characteristic of orthotropic material at superconducting coil is eliminated and propagation of inclined crack is slowed. Therefore, we recommend small elastic modulus ratio value for high field superconducting magnets.

5. CONCLUSION

In this paper, we have studied the inclined crack problem under biaxial loading in epoxy resin and the transfer of crack propagation energy from epoxy to superconducting wire. Thus, we can explain quench phenomena related on epoxy cracking within superconducting coils and find meaning of elastic modulus ratio of superconducting coil as orthotropic material. From those results, we have obtained useful information related on development of new technology about epoxy impregnation. But, a detailed experimental study is needed to develop new technology for epoxy impregnation.

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