# Experimental Verification of a Kinetic Model of Zr-Oxidation

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#### ABSTRACT

It has long been known that the oxidation kinetics of Zr-based alloys undergoes a crossover from parabolic to cubic in the pre-transition period (before breakaway of the oxide scale). This kinetic crossover, however, is not fully understood yet. We have earlier proposed a model for the Zr-oxidation kinetics, in a closed form for the first time, by taking into account a compressive strain energy gradient as a diffusional driving force in addition to a chemical potential gradient of component oxygen across the ZrO<sub>2</sub> scale upon Zr [J. Nucl. Mater., 299 (2001) 235]. In this paper, we experimentally reconfirm the validity of the proposed model by using the thermogravimetric data on mass gain of Zr in a plate and wire form, respectively, in air atmosphere at different temperatures in the range of 500° to 800°C, and subsequently report on the numerical values for oxygen chemical diffusivity and strain energy gradient across the oxide scale.

Key words: Oxidation kinetics, Zr-based alloys, ZrO2, Oxygen chemical diffusivity, Strain energy

### 1. Introduction

National contents of Zr is extremely important to predict the life time of various Zr-based structural components, e.g., pressure tubes and claddings, particularly in nuclear power plants. It has been repeatedly observed that the oxidation kinetics of Zr-based alloys is very initially linear  $(\Delta m \sim t)$ , followed seldom, or briefly if any, by a parabolic kinetics  $(\Delta m \sim t)^{1/2}$ , but eventually turns to a cubic kinetics  $(\Delta m \sim t)^{1/3}$  until a final transition back to a linear kinetics  $(\Delta m \sim t)$  or breakaway of the oxide. Despite many attempts thus far, however, quantitative understanding of the kinetic shift from parabolic to cubic is still far from complete. 1.2)

Yoo et al.<sup>1)</sup> (henceforth referred to as Ref. 1) have earlier proposed a kinetic model in a closed form to explain the subparabolic kinetics by taking into account the compressive strain energy gradient as a diffusional driving force in addition to the omnipresent oxygen potential gradient. The origin of the compressive stress is not yet elucidated unequivocally, but it is highly likely attributed to the metal/oxide lattice mismatch and/or chemical strain due to oxygen nonstoichiometry across the oxide scale.<sup>1,2)</sup> In their treatment,<sup>1)</sup> the authors assumed that this strain energy gradient is constant for the sake of mathematical simplicity and hence, they could establish a rate equation in a closed form. It was subsequently found that the rate equation surprisingly precisely describes the oxidation kinetics of Zr as observed via thermogravimetry in the temperature range of

 $400^\circ$  to  $800^\circ C$  by the authors themselves. Consequently, they could evaluate the mean chemical diffusion coefficient of component oxygen as a function of temperature. Furthermore, assuming the critical thickness  $\xi_C$  of the oxide scale, where its breakaway is about to start, to be the thickness where the compressive stress completely relieves, they could even evaluate the compressive stress at the metal/oxide interface. Nevertheless, the model was tested only for one specific case and hence, its general validity or reproducibility has remained untested.

This paper aims to test and reconfirm the validity of the proposed model with the kinetic data obtained from Zr samples of different sources and different shapes from the earlier study in Ref. 1. We will start by briefly summarizing the kinetic model.

#### 2. Kinetic Model

It is well known<sup>5-9)</sup> that the oxide scale until its breakaway consists of tetragonal t-ZrO $_2$  and monoclinic m-ZrO $_2$  but with gradually decreasing fraction of the former from the Zr/ZrO $_2$  interface even at temperatures where m-ZrO $_2$  would otherwise be thermodynamically stable. This fact already indicates that there must be a compressive stress to stabilize t-ZrO $_2$ , and the existence of a stress gradient has been confirmed experimentally. $^{2,10)}$ 

The strain energy per mole-O in the oxide may be written

$$\mathbf{u} = \frac{\mathbf{V}_{\mathbf{m}} \mathbf{Y}}{2 \cdot 1 - \mathbf{v}} \mathbf{\varepsilon}(\mathbf{x})^2 \tag{1}$$

where  $V_m$ , Y, and v denote the molar volume, Young's modulus, and Poisson's ratio of  $ZrO_2$ , respectively, and  $\epsilon(x)$  strain

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at position x in the reaction product ZrO<sub>2</sub>.

By taking into account the strain energy gradient  $\nabla u$  as a diffusional driving force in addition to the omnipresent oxygen potential gradient  $\nabla \mu_o$  across the oxide scale, one may obtain a kinetic equation for a planar geometry as

$$\frac{d\xi}{dt} = \frac{K_p}{\xi} - \beta \tag{2a}$$

or

$$t = -\frac{\xi}{\beta} - \frac{K_p}{\beta^2} ln \left( 1 - \frac{\beta \xi}{K_p} \right) \tag{2b}$$

with

$$K_{p} = \frac{\overline{D}_{o}}{2RT}(-\Delta G); \quad \beta = \frac{\overline{D}_{o}}{RT}(-\nabla u)$$
 (3)

where  $\xi$  is the thickness of the oxide scale as measured from the  $Zr/ZrO_2$  interface  $(x\!=\!0),\,t$  the length of oxidation duration,  $\overline{D}_0$  the spatial average of the oxygen diffusivity across  $\xi,\,\Delta G$  the reaction free energy of the reaction  $Zr+O_2\!=\!ZrO_2,$  and the rest have their usual significance. It is reminded that in order to reach the integral form, Eq. (2b), the strain energy gradient  $\nabla u$  was assumed to be constant. It is noted that Eq. (2) reduces to the conventional parabolic rate law  $\xi^2\!=\!2K_pt$  as  $\nabla u\!\to\!0$ . For detailed derivation, the reader is referred to Ref. 1.

#### 3. Experimental

In the earlier work, Ref. 1, the specimens were Zr-plates of 99.9% purity, produced by CEZUS, France. In the present work, we chose Zr in a plate form with 1 mm thickness as well as in a wire form with 1 mm diameter, both with 99.2% purity from Alfa Aesar, USA. Each type specimen was oxidized in dry air atmosphere in a thermo-balance (Setaram TG/DTA 92-18, France) at different temperatures in the range of 500° to 800°C while monitoring mass increase  $\Delta m$  with time t with a resolution of  $\pm 1~\mu g$ . For experimental details, the reader is referred to Ref. 1.

#### 4. Results and Discussions

The results from the plate specimens are as illustrated in

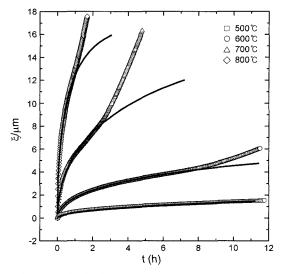


Fig. 1. Oxide scale thickness vs. time of Zr-plate specimens at different temperatures. The solid lines are the best-fitted to Eq. (2b) in the text.

Fig. 1, where the mass gain vs. time was translated into the oxide thickness  $\xi$  vs. t, assuming that in-diffusing oxygen atoms are all consumed for the oxidation of underlying Zr or in other words, neglecting the dissolution of oxygen into the base metal. (The correction against latter may be rather trivial at the steady state). As is seen, the oxide scales eventually exhibit a breakaway by turning back to the linear kinetics. (For the 500°C data shown, the breakaway is not shown in the figure because it occurs far outside the time scale shown in the figure). The kinetic behavior appears to be in excellent agreement with what was observed earlier in Ref. 1.

The experimental data of  $\xi$  vs. t has been non-linear-least-squares fitted to Eq. (2b). As depicted by the solid lines in Fig. 1, Eq. (2b) describes precisely enough the kinetics until breakaway at all temperatures with the fitting parameters  $K_p$  and  $\beta$  evaluated as listed in the  $2^{nd}$  and  $3^{rd}$  row, respectively, in Table 1. It is emphasized that the fitting errors are no greater than 2% for  $K_p$  and 6% for  $\beta$  even in the worst case (700°C).

By using the numerical values for the reaction free energy  $\Delta G^{13}$  (4<sup>th</sup> row of Table 1), one can evaluate, via Eq. (3), the

Table 1. All the Numerical Values as Evaluated from the Oxidation Kinetics of Zr-Plates

Temperature (°C)	500	600	700	800
$K_{p}/m^{2}s^{-1}$	$(3.485\pm0.007)\times10^{-17}$	$(5.368\pm0.008)\times10^{-16}$	$(5.48\pm0.09)\times10^{-15}$	$(4.65\pm0.03)\times10^{-14}$
$\beta/\text{ms}^{-1}$	$(7.56 \pm 0.05) \times 10^{-12}$	$(7.59\pm0.03)\times10^{-11}$	$(3.07\pm0.18)\times10^{-10}$	$(2.71\pm0.04)\times10^{-9}$
$\nabla \mu_{O_2}/J$	$-9.40 \times 10^{5}$	$-9.20 \times 10^{5}$	$-9.00 \times 10^{5}$	$-8.81\times10^{5}$
$\overline{\mathrm{D}}_{\mathrm{o}}/\mathrm{m}^{2}\mathrm{s}^{-1}$	$(4.77 \pm 0.01) \times 10^{-19}$	$(8.470\pm0.013)\times10^{-18}$	$(9.85 \pm 0.16) \times 10^{-16}$	$(9.42 \pm 0.06) \times 10^{-16}$
$\nabla u/Jm^{-1}mol\cdot O^{-1}$	$(1.020\pm0.007)\times10^{11}$	$(6.50\pm0.03)\times10^{10}$	$(2.52\pm0.15)\times10^{10}$	$(2.57 \pm 0.04) \times 10^{10}$
$\xi_{\rm C}/{ m m}$	$2.48 \times 10^{-6}$	$4.12 \times 10^{-6}$	$7.79\times10^{-6}$	$1.20\times10^{-5}$
$u(0)/Jmol \cdot O^{-1}$	$(2.530\pm0.017)\times10^{5}$	$(2.682 \pm 0.011) \times 10^{5}$	$(1.96\pm0.12)\times10^{5}$	$(3.08\pm0.05)\times10^5$

mean oxygen diffusivity  $\overline{D}_o$ .

The strain energy gradient, that has been assumed to be constant, is also evaluated from the values for  $\beta$  due to Eq. (3). The critical thickness  $\xi_C$ , corresponding to the position where  $\epsilon$ =0, may also be assumed to be the thickness just before the breakaway, and hence, estimated to be the position where the extensions of the fitted Eq. (2) and final linear kinetics (after breakaway) in Fig. 1 meet each other. Subsequently, the strain energy at the  $Zr/ZrO_2$  interface u(0) is evaluated as u(0) =  $\nabla u \cdot \xi_C$ . All the numerical values thus evaluated are summarized in Table 1.

On the other hand, the oxidation kinetics,  $\xi$  vs. t of the wire specimens are as shown in Fig. 2. As the critical thickness of oxide scale  $\xi_C$  (or the thickness up to the beginning of breakaway) is of the order of 20  $\mu$ m at the highest temperature of examination, but normally much less, one may approximate the cylindrical geometry to be the planar geometry with the maximum relative error of  $\xi_C/d=0.02$  (d=1 mm, the initial diameter of the wire specimen) in the worst case of 800°C. We have, thus, non-linear-least-squares-fitted the kinetic data in Fig. 2 to Eq. (2b). The results are depicted by solid lines in Fig. 2. As is clearly

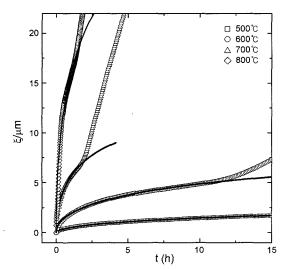


Fig. 2. Oxide scale thickness vs. time of Zr-wire specimens at different temperatures. The solid lines are the best-fitted to Eq. (2b) in the text.

seen, Eq. (2) again elegantly describes all the kinetics.

The fitting parameters  $K_p$  and  $\beta$  in Eq. (2b) have been evaluated as listed in Table 2. As expected, the results at 800°C are subjected to the highest uncertainty, but the error bounds are still tolerable (2% and 5%, respectively). Following the same procedures as in the Zr-plate case (Table 1),  $\overline{D}_o$ ,  $\nabla u$ , and u(0) are subsequently evaluated and listed in Table 2.

In the present work, the parabolic rate law constant  $K_p$  has turned out to take a value in the range of  $10^{-17}$  to  $10^{-14}$  m<sup>2</sup>/s and  $\beta$  in the range of  $10^{-11}$  to  $10^{-9}$  m/s depending on temperature for both plate and wire specimens. These values are quite close to those in Ref. 1 as expected.

The mean diffusivities of component oxygen are plotted against reciprocal temperature in Fig. 3 and best estimated as.

$$\overline{D}_{o}/m^{2}s^{-1} = (2.3^{+2.3}_{-1.2}) \times 10^{-7} \exp\left(-\frac{1.80 \pm 0.05 \text{ eV}}{\text{kT}}\right)$$
(4)

for the plate specimens, and

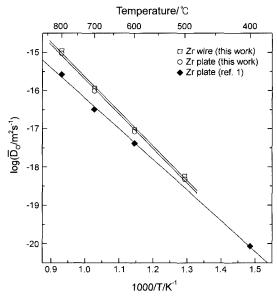


Fig. 3. Mean oxygen diffusivity vs. reciprocal temperature as obtained from Zr in plate forms and in wire forms, respectively.

Table 2. All the Numerical Values as Evaluated from the Oxidation Kinetics of Zr-Wires

Temperature (°C)	500	600	700	800
$K_{p}/m^{2}s^{-1}$	$(4.184 \pm 0.002) \times 10^{-17}$	$(5.971\pm0.019)\times10^{-16}$	$(6.53\pm0.06)\times10^{-15}$	$(5.48\pm0.11)\times10^{-14}$
β/ms <sup>-1</sup>	$(1.1801 \pm 0.0017) \times 10^{-11}$	$(7.57\pm0.05)\times10^{-11}$	$(5.66\pm0.12)\times10^{-10}$	$(1.79\pm0.09)\times10^{-9}$
$ abla \mu_{\mathrm{O}_2}$ /J	$-9.40 \times 10^5$	$-9.20 \times 10^{5}$	$-9.00 \times 10^5$	$-8.81 \times 10^5$
$\overline{\mathrm{D}}_{\mathrm{o}}$ / $\mathrm{m}^{2}\mathrm{s}^{-1}$	$(5.721 \pm 0.003) \times 10^{-19}$	$(9.42\pm0.03)\times10^{-18}$	$(1.174\pm0.011)\times10^{-16}$	$(1.11\pm0.02)\times10^{-15}$
$\nabla u/Jm^{-1}mol\cdot O^{-1}$	$(1.326 \pm 0.002) \times 10^{11}$	$(5.83\pm0.04)\times10^{10}$	$(3.90\pm0.09)\times10^{10}$	$(1.44\pm0.08)\times10^{10}$
$\xi_{\rm C}/{ m m}$	-	$5.12 \times 10^{-6}$	$6.93 \times 10^{-6}$	$1.86 \times 10^{-5}$
$u(0)/Jmol\cdot O^{-1}$	-	$(2.99\pm0.02)\times10^5$	$(2.70\pm0.06)\times10^{5}$	$(2.68\pm0.14)\times10^{5}$

$$\overline{D}_{o}/m^{2}s^{-1} = (2.7^{+3.3}_{-1.5}) \times 10^{-7} \exp\left(-\frac{1.80 \pm 0.06 \text{ eV}}{kT}\right)$$
 (5)

for the wire specimens. They are in excellent agreement with each other.

In the earlier work (Ref. 1) with a Zr-plate of different origin, the same diffusivity was estimated as

$$\overline{D}_{o}/m^{2}s^{-1} = (6.6^{+4.3}_{-2.6}) \times 10^{-9} \exp\left(-\frac{1.59 \pm 0.04 \text{ eV}}{kT}\right). \tag{6}$$

This is somewhat smaller than the present values, Eqs. (4) and (5), as compared in Fig. 3, probably reflecting a different quality of the Zr specimens in terms of impurity types and contents.

It was earlier found, from the open circuit voltage as measured across the growing oxide scale in Ref. 1, that the growing oxide is a mixed ionic-electronic conductor with a mean electronic transference number of ca. 0.5. This fact indicates that the diffusion coefficient of component oxygen must be of the Nernst-Planck type or

$$\overline{D}_{o} = \frac{RTV_{m}}{8F^{2}} \cdot \frac{\sigma_{i}\sigma_{e}}{\sigma_{i} + \sigma_{e}} \tag{7}$$

where  $\sigma_i$  and  $\sigma_e$  stand for the ionic conductivity by  $O^{2-}$  ions and electronic conductivity, respectively, of the oxide scale that is polycrystalline with gradually decreasing fraction of t-ZrO<sub>2</sub> with increasing thickness.

The strain energy at the Zr/ZrO<sub>2</sub> interface u(0) in the present case has been found to be of the order of magnitude of 10<sup>5</sup> J/mol-O, that is the same as in Ref. 1. When taking the Young's modulus of the oxide as that of sintered yttriastabilized zirconia<sup>14)</sup> or Y=200 GPa, and v=0.3, and assuming the oxide breakaway to start as soon as the compressive strain fully relaxes to nill at  $\xi = \xi_C$ , the present strain energy values again lead, due to Eq. (1), to a compressive stress of the order of magnitude of 60 GPa as in Ref. 1. According to a recent measurement,10) the compressive stress at the Zr/ ZrO<sub>2</sub> interface increases with increasing thickness of the oxide scale and takes a value of 1~1.5 GPa when  $\xi = \sim 2 \mu m$ . In this light, the stress evaluated in the present work is obviously too much overestimated and it may be attributed to the overestimation of the critical thickness of the oxide scale  $\xi_c$ . For the origins of the stress and more discussions, the reader is referred to Ref. 1.

## 5. Conclusion

We, thus, conclude that the validity of the proposed kinetic model of Zr-oxidation, Eq. (2) is reconfirmed. It is further supported that the sub-parabolic or cubic kinetics is attributed to the compressive strain energy gradient that counteracts the oxygen potential gradient as the driving forces for oxygen chemical diffusion. The mean chemical diffusion coefficient of oxygen across the oxide scale is on the order of magnitude of  $10^{-19}$  to  $10^{-15}$  m<sup>2</sup>/s depending on temperature with an activation energy of  $1.6-1.8 \, \mathrm{eV}$  in the range of  $500^{\circ}$  to  $800^{\circ}$ C. The strain energy gradient, which is

assumed to be constant in the present model, takes a value in the range of  $10^{11}$  to  $10^{10}$  J/m/mol-O, decreasing with increasing temperature.

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