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Forecasting Accidents by Transforming Event Trees into Influence disgrams

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Event trees are widely used graphical tool to denote the accident inintiation and escalation to more severe accident. But they have some drawbacks in that they do not have efficient way of updating model parameters and also they can not contain the information about dependency or independency among model parameters. A tool that can cure such drawbacks is an influence diagram. We introduce influence diagrams and explain how to update model parameters and obtain predictive distributions. We show that an event tree can be converted to a statistically equivalent influence diagram, and bayesian prediction can be made more effectively through the use of influence diagrams.

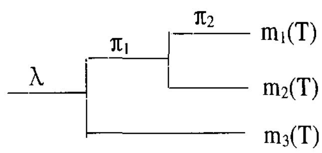
1. Introduction

Event tree is one of the most frequently used graphical tools in the safety analysis of large complex systems such as weapon systems, large chemical plants, and nuclear power plants, and so forth. Event tree is generally composed of many branches, and the probability of passing through a specific branch is usually estimated from the analysis of fault trees. Therefore the rate of accidents or probability of accidents following a specific path in an event tree is described by many number of parameters. The estimation and updating of parameters based on observed data is very much complicated and sometimes even impossible. So, event trees are usually used just to analyze how an accident is initiated and through what path it is escalated to more severe accident.

This paper introduces influence diagrams as a superior substitution to event trees. We adopt the philosophy of bayesian statistics. According to bayesian concept, we show how influence diagrams are manipulated to proceed the process of parameter updating to get posterior distributions, and node absorption to get predictive distributions. We try to find the graphical and statistical relationship between event trees and influence diagrams, and thereby gets the theoretical basis that makes the safety analysis and forecasting possible through the use of influence diagrams. We briefly present a numerical example where bayesian prediction is performed with the help of influence diagrams.

2. Event trees

Event tree is used to clearly show the accident initiation and escalation to more severe accident. Event tree shows paths from the initiation of accident to escalation of various levels of accidents depending on whether following subsystems are properly functioning or not. In figure 1, the leftmost branch denotes the initiation of abnormal event, and the rate of event, λ, is assigned on the branch. The next branch right hand side of it denotes whether the following subsystem is functioning or not. The up branch denotes the failure of a following subsystem and the corresponding probability, π_1, is assigned on the branch. Therefore the path passing through down branch from the fork means the following subsystem is functioning successfully and the probability of it is $1-\pi_1$. λ and π's are generally estimated from the analysis of fault trees. The uppermost node of fault is an estimate of λ or π's. Same explanation applies to the second subsystem. Figure 1 shows the case where only two subsystems exist, but we can easily extend the model with the same logic to explain the case where many subsystems exist.



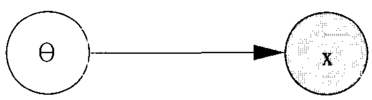
<Figure 1> An example of an Event tree

When an abnormal accident is initiated and the following subsystem functions properly, it does not escalate to more severe accident and end up with minor accident. In figure 1, $m_3(T)$ denotes the number of minor accidents during the time period (0, T). If the first subsystem fails to function but the second subsystem functions successfully, the accident escalates to significant accident. In figure 1, $m_2(T)$ denotes the number of significant accidents during the time period (0, T). Similarly, $m_1(T)$ denotes the number of the most severe accidents due to the malfunction of both following subsystems 1 and 2.

Once we draw an event tree like this we can now clearly see the process of accident initiation and propagation to more severe accidents. But there are several drawbacks on the event trees. Firstly, we cannot see the statistical dependencies or independencies among branch parameters. Secondly and more importantly, there is no way to adaptively update model parameters as we observe data. To improve such drawbacks, we introduce influence diagrams.

3.Influence diagrams

An Influence diagram is a graphical tool that explains the statistical dependency or independency among model parameters. This is often used in bayesian analysis since it follows the bayesian philosophy where model parameters are treated as unobservable random variables. An Influence diagram is composed of arcs and circles. Circles denote random variables and arcs denote possible statistical dependencies. Shaded circles denote observable random variables and unshaded circles denote unobservable model parameters. Figure 2 is a simple example of an influence diagram where model parameter Θ influences random variable x. Model parameter Θ is unobservable and random variable x is observable.



<Figure 2> An Example of an Influence diagram

We can assume prior distribution on model parameter Θ, p(Θ), and likelihood on x given Θ, p(x|Θ). Then the joint probability distribution of random variables involved in a model, p(Θ,x) is expressed as p(Θ|x)p(x).

When we observe data on x, we update model parameter Θ based on observed data x. This is the process of arc reversal in influence diagrams like in figure 3 so that arrow now goes from observed x to unobserved parameter Θ. Such graphical manipulation is equivalent to obtain a posterior distribution on model parameter Θ.

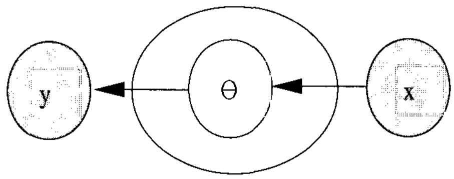
$$p(\Theta|x) = C\ p(\Theta)\ p(x|\Theta)$$

<Figure 3> An Influence diagram that shows an arc reversal

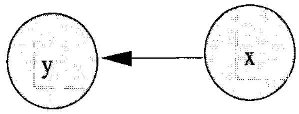
The predictive distribution on y is obtained by integrating out unobservable model parameter Θ so that the prediction is made based only on the observed value x;

$$p(y|x) = \int p(y|\Theta)\ p(\Theta|x)d\Theta$$

where p(y|Θ) is a likelihood and p(Θ|x) is a posterior distribution obtained in the above process. Graphically this is equivalent to the node absorption process as in figure 4.



(a)



(b)

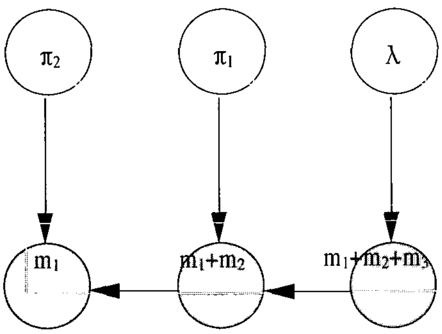
<Figure 4> Node absorption

4. Getting Influence diagram from event tree

The event tree in figure 1 can be converted to an influence diagram as in figure 5. m_i in figure 5 is a simple expression of $m_i(T)$, i=1,2,3. Figure 5 assumes that model parameters λ and π's are independent each other. If it is inadequate to assume independency among model parameters, we add arcs among parameter nodes so that possible dependency among model parameters are considered. The rate λ influences the number of initial abnormal event $m_1(T)+m_2(T)+m_3(T)$, so the arc is connected from λ node to event count node. In the event tree, if the first sub-system fails to operate with the probability of π_1, the initiating event escalates to more severe accident. The count of such accident is $m_1(T)+m_2(T)$. In the influence diagram, we can see that $m_1(T)+m_2(T)$ accidents out of $m_1(T)+m_2(T)+m_3(T)$ accidents escalate to more severe accident with probability π_1. Similarly, when the second sub-system fails to operate with probability of π_2, $m_1(T)$ accidents out of $m_1(T)+m_2(T)$ accidents escalate to more severe accident.

The information contained in an event tree can be transferred to an influence diagram. Furthermore, the statistical dependency or independency among model parameters, that cannot be expressed in an event tree, can be explicitly denoted in an influence diagram.

Another crucial advantage of an influence diagram over an event tree is that model



<Figure 5>An equivalent influence diagram

parameters can be adaptively updated as real data is observed in an influence diagram.

5. Numerical Example

We illustrate with a numerical example based on the event tree in 1. The growth of accident counts over time is shown in table 1. Notice that the large majority of accident sequences are included in the bottom bath of the tree which, in our example, corresponds to the least severe sequence.

<Table 1> Counts of Accidents Sequences over time

T	m1	m2	m3
0	0	0	0
20	25	3	0
40	40	3	0
60	66	10	1
80	85	11	1
100	110	15	2
120	130	15	3
140	155	15	3
160	167	20	3
180	189	22	3
200	205	25	4

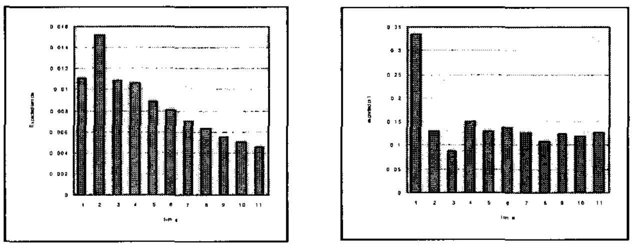
The prior distribution of λ is assumed to follow gamma distribution with parameter α=10, β=30. We follow the convention that if a random variable λ follows a gamma distribution with parameters α and β, the probability density function is expressed as follows;

$$p(\lambda) = \frac{\beta}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

The prior distributions of conditional probabilities of π's are assumed to have a beta distributions with parameters $a_1=1$, $b_1=2$, $a_2=1$, $b_2=20$. If π follows beta distribution with parameters a and b, the probability density function is expressed as follows;

$$p(\pi) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \pi^{a-1}(1-\pi)^{b-1}$$

We assume that counts on branches have the binomial distribution conditional on the total count of accidents on the immediately preceding upstream branch, and the probability of failure of a sub-system. The posterior expected values of the branch parameters are plotted in figure 6 as a function of time. We can see that the posterior expected values are fluctuating as we get more data.



<Figure 6> Posterior Expected Values of λ and π2

As we get more counts over time, the posterior distributions of the branch parameters often becomes sharper due to the availability of more information. Figure 7 shows this effect by comparing the prior and posterior distributions on π_1 at T= 100 and 200.

The predictive distribution of time to next abnormal accident x can be obtained by integrating out the unobservable model parameters;

$$p(x|data) = \int p(x|\lambda)\ p(\lambda|data)\ d\lambda$$
$$= \left(\frac{\beta + T}{\beta + T + x}\right)^{\alpha} \frac{\alpha + n}{\beta + T + x}$$

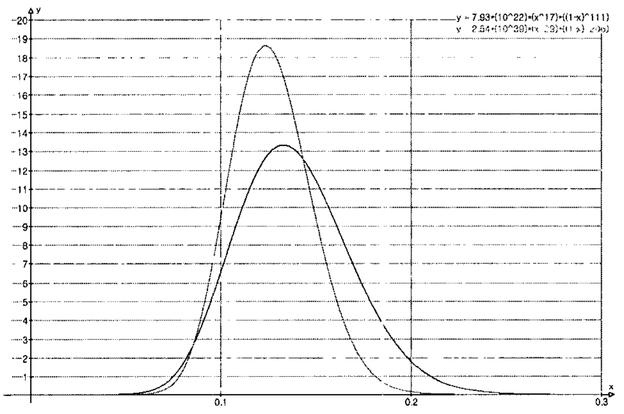


Figure 7 : Prior and Posterior distribution of π

Luckly enough, we get the closed form distribution of shifted pareto for the time to next accident.

5. Summary

We can show how an abnormal event is initiated and escalated to more severe accident using an event tree. But there is no way to adaptively update model parameters as we acquire data. Also dependency cr independency among model parameters can not be explicitly expressed in an event tree. An influence diagram make up for the above mentioned drawbacks of event trees while it contains statistically equivalent information.

We proposed the way of updating model parameters and obtaining predictive distributions using influence diagrams. Based on such paradigm, influence diagrams can be widely used in the area of safety analysis.

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