

# 웨이블릿 신경망을 이용한 한발지지상태에서의 5 링크 이족 로봇의 하이브리드 슬라이딩 모드 제어

## Hybrid Sliding Mode Control of 5-link Biped Robot in Single Support Phase Using a Wavelet Neural Network

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**Abstract :** Generally, biped walking is difficult to control because a biped robot is a nonlinear system with various uncertainties. In this paper, we propose a hybrid sliding-mode control method using a WNN uncertainty observer for stable walking of the 5-link biped robot with model uncertainties and the external disturbance. In our control system, the sliding mode control is used as main controller for the stable walking and a wavelet neural network (WNN) is used as an uncertainty observer to estimate uncertainties of a biped robot model, and the error compensator is designed to compensate the reconstruction error of the WNN. The weights of WNN are trained by adaptation laws that are induced from the Lyapunov stability theorem. Finally, the effectiveness of the proposed control system is verified through computer simulations.

**Keywords :** hybrid adaptive control, wavelet neural network, biped robot, sliding mode control

### I. Introduction

Since a biped robot has become more anthropomorphic and performs various tasks on behalf of human, the research on biped robots gradually has attracted much attention and is progressing dynamically. However, the control of biped robots is difficult due to their nonlinear and coupled dynamics. First, inverted pendulum concept is applied to interpret some characteristics of human walking [1]. Later, researchers construct a 3-link biped robot model [2], and a 5-link biped robot model [3,4].

In controlling biped robots, we face some problems such as instability of locomotion, high-order dynamic equation, existence of different phases of the walking cycle and various uncertainties. Due to these constraints, a biped robot requires a robust control technique having higher performance compared with standard PD control. So, a computed torque or inverse dynamics technique using feedback linearization [5,6] is proposed to control a biped robot. However, such methods are difficult to control a biped robot model with the model uncertainties. Therefore, the sliding mode technique is proposed for the robust control of a biped robot with uncertainties [7]. However, the sliding mode control (SMC) requires a priori knowledge of the exact mathematical model and uncertainty bounds.

On the other hand, recently, wavelet neural networks (WNNs), which combine the learning of neural network [8,9] and the advantages of the wavelet decomposition [10], are proposed and used as a good estimation tool for the identification and control of dynamic system [11]. Also, the gradient-descent (GD) method is used as conventional on-line training technique. However, the GD

method is difficult to acquire sensitivity information about unknown or highly nonlinear dynamics and has the problem, which settles to the local minimum. So, training methodology, which is induced by Lyapunov stability theorem [12], has been researched to ensure the stability, robustness, and performance of system.

Accordingly, we propose a hybrid sliding-mode control method using a WNN uncertainty observer for stable walking of the 5-link biped robot with model uncertainties and the external disturbance. The proposed hybrid control system consists of the SMC and the error compensation controller [14]. In our control system, a WNN is utilized to estimate both the internal uncertainties and the external disturbance. The compensation controller is designed to compensate the approximation error of a WNN. The adaptation laws for the weights of a WNN and those for the compensation controller are induced from the Lyapunov stability theorem, which are used to guarantee the stability of control system. Finally, in order to verify the effectiveness and robustness of the proposed control technique, the tracking performance of the hybrid SMC is compared with that of the computed torque control (CTC).

### II. The 5-link biped robotic model

#### 1. Kinematic model

The 5-link biped robot model used in this paper is shown in Fig. 1. Each link is connected by a rotating joint, which is driven by an independent DC motor. The motion of the biped robot is assumed to be constrained within the sagittal plane. The parameters for the 5-link biped robot shown in Fig. 1 are as follows [7]:

$m_i$  : Mass of link  $i$ ,

$l_i$  : Length of link  $i$ ,

$d_i$  : Distance between the mass center of link  $i$  and its lower joint,

$I_i$  : Moment of inertia with respect to an axis passing through the mass center of link  $i$  and being perpendicular to the motion plane,

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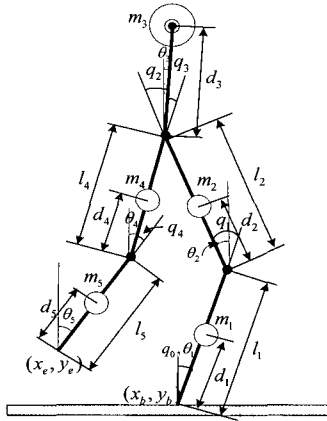


그림 1. 한발 지지 상태에서의 이족 로봇.  
Fig. 1. Biped robot in single support phase.

$\theta_i$ : Angle of link  $i$  with respect to vertical (the positive direction of  $\theta_i$ ,  $i = 1, 2, 3, 4, 5$ , is the one shown in the figure),  
 $(x_e, y_e)$ : position of the free end,  
 $(x_b, y_b)$ : position of the point of support.

From Fig. 1, the relationship of links is expressed as

$$\begin{aligned} x_e &= x_b + l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_4 \sin \theta_4 + l_5 \sin \theta_5, \\ y_e &= y_b + l_1 \cos \theta_1 + l_2 \cos \theta_2 - l_4 \cos \theta_4 - l_5 \cos \theta_5. \end{aligned} \quad (1)$$

Differentiating (1) with respect to time, we obtain

$$\begin{aligned} v_e = \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \end{pmatrix} &= \begin{pmatrix} l_1 \cos \theta_1 \\ -l_1 \sin \theta_1 \end{pmatrix} \dot{\theta}_1 + \begin{pmatrix} l_2 \cos \theta_2 \\ -l_2 \sin \theta_2 \end{pmatrix} \dot{\theta}_2 \\ &+ \begin{pmatrix} l_4 \cos \theta_4 \\ -l_4 \sin \theta_4 \end{pmatrix} \dot{\theta}_4 + \begin{pmatrix} l_5 \cos \theta_5 \\ -l_5 \sin \theta_5 \end{pmatrix} \dot{\theta}_5. \end{aligned} \quad (2)$$

2. Dynamic model with model uncertainties

It is assumed that the biped robot does not slip at the end of supporting without a friction. The biped robot model, which induced by the Lagrange dynamic model describing the motion of the biped in single support phase, represents as follows:

$$H(\theta)\ddot{\theta} + B(\theta, \dot{\theta}) + G(\theta) = \tau_\theta \quad (3)$$

where  $\theta = [\theta_1, \theta_2, \dots, \theta_5]^T$ ,  $\tau_\theta = [\tau_{\theta 1}, \tau_{\theta 2}, \dots, \tau_{\theta 5}]^T$ ,

$$B(\theta, \dot{\theta}) = \text{col} \left[ \sum_{j=1}^5 \left( h_{ij} (\dot{\theta}_j)^2 \right) \right],$$

$$G(\theta) = \text{col} [G_i(\theta)], \quad H(\theta) = [H_{ij}(\theta)] \quad (i, j = 1, 2, \dots, 5),$$

$\tau_\theta$  is generalized torque vector which is corresponding to each joint angle, and  $\text{col}[a_i]$  is a column vector with elements  $a_i$ .  $H(\theta)$  is a  $5 \times 5$  symmetric positive-definite inertia matrix,  $B(\theta, \dot{\theta})$  is a  $5 \times 1$  column vector with respect to the Coriolis and centripetal torque, and  $G(\theta)$  is a  $5 \times 1$  gravity vector.

Relative joint angles ( $q_1, q_2, q_3$ , and  $q_4$ ) are represented as

$$q_1 = \theta_1 - \theta_2, q_2 = \theta_2 - \theta_3, q_3 = \theta_3 - \theta_4, q_4 = \theta_4 - \theta_5.$$

The torques are transformed as  $\tau_q = E\tau_\theta$ ,

$$\text{where, } E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Transformed by relative angles, (3) is expressed as

$$H(q)\ddot{q} + B(q, \dot{q}) + G(q) = \tau_q. \quad (4)$$

Here, only four of this five relative degrees  $q = [q_1, q_2, \dots, q_4]^T$  can be controlled directly by the four driving torques at each joints. That is, the hypothetical joint  $q_0$  which is contacted with the surface without the gravitational effects is an uncontrollable joint.  $H(q)$ ,  $B(q, \dot{q})$ , and  $G(q)$  proposed in [7] are used. A biped robot gets in trouble with various bad conditions like expansion of frame, uncertain measurement of biped part and disturbance. Therefore, in this paper, we consider dynamic model with model uncertainties expressed as following form [15]:

$$H(q)\dot{q} + B(q, \dot{q}) + G(q) + \Xi(q, \dot{q}, \tau_q) = \tau_q, \quad (5)$$

where,

$$\begin{aligned} \Xi(q, \dot{q}, \tau) &= -H(q)\bar{H}(q)\{\tau_q - \tau_d - \bar{B}(q, \dot{q}) + \bar{G}(q)\} \\ &+ \{\tau_q - B(q, \dot{q}) - G(q)\} \end{aligned}$$

denotes the uncertainty of the robot system, and  $\bar{H}(q)$ ,  $\bar{B}(q, \dot{q})$ , and  $\bar{G}(q)$  are actual values with uncertainties in nominal values  $H(q)$ ,  $B(q, \dot{q})$ , and  $G(q)$ . Also,  $\tau_d$  is the external disturbance. It is assumed that the nominal values are only known for a given robot system.

Then, (5) can be rewritten as

$$\begin{aligned} \ddot{q} &= H^{-1}(q)(\tau_q - B(q, \dot{q}) - G(q) - \Xi(q, \dot{q}, \tau_q)) \\ &= H^{-1}(q)(\tau_q - B(q, \dot{q}) - G(q)) + \Upsilon(q, \dot{q}, \tau_q), \end{aligned} \quad (6)$$

where  $\Upsilon(q, \dot{q}, \tau_q) = -H^{-1}(q)\Xi(q, \dot{q}, \tau_q)$  is the uncertainty term. Here,  $\tau_q$  is a function of  $q$ ,  $\dot{q}$ , and  $Q_d = (q_d, \dot{q}_d, \ddot{q}_d)$  which denotes the reference position, velocity, and acceleration. Accordingly, the uncertainty term can be represented as  $\Upsilon(q, \dot{q}, \tau_q) = \Upsilon(q, \dot{q}, Q_d)$ . However, since  $\bar{H}(q)$ ,  $\bar{B}(q, \dot{q})$ ,  $\bar{G}(q)$ , and  $\tau_d$  are unknown values, the uncertainty term  $\Upsilon(q, \dot{q}, Q_d)$  can not be computed directly.

III. Control of the biped robot

In order to control the biped robot with the uncertainty term  $\Upsilon(q, \dot{q}, Q_d)$ , the hybrid SMC method is proposed in this Section.

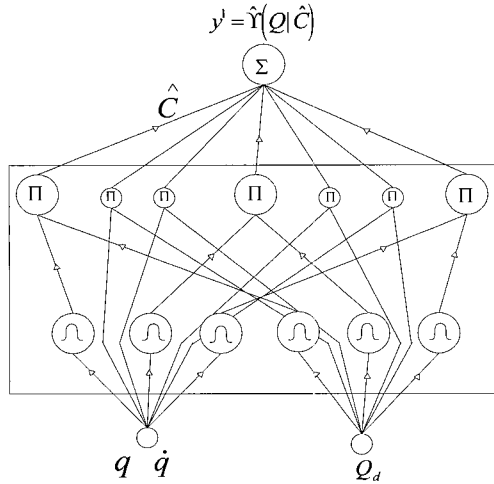


그림 2. 웨이블릿 신경 회로망의 구조.  
Fig. 2. Structure of wavelet neural network.

First, we present a WNN structure used as uncertainty observer. Second, the design methodology of the hybrid SMC system is discussed.

1. Wavelet neural network

In this paper, we use a WNN to observe the uncertainty term  $Y(q, \dot{q}, Q_d)$  in each joint. A proposed WNN structure is shown in Fig. 2.

The signal propagation and basic function in the product layer are expressed as

$$y_i = \prod_j \phi(net_{jp}) \text{ with } net_{jp} = \frac{x_p - m_{jp}}{d_{jp}}, \text{ and } p=1, \dots, 4 \quad (7)$$

where  $x_p$  denotes the input of the WNN, and  $m_{jp}$ ,  $d_{jp}$  are translation and dilation vector in the product layer respectively. In this paper, we select the first derivatives of a Gaussian function,  $\phi(x) = -x \exp(-x^2/2)$  as a mother wavelet function. It may be regarded as a differentiable version of a Harr mother wavelet, just as the sigmoid is a differentiable version of a step function, and it has the universal approximation property [16]. Then, the WNN output  $y^o$  is composed of the training weights  $w_{jo}$  and the output of the product layer  $y_j$  as follows:

$$y^o = \sum_j w_{jo} y_j, \text{ with } j=1, 2, \dots, n \text{ and } o=1, \quad (8)$$

where,  $w_{jo}$  is the weight vector between product layer and the output layer,  $j$  is the number of wavelet node,  $o$  is the node number of output layer. Output layer  $y^o$  is expressed as follows:

$$y^o = \hat{Y}(Q|\hat{C}) = \hat{C}\Gamma, \quad (9)$$

where,  $\Gamma = [y^{o1}, y^{o2}, \dots, y^{on}]$  is output vector of wavelet function and  $\hat{C} = [w_{11}, w_{21}, \dots, w_{n1}]^T$  is weight vector trained by tuning algorithm. By universal approximation theorem [16], there

exists an optimal WNN in the form of (9) such that

$$Y = Y^*(Q|C^*) + \varepsilon = C^*\Gamma + \varepsilon. \quad (10)$$

where  $C^*$  is optimal weighting vector that achieves the minimum reconstruction error and it is assumed that the elements of reconstruction error vector  $\varepsilon$  are bounded by  $|\varepsilon| \leq E$ .

2. Hybrid SMC

In the proposed control system, the controller based on the SMC technique is designed as  $u = \hat{u} - k \text{sgn}(s)$ .  $\hat{u}$  consists of a main controller and compensation controller. A main controller guarantees stable walking of a biped robot with uncertainties by means of sliding mode technique and WNN uncertainty observer. A compensation controller reduces the approximation error between actual uncertainties and observer uncertainties. A hybrid SMC system is shown in Fig. 1.

First, in order to follow the reference trajectory  $q_d(t)$ , the tracking error  $e$  and the sliding surface vector  $s$  are selected as [7]

$$\begin{aligned} e &= q - q_d, \\ s &= \dot{e} + \lambda_1 e + \lambda_2 \int e. \end{aligned} \quad (11)$$

Differentiating  $s$  and using (6), we can obtain

$$\begin{aligned} \dot{s} &= \ddot{e} + \lambda_1 \dot{e} + \lambda_2 e \\ &= \ddot{q} - \ddot{q}_d + \lambda_1 \dot{e} + \lambda_2 e \\ &= H^{-1}(q)(\tau_q - B(q, \dot{q}) - G(q)) + Y(q, \dot{q}, Q_d) \\ &\quad - \ddot{q}_d + \lambda_1 \dot{e} + \lambda_2 e. \end{aligned} \quad (12)$$

The controller which would achieve  $\dot{s} = 0$  for the best tracking performance is designed as the hybrid controller  $\tau_h$  using WNN uncertainty observer,

$$\tau_q = \tau_s + \tau_c, \quad (13)$$

where, two vectors  $\tau_s$  and  $\tau_c$  denote a sliding controller using WNN observer and a compensation controller, respectively, defined as follows:

$$\begin{aligned} \tau_s &= B + G + H(q)^{-1}(\ddot{q}_d - \lambda_1 \dot{e} - \lambda_2 e - \hat{Y}(q, \dot{q}, Q_d) - K \text{sgn}(s)) \\ \tau_c &= -H(q)^{-1} \hat{E} \text{sgn}(s). \end{aligned}$$

where,  $K = \text{diag}[k_i]$ ;  $\text{diag}[\bullet]$  denotes the diagonal matrix and  $k_i$  is  $i$ th positive gain. In biped model (4), since  $q_0$  of the biped robot model is the uncontrollable joint, our control law is redefined as follows:

$$\tau_q = H(q)U + B(q, \dot{q}) + G(q), \quad (14)$$

where  $U \in R^5$  is the vector with following components:

$$\begin{aligned} u_1 &= -\frac{1}{H_{11}(q)} \left[ \sum_{l=1}^4 (H_{1l+1}(q)u_{l+1}) + B_1(q, \dot{q}) + G_1(q) \right] \\ u_{l+1} &= \ddot{q}_{dl} - \lambda_{1l} \dot{e}_l - \lambda_{2l} e_l - \hat{Y}(q, \dot{q}, Q_d) - (\hat{E}_l + k_l) \text{sgn}(s_l) \end{aligned}$$

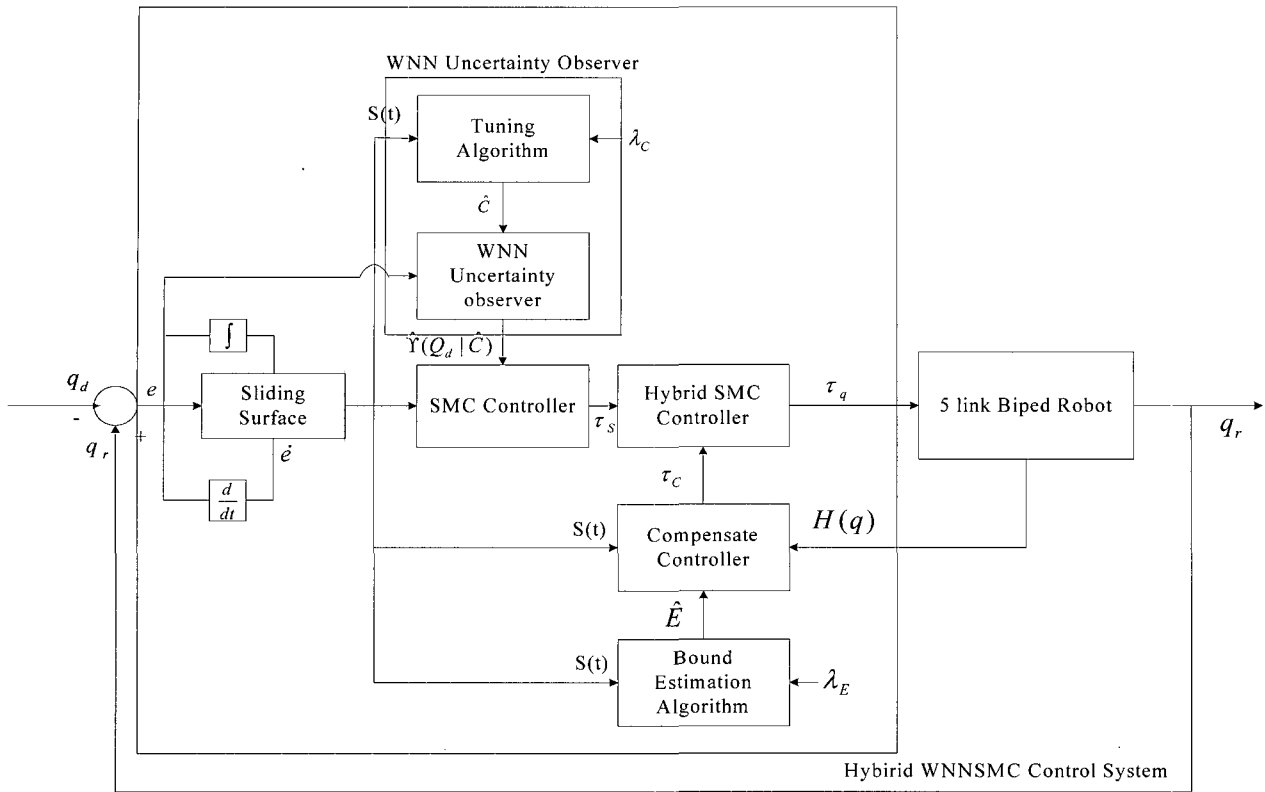


그림 3. 하이브리드 SMC 제어 시스템.  
Fig. 3. A hybrid SMC control system.

Theorem 1: Assume that the biped robot model (6) and the controller (14) are used for our control system. If the weights of the WNN and the error compensator are trained by adaptation laws (15) and (16), respectively:

$$\dot{\hat{C}}_i = \lambda_c s_i \Gamma_i, \tag{15}$$

$$\dot{\hat{E}}_i = \lambda_E |s_i|, \tag{16}$$

where  $\lambda_c$  and  $\lambda_E$  are positive tuning gains, then the stability of our control system is guaranteed.

Proof: Lyapunov function candidate is as follows:

$$V(S, \tilde{C}, \tilde{E}) = \frac{1}{2} s^T s + \frac{1}{2\lambda_c} \tilde{C}^T \tilde{C} + \frac{1}{2\lambda_E} \tilde{E}^T \tilde{E}, \tag{17}$$

where,  $\tilde{C} = C^* - \hat{C}$ ,  $\tilde{E} = E - \hat{E}$ . Taking the time derivative of the Lyapunov function, we obtain

$$\dot{V} = s^T \dot{s} - \frac{1}{\lambda_c} \tilde{C}^T \dot{\hat{C}} - \frac{1}{\lambda_E} \tilde{E}^T \dot{\hat{E}}. \tag{18}$$

Substituting (12) into (18),  $\dot{V}$  can be rewritten as:

$$\begin{aligned} \dot{V} = & s[H^{-1}(q)(\tau_q - B(q, \dot{q}) - G(q)) + Y(q, \dot{q}, \ddot{q}_d) \\ & - \ddot{q}_d + \lambda_1 \dot{e} + \lambda_2 e] - \frac{1}{\lambda_c} \tilde{C}^T \dot{\hat{C}} - \frac{1}{\lambda_E} \tilde{E}^T \dot{\hat{E}}. \end{aligned} \tag{19}$$

Using proposed controller (14) and optimal WNN (10), we can obtain

$$\begin{aligned} \dot{V} = & \sum_{i=1}^4 s_i \left[ C_i \Gamma - \hat{C}_i \Gamma - \hat{E}_i \text{sgn}(s_i) - k_i \text{sgn}(s_i) \right] - \frac{1}{\lambda_c} \tilde{C}_i \dot{\hat{C}}_i - \frac{1}{\lambda_E} \tilde{E}_i \dot{\hat{E}}_i \\ = & \sum_{i=1}^4 s_i \left[ C_i^* \Gamma - \hat{C}_i \Gamma + \varepsilon_i - \hat{E}_i \text{sgn}(s_i) - k_i \text{sgn}(s_i) \right] - \frac{1}{\lambda_c} \tilde{C}_i \dot{\hat{C}}_i - \frac{1}{\lambda_E} \tilde{E}_i \dot{\hat{E}}_i \\ = & \sum_{i=1}^4 s_i \tilde{C}_i \Gamma + \varepsilon_i s_i - \hat{E}_i |s_i| - \frac{1}{\lambda_c} \tilde{C}_i \dot{\hat{C}}_i - \frac{1}{\lambda_E} \tilde{E}_i \dot{\hat{E}}_i - k_i |s_i| \\ = & \sum_{i=1}^4 \tilde{C}_i (\Gamma s_i - \frac{1}{\lambda_c} \dot{\hat{C}}_i) + \varepsilon_i s_i - \hat{E}_i |s_i| - \frac{1}{\lambda_E} \tilde{E}_i \dot{\hat{E}}_i - k_i |s_i|. \end{aligned} \tag{20}$$

Using (15) and (16), (20) can be derived as:

$$\begin{aligned} \dot{V} = & \sum_{i=1}^4 -\hat{E}_i |s_i| + \varepsilon_i s_i - (E_i - \hat{E}_i) |s_i| - k_i |s_i| \\ = & \sum_{i=1}^4 -E_i |s_i| + \varepsilon_i s_i - k_i |s_i| \\ \leq & \sum_{i=1}^4 -E_i |s_i| + |\varepsilon_i| |s_i| - k_i |s_i| \\ = & \sum_{i=1}^4 -(E_i - |\varepsilon_i|) |s_i| - k_i |s_i| \leq 0. \end{aligned} \tag{21}$$

Since  $\dot{V} \leq 0$ ,  $\dot{V}$  is negative semi-definite. Thus,  $\tilde{C}$ ,  $\tilde{E}$  and  $s(t)$  are bounded. We define a function

$$M(t) = -\dot{V}(s(t), \tilde{C}, \tilde{E}). \tag{22}$$

Integrating  $M(t)$  with respect to time, then we obtain

$$\int_0^t M(\rho) d\rho \leq V(s(0), \tilde{C}, \tilde{E}) - V(s(t), \tilde{C}, \tilde{E}). \quad (23)$$

Because  $V(s(0), \tilde{C}, \tilde{E})$  is bounded, and  $V(s(t), \tilde{C}, \tilde{E})$  is nonincreasing and bounded, the following result can be concluded

$$\lim_{t \rightarrow \infty} \int_0^t M(\rho) d\rho < \infty. \quad (24)$$

Also,  $\dot{M}(t)$  is bounded, so by Barbalat's lemma,  $\lim_{t \rightarrow \infty} M(t) = 0$ .

That is,  $s(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Consequently, the Lyapunov stability can be guaranteed.

Remark 1: In [6], the SMC technique was applied to the robust control of the five-link. This paper requires the upper bounds of the internal uncertainties and external disturbances to compute the gains of the sliding controllers. However, it is difficult to satisfy this condition in the real bipedal systems because the exact values of the internal uncertainties and external disturbances are unavailable to measure or to know in advance. Therefore, the SMC method cannot be applied to the robust control of biped robot systems with the unknown uncertainties and disturbances. Hence, in the proposed control method, we employ the hybrid controller using the WNN uncertainty observer for the estimation of the uncertainty term of the bipedal systems. Accordingly, in our control system, any information for the model uncertainties and external disturbances is not needed.

#### IV. Simulations

The 5-link biped model shown in Fig. 1 is used in simulation. The parameter of a biped robot is small sized, whose values are shown in Table 1. Desired references are a steady stable gait on horizontal plane and such a gait can be obtained by feeding to the control system repeatedly at every step the same reference signal. Actually, the complete motion of the biped robot can be explained by a single support phase, a double support phase, double impact, switching and transformation [14]. Thus, there is a need to switch the dynamic equations and controllers during the iterative computation of the simulation program. However, this method causes the complex programming problems.

Accordingly, in this paper, we apply our control system for stable walking control of the planar five-link biped robot with only a single support phase. The used walking pattern is shown in Fig. 4[7]. To examine the robustness and performance of the proposed control method, we compare the hybrid SMC method using WNN uncertainty observer with the computed torque control (CTC) method. To compare the performance of a hybrid SMC system and CTC system, it is assumed that the same disturbances and uncertainties have influence on the biped robot system. We simulate the biped robot system with 100% parametric uncertainties of  $m_1, m_2, m_4,$  and  $m_5,$  and 20% time varying parametric uncertainty of  $m_3,$  and 10% parametric uncertainty of each length of link.

In addition, it is assumed that the external disturbances  $\tau_d = [0.4\sin(10t) \ 0.1\cos(10t) \ 0.2\sin(10t) \ 0.2\cos(10t) \ 0.3\sin(10t)]$

표 1. 이족 로봇의 파라미터들.

Table 1. Parameters of biped robot.

Link	Mass $m_i$ (kg)	Moment of inertia $I_i$ (kg m)	Length $l_i$ (m)	Location of center of mass $d_i$ (m)
Torso	14.79	$3.30 \times 10^{-2}$	0.486	0.282
Thigh	5.28	$3.30 \times 10^{-2}$	0.302	0.236
Leg	2.23	$3.30 \times 10^{-2}$	0.332	0.189

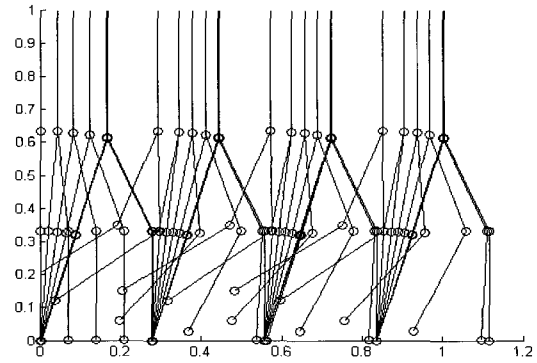


그림 4.5 링크 이족 로봇의 걸음새 형태.

Fig. 4. Locomotion mode of 5-link biped robot.

apply to a biped robot. We set up with CTC gain as follows:

$$K_d = \text{diag}[500 \ 500 \ 500 \ 500]$$

$$K_p = \text{diag}[1000 \ 1000 \ 1000 \ 1000]$$

where  $K_d$  and  $K_p$  are the proportional and derivative gain diagonal matrices, respectively. And parameters of a hybrid SMC are given by

$$\lambda_1 = \text{diag}[500 \ 500 \ 500 \ 500]$$

$$\lambda_2 = \text{diag}[10 \ 10 \ 10 \ 10]$$

$$\lambda_c = \text{diag}[0.1 \ 0.1 \ 0.1 \ 0.1]$$

$$\lambda_E = \text{diag}[0.001 \ 0.001 \ 0.001 \ 0.001]$$

$$K = [200 \ 200 \ 200 \ 200],$$

where  $\lambda_1$  and  $\lambda_2$  are hybrid SMC gains,  $\lambda_c$  is a learning rate matrix of WNN uncertainty observer, and  $\lambda_E$  is a learning rate matrix of compensation controller. Also, to eliminate the chattering phenomenon in the control input, we use saturation function instead of signum function [5].

$$k_i \text{sat}(s_i / \Phi) = \begin{cases} k_i s_i / \Phi & \text{if } |s_i / \Phi| \leq 1 \\ k_i \text{sgn}(s_i / \Phi) & \text{if otherwise} \end{cases}$$

where  $\Phi$  is the boundary layer thickness. In this paper,  $\Phi$  is chosen 0.1. WNNs are used as the uncertainty observer where each WNN estimator has five inputs, three wavelet nodes, four product nodes and one output node. WNN parameters of translation and dilation are randomly selected and weights are tuned by the adaptation laws induced from the Lyapunov stability theorem. Sampling time is chosen as 0.001 sec. Fig. 5 compares

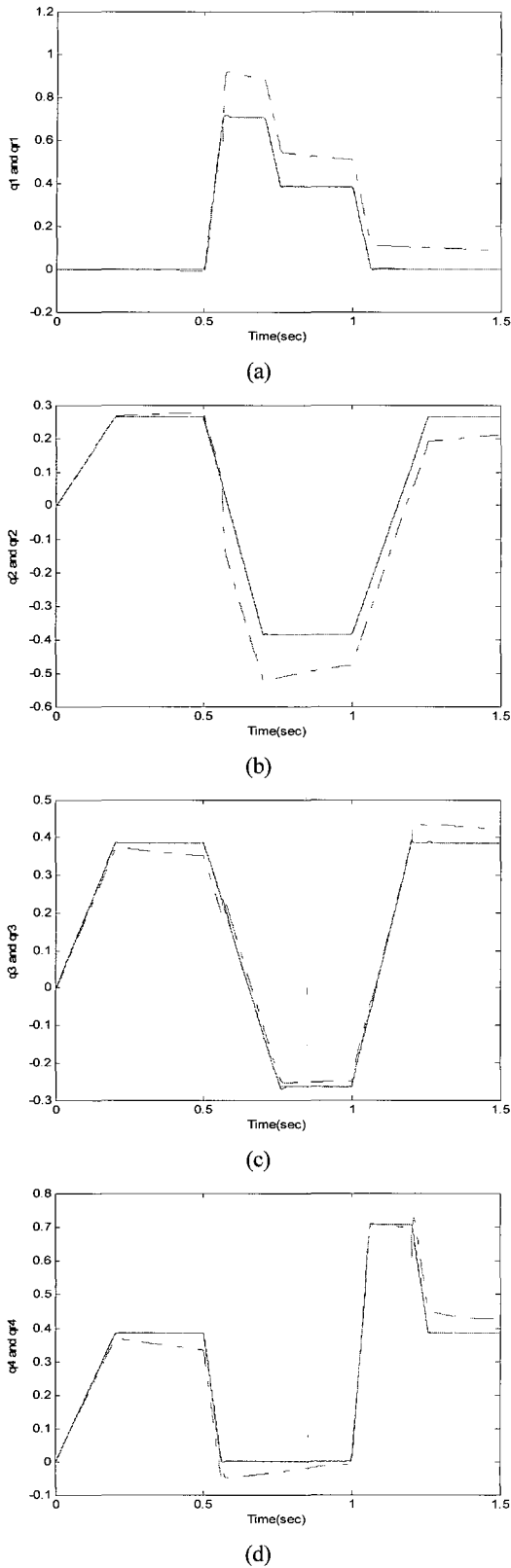


그림 5. 5 링크 이족 로봇의 제어 결과 비교 (a)  $q_1$  (b)  $q_2$  (c)  $q_3$  (d)  $q_4$  (실선: 하이브리드 SMC, 실점선: CTC, 점선: 원하는 궤적).

Fig. 5. Comparison of tracking results for five-link biped robot (a)  $q_1$  (b)  $q_2$  (c)  $q_3$  (d)  $q_4$  (solid line : a hybrid SMC, dash-dotted line : a CTC, dash line : reference trajectory).

표 2. 하이브리드 SMC와 CTC의 오차 비교.

Table 2. The error comparison of a hybrid SMC with a CTC.

	$e_1$ (rad)	$e_2$ (rad)	$e_3$ (rad)	$e_4$ (rad)
CTC	0.0835	0.0631	0.0233	0.0292
Hybrid SMC	$5.7 \times 10^{-8}$	$5.7 \times 10^{-8}$	$1.3 \times 10^{-7}$	$2.6 \times 10^{-8}$

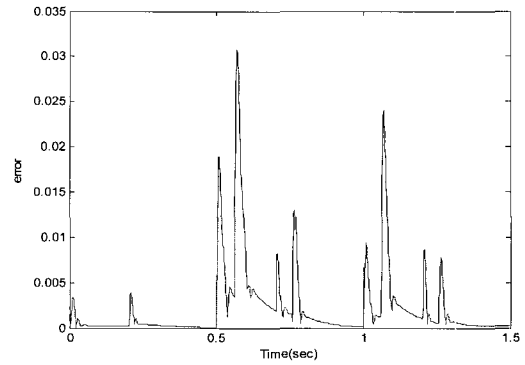


그림 6. 하이브리드 SMC의 절대 에러.

Fig. 6. Absolute error of hybrid SMC.

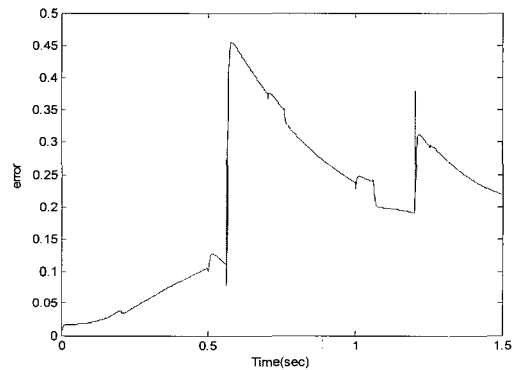


그림 7. CTC의 절대 에러.

Fig. 7. Absolute error of CTC.

the simulation results of a hybrid SMC which is solid line and CTC which is dash dotted line. Therefore, a hybrid SMC shows tracking performances compared with CTC in uncertainty case.

Table 2 compares the performance of the hybrid SMC with that of the CTC. In Figs. 6 and 7, absolute errors of a hybrid SMC and CTC are compared. In Fig. 7 Absolute summed error of CTC is gradually elevated. As a result, we can confirm that the hybrid SMC overcomes biped robot parameter uncertainties and external disturbances.

### V. Conclusion

In this paper, we have designed a hybrid SMC system using a WNN uncertainty observer for the 5-link biped robot model with model uncertainties and external disturbances. In our control system, SMC was used as a main controller and WNN observer estimated uncertainty terms of the 5-link biped robot. Also, the compensation controller has been used to compensate estimation errors of the WNN. Weights of WNN observer and compensation controller are trained by the adaptation laws induced from the Lyapunov stability theorem, which are used to guarantee the

stability of the proposed control scheme. Through computer simulations, we have confirmed that the performance of the hybrid SMC is superior to that of the CTC for the 5-link biped robot with uncertainties.

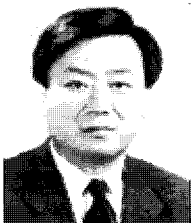
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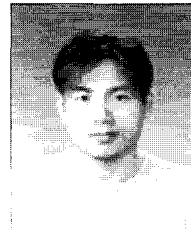
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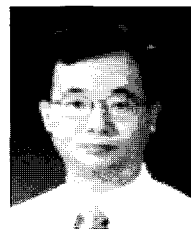
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