Design of Sliding Mode Controller with Uncertainty Adaptation

Min-Chan Kim, Jing-Rak Nam, Seung-Kyu Park, and Gun-Pyong Kwak, *Member, KIMICS*

Abstract—In this paper, a sliding mode control method with uncertainty adaptation is proposed by introducing the virtual state. Because upper bound of the uncertainty is very difficult to know, we estimate this by using the simple adaptation law and design the sliding surface which has dynamic of nominal system. An optimal controller is used by nominal controller. And if initial values of the virtual state are chosen properly, the reaching phase is removed.

Index Terms—Robust control , Sliding Mode Control, Parameter Adaptation

I. INTRODUCTION

Usually, when upper bound of parameter uncertainty norm is known, sliding mode control has robust performance.[1][2][3][4] However the dynamics of sliding mode controller cannot has the nominal system dynamics because it has dynamics behavior of predetermined sliding surface. Therefore, in this paper we prove the equivalence of the proposed sliding surface dynamics and nominal system dynamics, and verify the performance. For the designing of sliding mode controller, it is required to know the upper bound of parameter uncertainty. It is the one of disadvantage to construct sliding mode controller. However, we can not know upper bound easily, so control input can grow and influence in stability of system. This problem can be solved through estimation of upper bound using the simple adaptation law. Finally, we propose the sliding mode controller with the virtual state that estimate the upper bound over parameter uncertainty, and it can be applied to the system whose dynamics satisfies nominal system dynamics. By choosing properly initial value of the virtual state, reaching phase is eliminated. To construct the proposed sliding surface, an optimal controller is used by nominal controller. This controller can minimize a cost function.

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II. PROBLEM FORMULATION

Consider n-th order system including the following uncertainty.

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + Df(t) \quad (1)$$

where $x(t) \in R^n$, $u(t) \in R^m$, $f(t) \in R^r$, the bounded uncertainties ΔA , ΔB and the disturbance matrix D satisfy the following matching condition.

$$\text{rank}([B : \Delta A : \Delta B : D]) = \text{rank}B \quad (2)$$

With the above condition, the uncertainties and disturbance can be expressed as

$$\begin{aligned} \Delta Ax(t) &= B\Delta A_1 x(t) \\ \Delta Bu(t) &= B\Delta B_1 u(t) \\ Df(t) &= BD_1 f(t) \end{aligned} \quad (3)$$

The system (1) can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(x,t) + Bh(x,t) \quad (4)$$

where $h(x,k) = \Delta A_1 x(k) + \Delta B_1 u(k) + D_1 f(k)$ is lumped uncer- tainty and bounded as follows.

$$\|h(x,k)\| \le \rho(x,k) \quad (5)$$

Typical sliding surface of Eq.(4) is given as follows.

$$\sigma(x,t) = Cx(t) = 0 \quad (6)$$

where $C = [c_1 \cdots c_n]$ and $c_1 \cdots, c_n$ are given so that dynamic of sliding mode must be stable.

If candidate function of Lyapunov function is

$$V(x,t) = \frac{1}{2}\sigma(x)^2$$

And then the following condition guarantees sliding mode.

$$\dot{V}(x,t) < 0,\ \sigma \neq 0 \Rightarrow \sigma(x,t)\dot{\sigma}(x,t) < 0 \qquad (7)$$

The sliding mode control input which satisfies above condition is constructed. However, the size of parameter uncertainty can not be known easily in Eq.(4). Therefore, purpose of this paper is designing a sliding surface which has dynamic of nominal system and estimated a upper bound of uncertainty by simple adaptation law. If the virtual state's initial value is chosen properly, the reaching phase is removed.

III. DESIGN OPTIMAL CONTROLLER WITH SLIDING SURFACE AND PARAMETER ADAPTATION LAW

Nominal system about Eq.(1) is as follows.

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t) \qquad (8)$$

Above equation can change to the following controllable canonical form using transformation matrix. $(z(t) = Px(t))$

$$\dot{z}_0(t) = A_c z_0(t) + B_c u_0(t) \qquad (9)$$

where $A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & \cdots & -\alpha_n \end{bmatrix}$, $B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

The virtual state of nominal system is defined by z_{0n} 's derivative and its dynamic is as follows.

$$\dot{z}_{0v} = -\alpha_n z_{0v}(t) - \cdots\cdots - \alpha_2 z_{03}(t) - \alpha_1 z_{02}(t) + \dot{u}_0(x_0,t) \qquad (10)$$

Therefore, the virtual state of the actual system changes the nominal state z_0 to actual state z as follows.

$$\dot{z}_v = -\alpha_n z_v(t) - \cdots\cdots - \alpha_2 z_3(t) - \alpha_1 z_2(t) + \dot{u}_0(x,t) \qquad (11)$$

The augmented system including the virtual state is as follows.

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + Df(t) \\ \dot{z}_v &= -\alpha_n z_v(t) - \cdots\cdots - \alpha_2 z_3(t) - \alpha_1 z_2(t) + \dot{u}_0(x,t) \end{aligned} \qquad (12)$$

where $u(t)$ is input that guarantees sliding mode on sliding surface, and $u_0(x,t)$ is the differentiable nominal control input.

Under the nominal control input of Eq.(12)., the following optimal controller can be obtained.[5][6][7]

Cost function of Eq.(8) is as follows.

$$J = \int_0^\infty (x_0^T Q x_0 + u_0^T R u_0)dt \qquad (13)$$

where Q is state weighting and R is control weighting

Therefore, the optimal control input which can minimize a cost function is calculated out from the following formula.

$$u_0(x_0) = -R^{-1}B^T S \qquad (14)$$

S is the following Riccati equation's solution.

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \qquad (15)$$

An optimal gain in Eq.(14) is $K = R^{-1}B^T S$. Then the nominal control input is expressed as follows.

$$u_0(x_0) = -Kx_0(t) \qquad (16)$$

Sliding surface is defined by the following formula with the virtual state and nominal control input.

$$\sigma(z, z_v) = z_v(t) + \alpha_n z_n(t) + \cdots\cdots + \alpha_1 z_1(t) - u_0(x,t) = 0 \qquad (17)$$

If initial value of the virtual state is selected as following, reaching phase is eliminated.

$$z_v(t_0) = -\alpha_n z_n(t_0) - \cdots\cdots - \alpha_1 z_1(t_0) - u_0(t_0) \qquad (18)$$

Therefore, the following theorem can be obtained.

Theorem

Dynamic of sliding surface such as Eq.(17), has the same dynamic, such as nominal system of Eq.(8) controlled by nominal control input. [8][9]

And estimate parameter uncertainty and construct control input that guarantees sliding mode by the following condition. [10]

$$\|p(t,x)\| \le c_0 + c_1\|x\| \quad \text{for all } (t,x) \qquad (19)$$

where c_0, c_1 are positive constants.

Also the sliding mode input is given as follows.

$$u(t) = u_{eq}(t) + u_s(t) + u_n(t) \qquad (20)$$

where $u_{eq}(t) = -(CB)^{-1}CAx(t)$ is equivalent control input and C includes coefficients of Eq. (17).

$u_s(t) = -(CB)^{-1}K\sigma(z,t)$ is a positive definite matrix.

$$u_n(t) = \begin{cases} -\dfrac{B^T C^T \sigma}{\|B^T C^T \sigma\|}\bar{\rho}(t,x) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \end{cases}$$

And $\bar{\rho}(t,x)$ is defined by the following condition when the norm's adaptive upper bound is in $u_n(t)$

$$\bar{\rho}(t,x) = \bar{c}_0(t,x) + \bar{c}_1(t,x)\|x\| \qquad (21)$$

where $\bar{c}_0(t,x)$ and $\bar{c}_1(t,x)$ are adaptation parameters of c_0 and c_1.

Therefore, the adaptation control law for upper bound of $\|\rho(t,x)\|$ is as follows.

$$\dot{\tilde{c}}_0(t,x) \equiv q_0 \left\|B^T C^T \sigma\right\|$$
$$\dot{\tilde{c}}_1(t,x) \equiv q_1 \left\|B^T C^T \sigma\right\| \|x\| \qquad (22)$$

where $\tilde{c}_0(t,x) = \bar{c}_0(t,x) - c_0$ and $\tilde{c}_1(t,x) = \bar{c}_1(t,x) - c_1$ are parameter adaptation errors. q_0 and q_1 are adaptation gains that have positive constants respectively. Also, because c_0 and c_1 are constants, the adaptation control law is changed as follows.

$$\dot{\bar{c}}_0(t,x) \equiv q_0 \left\|B^T C^T \sigma\right\|$$
$$\dot{\bar{c}}_1(t,x) \equiv q_1 \left\|B^T C^T \sigma\right\| \|x\| \qquad (23)$$

The adaptation parameter $\bar{c}_0(t,x)$ and $\bar{c}_1(t,x)$ can be get from Eq.(24),Eq.(25).

$$\bar{c}_0(t,x) = \bar{c}_{0i} + q_0 \int_{t_0}^{t} \left\|B^T C^T \sigma\right\| dt \qquad (24)$$

$$\bar{c}_1(t,x) = \bar{c}_{1i} + q_1 \int_{t_0}^{t} \left\|B^T C^T \sigma\right\| \|x\| dt \qquad (25)$$

where $\bar{c}_{0i}$ and $\bar{c}_{1i}$ are initial values of $\bar{c}_0(t,x)$ and $\bar{c}_1(t,x)$ respectively. And they can control adaptation ratio of parameter by selecting $\{\bar{c}_{0i}, \bar{c}_{1i}\}$ and $\{q_0, q_1\}$ properly.

IV. Numerical Example and Computer Simulation

Consider the second-order system.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u(t) + p(t,x))$$

where $p(t,x) = 0.1 + 0.2\|x(t)\|$

Nominal control input of the optimal controller can be obtained.

$$u_0(z,t) = [-0.0048 \quad -0.0048] \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Proposed sliding plane is constructed by

$$\sigma(z,t) = z_v(t) + [1 \quad 2] \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} - u_0(z,t)$$

If choose initial value of the virtual state as follows, the reaching phase is removed.

$$z_v(t_0) = -[1 \quad 2] \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + u_0(z,t)$$

Finally, the following sliding mode control input for system is obtained.

$$u(t) = u_{eq}(t) + u_s(t) + u_n(t)$$

where $u_{eq}(t) = [0.9750 \quad 1.9512 \quad 0.9762] \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

$$u_s(t) = -3.1238 \cdot \sigma(z,t)$$

$$u_n(t) = \begin{cases} -\dfrac{2.0488 \cdot \sigma(z,t)}{\|2.0488 \cdot \sigma(z,t)\|} \bar{\rho}(t,x) & \text{if } \sigma \neq 0 \\ 0 & \text{if } \sigma = 0 \end{cases}$$

The following figures are results of computer's simulation.

Fig.1 is optimal trajectory of state that is controlled by optimal control input. Even if parameter uncertainty exists in fig.2, we can know that trajectories of states have same dynamic of nominal system such as fig.1. This means that state has same dynamic of nominal system on proposed sliding surface. Fig. 3 is discontinuous control input, Fig.4 is proposed sliding plane. And Fig.5 is trajectory of the virtual state. In fig.6, we can see that adaptation parameter is estimated well than the actual parameter.

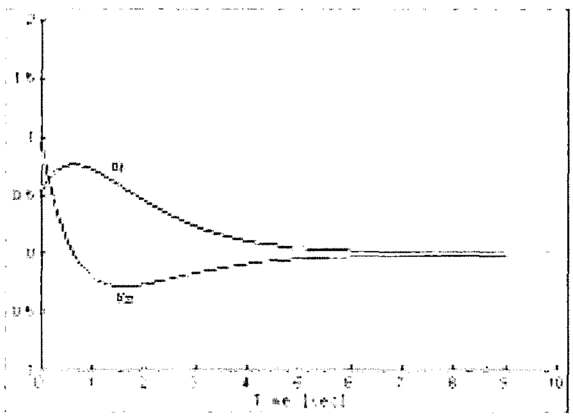

Fig.1 Optimal state trajectory of nominal system

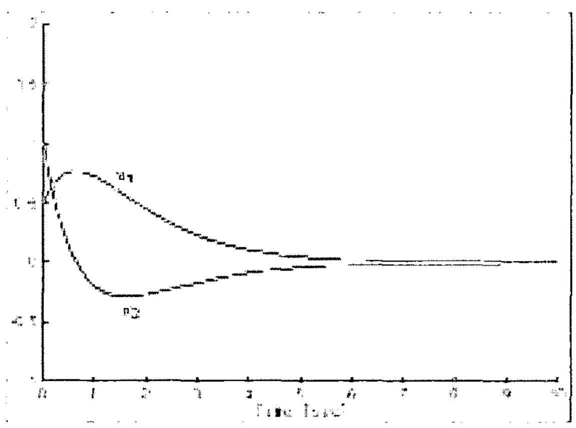

Fig.2 State trajectory of proposed sliding mode control

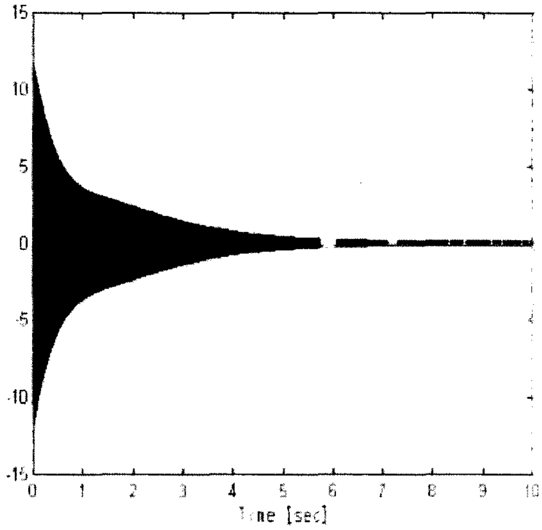

Fig.3 Input of proposed sliding mode control

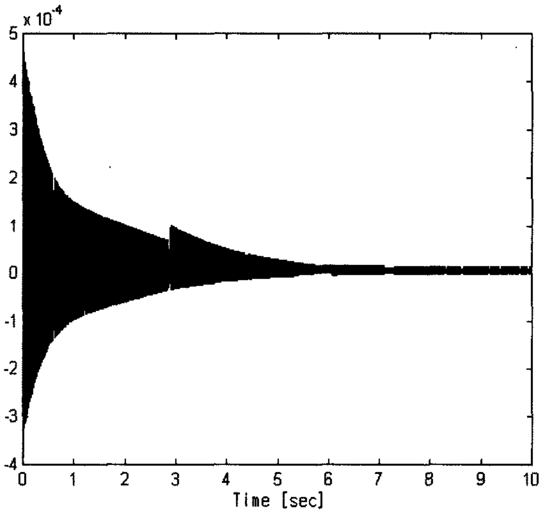

Fig.4 Sliding surface of proposed sliding mode control

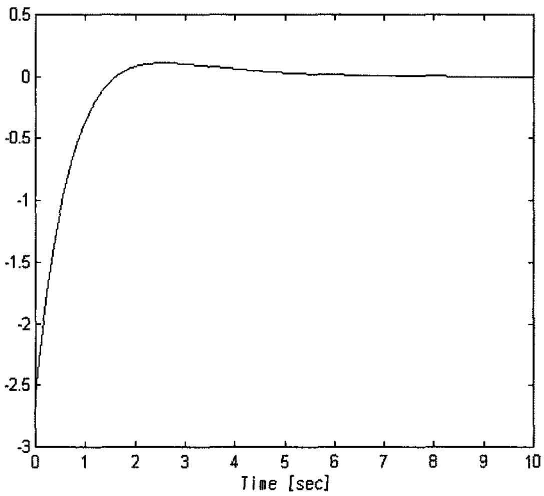

Fig.5 The virtual state trajectory of proposed sliding mode control

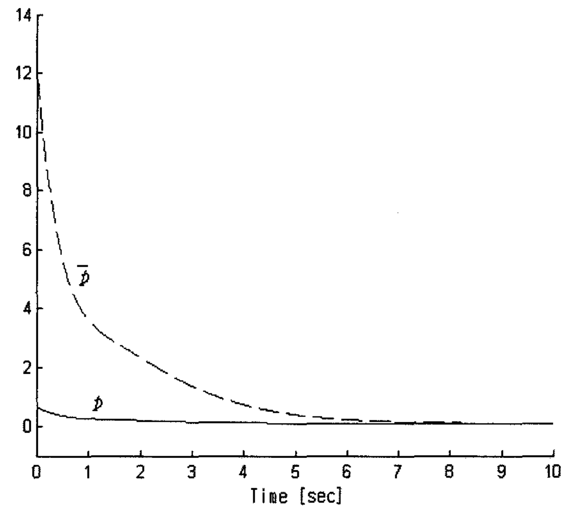

Fig.6 actual parameter($\mathbf{P}$) and adaptation parameter($\bar{\mathbf{P}}$)

V. CONCLUSIONS

When the norm of upper bound of parameter uncertainty existence is not known, it should be estimated by adaptation control technique. The proposed sliding surface that introduces the virtual state can have dynamics which is the same as nominal system. The designed sliding mode controller can have robust performance about parameter uncertainty. Optimal controller is used in the nominal system. By choosing initial value of the virtual state properly, the reaching phase can be removed.

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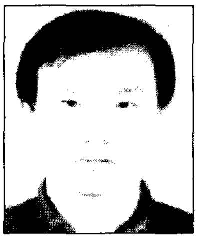

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