Systematic Isotropy Analysis of Caster Wheeled Mobile Robot with Steering Link Offset Different from Wheel Radius

Sung-bok Kim*

Abstract

This paper presents the systematic isotropy analysis of a fully actuated caster wheeled omnidirectional mobile robot (COMR) with the steering link offset different from the wheel radius, which can be considered as the generalization of the previous analysis. First, with the characteristic length introduced, the kinematic model of a COMR is obtained based on the orthogonal decomposition of the wheel velocities. Second, the necessary and sufficient conditions for the isotropy of a COMR are derived and examined to categorize three different groups, each of which can be dealt with in a similar way. Third, for each group, the isotropy conditions are further explored so as to identify four different sets of all possible isotropic configurations. Fourth, for each set, the expressions of the isotropic characteristic length required for the isotropy of a COMR are elaborated.

Keywords : Caster wheeled mobile robot, Steering link offset, Wheel radius, Isotropy analysis, Isotropic configuration, Isotropic characteristic length

I. Introduction

There are a variety of mobile robot systems having different structures to provide the mobility: wheeled, legged, wheel-leg hybrid, tracked, and so on [1]. Among them, wheels are widely accepted as a practical means due to the simplicity in design and control, specially for indoor applications. When a mobile robot is requested to navigate in a restricted space cluttered with obstacles, the omnidirectional mobility becomes a must. Several omnidirectional wheel mechanisms have been proposed, including universal wheels, Swedish wheels, orthogonal wheels, ball wheels, and so on [2]. Caster wheels were successfully employed to develop an omnidirectional mobile robot, which was later commercialized Nomadic Technologies XR4000 [3]. Since caster wheels operate without additional peripheral rollers or support structure, a caster wheeled omnidirectional mobile robot (COMR) can maintain good performance even though payload or ground condition changes.

There have been several works on the kinematics of a COMR. For a general form of wheeled mobile robots, a systematic procedure for kinematic modeling was presented [4,5]. Regarding the minimal actuation set, it was shown that at least four joints out of two caster wheels should be actuated to avoid the singularity [6]. For a COMR under partial and full actuation, the isotropy analysis was made to identify all possible isotropic configurations [7]. The isotropy index of a COMR was defined and examined to determine the optimal design parameters [8,9]. On the other hand, for an omnidirectional mobile robot employing Swedish wheels, the isotropy analysis was made but the results are incomplete and need further elaboration [10].

With nonzero steering link offset, the omnidirectional mobility of a COMR is guaranteed independently of a given wheel configuration. In contrast, the singularity and the isotropy of a COMR may happen depending on the current instance of the wheel configurations. At singular configurations, a COMR becomes instantaneously movable even when all the actuated joints are locked [11]. On the other hand, at isotropic configurations, the velocity transmission ratios from the joint to the task spaces becomes uniform in all directions [12]. Obviously, it is desirable for robust motion control to keep a COMR away from the singularity but close to the isotropy, as much as possible [10].

Previous isotropy analysis was made only for a COMR in which the steering link offset is equal to the wheel radius [7-9]. In fact, the restriction of the steering link offset equal to the wheel radius constitutes a necessary condition for globally optimal isotropic characteristics of a COMR. Nevertheless, many practical COMR's in use take advantage of the steering link offset which is different from the wheel radius, mainly because of the

* School of Electronics and Information Eng.
University of Foreign Studies
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tipover stability. The tipover stability measure was proposed [13, 14], which takes into account the height of the center of gravity (COG) above the ground as well as the distance from the projected COG to the support pattern boundary. For a given wheel radius, the relative scale of the steering link offset should be determined to achieve a required level of the tipover stability. The purpose of this paper is to present a systematic isotropy analysis of a fully actuated COMR with the steering link offset different from the wheel radius. The key to the systematic analysis is to incorporate the ratio of the steering link length to the wheel radius into the isotropy conditions. This paper is organized as follows. With the characteristic length introduced [10], Section II develops the kinematic model based on the orthogonal decomposition of the wheel velocities. Sections III derives and examines the necessary and sufficient conditions for the isotropy to categorize three different groups, each of which can be treated in a similar way. For each group, Section IV explores the isotropy conditions so as to identify four different sets of all possible isotropic configurations. For each set, Section V elaborates the expressions of the isotropic characteristic length required for the isotropy of a COMR. Finally, the conclusion and some discussions are made in Section VI.

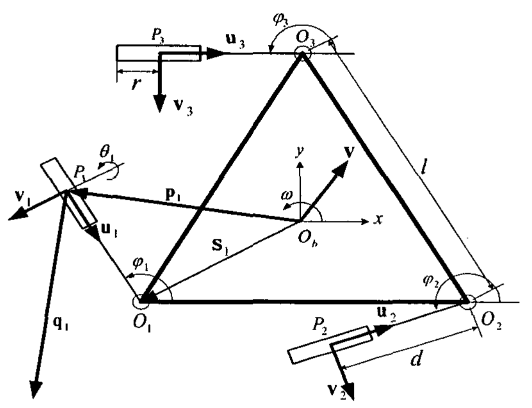

Fig. 1. A caster wheeled omnidirectional mobile robot.

II. Kinematic Model

Consider a COMR with three identical caster wheels of attached to a regular triangular platform moving on the xy-plane, as shown in Fig. 1. For each wheel, it is assumed that the steering link offset, d (>0), can be different from the wheel radius, r (>0), that is, $d \neq r$. Note that with $d=0$, a caster wheel reduces to a conventional wheel. In Fig. 1, let l be the side length of the platform with the center denoted by O_b, and the vertices denoted by O_i, $i=1,2,3$. Without loss of generality, the side length is assumed to be unity, that is, $l=1.0$ [m]. For the i th caster wheel with the center denoted by P_i, $i=1,2,3$, we define the following. Let d and r be the offset of the steering link and the radius of the wheel, respectively. Let ϕ_i and θ_i be the steering and the rotating angles of the caster wheel, respectively.

Let $\mathbf{u}_i$ and $\mathbf{v}_i$, $i=1,2,3$, be two orthogonal unit vectors along the steering link and the wheel axis, respectively:

$$\mathbf{u}_i = \begin{bmatrix} -\cos\varphi_i \\ -\sin\varphi_i \end{bmatrix}, \mathbf{v}_i = \begin{bmatrix} -\sin\varphi_i \\ \cos\varphi_i \end{bmatrix} = \mathbf{R}\mathbf{u}_i \tag{1}$$

where $\mathbf{R}=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Let $\mathbf{s}_i$, $i=1,2,3$ be the vector from O_b to O_i:

$$\mathbf{s}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}, \mathbf{s}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}, \mathbf{s}_3 = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

Let $\mathbf{p}_i$, $i=1,2,3$ be the vector from O_b to P_i, and $\mathbf{q}_i$ be the rotation of $\mathbf{p}_i$ by 90^o counterclockwise:

$$\mathbf{p}_i = \mathbf{s}_i - d\,\mathbf{u}_i, \quad \mathbf{q}_i = \mathbf{R}\,\mathbf{p}_i \tag{3}$$

Note that

$$\begin{aligned} \sum_1^3 \mathbf{u}_i = 0 \;&\Leftrightarrow\; \sum_1^3 \mathbf{v}_i = 0 \\ &\quad\;\; \sum_1^3 \mathbf{s}_i = 0 \\ \sum_1^3 \mathbf{p}_i = 0 \;&\Leftrightarrow\; \sum_1^3 \mathbf{q}_i = 0 \end{aligned} \tag{4}$$

Let $\mathbf{v}$ and ω be the linear and the angular velocities at O_b of the platform, respectively. The kinematic model of a COMR can be obtained based on the decomposition of the wheel velocities into two orthogonal components of the rotating and the steering joints [7]. With the introduction of the characteristic length, L [10], the kinematics of a COMR under full actuation is obtained by

$$\mathbf{A}\,\dot{\mathbf{x}} = \mathbf{B}\,\dot{\Theta} \tag{5}$$

where $\dot{\mathbf{x}} = [\,\mathbf{v}\;L\omega\,]^t \in \mathbf{R}^{3\times1}$ is the task velocity vector, and $\dot{\Theta} = [\dot{\theta}_1\;\dot{\theta}_2\;\dot{\theta}_3\;\dot{\phi}_1\;\dot{\phi}_2\;\dot{\phi}_3] \in \mathbf{R}^{6\times1}$ is the joint velocity vector, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{u}_1^t & \frac{1}{L} & \mathbf{u}_1^t\mathbf{q}_1 \\ \mathbf{u}_2^t & \frac{1}{L} & \mathbf{u}_2^t\mathbf{q}_2 \\ \mathbf{u}_3^t & \frac{1}{L} & \mathbf{u}_3^t\mathbf{q}_3 \\ \mathbf{v}_1^t & \frac{1}{L} & \mathbf{v}_1^t\mathbf{q}_1 \\ \mathbf{v}_2^t & \frac{1}{L} & \mathbf{v}_2^t\mathbf{q}_2 \\ \mathbf{v}_3^t & \frac{1}{L} & \mathbf{v}_3^t\mathbf{q}_3 \end{bmatrix} \in R^{6\times 3} \quad (6)$$

$$\mathbf{B} = \begin{bmatrix} r\,\mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & d\,\mathbf{I}_3 \end{bmatrix} \in R^{6\times 6} \quad (7)$$

are the Jacobian matrices. Notice that the introduction of L makes all three columns of $\mathbf{A}$ to be consistent in physical unit. Notice that By decomposing the instantaneous wheels motions into two orthogonal components of the rotating and the steering joints. Using (1) and (3), the expressions of $\mathbf{u}_i^t\mathbf{q}_i$ and $\mathbf{v}_i^t\mathbf{q}_i$, $i=1,2,3$, can be written as

$$\begin{aligned} \mathbf{u}_i^t\mathbf{q}_i &= \mathbf{v}_i^t\mathbf{p}_i = \mathbf{v}_i^t\mathbf{s}_i \\ \mathbf{v}_i^t\mathbf{q}_i &= -\mathbf{u}_i^t\mathbf{p}_i = -\mathbf{u}_i^t\mathbf{s}_i + d \end{aligned} \quad (8)$$

III. Three Isotropy Conditions

Based on (5), the necessary and sufficient condition for the isotropy of a COMR can be expressed as

$$\mathbf{Z}^t\mathbf{Z} = \sigma\,\mathbf{I}_3 \quad (9)$$

where

$$\mathbf{Z} = \mathbf{B}^{-1}\mathbf{A} \quad (10)$$

$$\sigma = \frac{3}{2}\left(\frac{1}{r^2} + \frac{1}{d^2}\right) \quad (11)$$

Note that previous isotropy analysis was made under the assumption that the steering link offset d equal to the wheel radius r, so that $\mathbf{B}$ becomes a multiple of $\mathbf{I}_6$.

Plugging (6) and (7) into (9), we obtain the following three isotropy conditions of a COMR:

$$\sum_{i=1}^{3}[\mu(\mathbf{u}_i\mathbf{u}_i^t) + (\mathbf{v}_i\mathbf{v}_i^t)] = \frac{3}{2}(\mu+1)\,\mathbf{I}_2 \quad (12)$$

$$\sum_{i=1}^{3}[\mu(\mathbf{u}_i^t\mathbf{q}_i)\mathbf{u}_i + (\mathbf{v}_i^t\mathbf{q}_i)\mathbf{v}_i] = 0 \quad (13)$$

$$\frac{1}{L^2}\sum_{i=1}^{3}[\mu(\mathbf{u}_i^t\mathbf{q}_i)^2 + (\mathbf{v}_i^t\mathbf{q}_i)^2] = \frac{3}{2}(\mu+1) \quad (14)$$

where

$$\mu = \left(\frac{d}{r}\right)^2 > 0 \quad (15)$$

which represents the square of the ratio of the steering link offset d to the wheel radius r. It should be mentioned that the incorporation of μ into the isotropy conditions is the key to the systematic isotropy analysis of a COMR with the steering link offset different from the wheel radius.

In general, the first and the second conditions, given by (12) and (13), are a function of the steering joint angles, ϕ_i, $i=1,2,3$, from which the isotropic configurations can be identified. With ϕ_i, $i=1,2,3$, known, the third condition, given by (14), determines the specific value of L, which is required for the isotropy of a COMR, called the *isotropic characteristic length*, L_{iso}.

Table 1. Three groups of ($\hat{\phi}_2$, $\hat{\phi}_3$).

Group I	Group II	Group III
$\mu=1$	$(-\frac{\pi}{3}, \frac{\pi}{3})$	$(\frac{\pi}{3}, -\frac{\pi}{3})$
	$(-\frac{\pi}{3}, -\frac{2\pi}{3})$	$(-\frac{2\pi}{3}, -\frac{\pi}{3})$
	$(\frac{2\pi}{3}, \frac{\pi}{3})$	$(\frac{\pi}{3}, \frac{2\pi}{3})$
	$(\frac{2\pi}{3}, -\frac{2\pi}{3})$	$(-\frac{2\pi}{3}, \frac{2\pi}{3})$

Using (1), the first isotropy condition (12) can be written as

$$\begin{aligned} (\mu-1)(1+\cos 2\hat{\phi}_2 + \cos 2\hat{\phi}_3) &= 0 \\ (\mu-1)(\sin 2\hat{\phi}_2 + \sin 2\hat{\phi}_3) &= 0 \end{aligned} \quad (16)$$

where $\hat{\phi}_2 = \phi_2 - \phi_1$ and $\hat{\phi}_3 = \phi_3 - \phi_1$. There are three different groups of the solutions to (16): $\mu=1$ and two groups of ($\hat{\phi}_2$, $\hat{\phi}_3$), as listed in Table 1.

Using (8), the second isotropy condition (13) can be written as

$$(\mu-1)\sum_{i=1}^{3}(\mathbf{v}_i^t\mathbf{s}_i)\,\mathbf{v}_i - d\sum_{i=1}^{3}\mathbf{u}_i = \mathbf{0} \quad (17)$$

In the next section, for three different groups satisfying (16), (17) will be further explored to identify all possible isotropic configurations of a COMR.

IV. Isotropic Configurations

A. Isotropy analysis for Group I

With $\mu=1$, (17) reduces to

$$\sum_{1}^{3}\mathbf{u}_i = \mathbf{0} \Leftrightarrow \begin{aligned} c_1 + c_2 + c_3 &= 0 \\ s_1 + s_2 + s_3 &= 0 \end{aligned} \quad (18)$$

where $c_i = \cos(\phi_i)$ and $s_i = \sin(\phi_i)$, $i=1,2,3$. Solving (18) leads to

$$\begin{aligned} \phi_2 &= \phi_1 + \frac{2\pi}{3}, \quad \phi_3 = \phi_1 - \frac{2\pi}{3} \\ \phi_2 &= \phi_1 - \frac{2\pi}{3}, \quad \phi_3 = \phi_1 + \frac{2\pi}{3} \end{aligned} \quad (19)$$

(19) tells that there are two sets of infinitely many isotropic configurations: $(\phi_1, \phi_1+\frac{2\pi}{3}, \phi_1-\frac{2\pi}{3})$ and $(\phi_1, \phi_1-\frac{2\pi}{3}, \phi_1+\frac{2\pi}{3})$. Note that $\mu=1$ corresponds to the case of the steering link offset d equal to the wheel radius r [7].

B. Isotropy analysis for Group II

First, consider the case of $(\hat{\phi}_2, \hat{\phi}_3) = (-\frac{\pi}{3}, \frac{\pi}{3})$, for which

$$\begin{aligned} \mathbf{v}_2^t \mathbf{s}_2 = \mathbf{v}_3^t \mathbf{s}_3 &= -\mathbf{v}_1^t \mathbf{s}_1 \\ \mathbf{v}_1 - \mathbf{v}_2 - \mathbf{v}_3 &= \mathbf{0} \end{aligned} \tag{20}$$

With (20), (17) reduces to (18), which cannot be satisfied, implying that there does not exist any isotropic configuration. Similar analysis can be made in the cases of $(\hat{\phi}_2, \hat{\phi}_3) = (-\frac{\pi}{3}, -\frac{2\pi}{3})$ and $(\frac{2\pi}{3}, \frac{\pi}{3})$, for which no isotropic configuration exists. Next, consider the case of $(\hat{\phi}_2, \hat{\phi}_3) = (\frac{2\pi}{3}, -\frac{2\pi}{3})$, for which

$$\begin{aligned} \mathbf{v}_1^t \mathbf{s}_1 = \mathbf{v}_2^t \mathbf{s}_2 &= \mathbf{v}_3^t \mathbf{s}_3 \\ \sum_{i=1}^{3} \mathbf{u}_i = \sum_{i=1}^{3} \mathbf{v}_i &= \mathbf{0} \end{aligned} \tag{21}$$

With (21), (17) is always satisfied independently of the values of the steering link offset d and the ratio μ. This implies that there exist a set of infinitely many isotropic configurations, $(\phi_1, \phi_1+\frac{2\pi}{3}, \phi_1-\frac{2\pi}{3})$.

C. Isotropy analysis for Group III

First, consider the case of $(\hat{\phi}_2, \hat{\phi}_3) = (\frac{\pi}{3}, -\frac{\pi}{3})$, for which (17) becomes

$$(\mu-1)\begin{bmatrix} \frac{3}{4}c_1^2+\frac{\sqrt{3}}{2}c_1s_1-\frac{3}{4}s_1^2 \\ -\frac{\sqrt{3}}{4}c_1^2+\frac{3}{2}c_1s_1+\frac{\sqrt{3}}{4}s_1^2 \end{bmatrix} - d\begin{bmatrix} -2c_1 \\ -2s_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{22}$$

For the existence of the solution to (22), it should hold that

$$c_1 - \sqrt{3}s_1 = 0 \tag{23}$$

which yields

$$\phi_1 = \frac{\pi}{6}, -\frac{5\pi}{6} \tag{24}$$

Plugging (24) into (22), we have

$$d = \frac{1}{2}(1-\mu)c_1 > 0 \tag{25}$$

From (24) and (25), it follows that

$$\begin{aligned} \phi_1 &= \frac{\pi}{6}, \quad d=\frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ \phi_1 &= -\frac{5\pi}{6}, \quad d=\frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{26}$$

(26) tells that there exists only a single isotropic configuration, $(\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6})$ or $(-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{5\pi}{6})$, depending on the value of the ratio μ relative to unity, which requires the specific value of the steering link offset d as a function of μ.

Similar analysis to the above can be made in the cases of $(\hat{\phi}_2, \hat{\phi}_3) = (-\frac{2\pi}{3}, -\frac{\pi}{3})$ and $(\frac{\pi}{3}, \frac{2\pi}{3})$: For the case of $(\hat{\phi}_2, \hat{\phi}_3) = (-\frac{2\pi}{3}, -\frac{\pi}{3})$, we obtain

$$\begin{aligned} \phi_1 &= -\frac{\pi}{6}, \quad d=\frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ \phi_1 &= \frac{5\pi}{6}, \quad d=\frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{27}$$

which results in a single isotropic configuration, given by $(-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2})$ or $(\frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2})$. And, for the case of $(\hat{\phi}_2, \hat{\phi}_3) = (\frac{\pi}{3}, \frac{2\pi}{3})$, we obtain

$$\begin{aligned} \phi_1 &= \frac{\pi}{2}, \quad d=\frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ \phi_1 &= -\frac{\pi}{2}, \quad d=\frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{28}$$

which results in a single isotropic configuration, given by $(\frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6})$ or $(-\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6})$.

Finally, consider the case of $(\hat{\phi}_2, \hat{\phi}_3) = (-\frac{2\pi}{3}, \frac{2\pi}{3})$, for which (17) becomes

$$(\mu-1)\begin{bmatrix} \frac{3}{4}c_1^2+\frac{\sqrt{3}}{2}c_1s_1-\frac{3}{4}s_1^2 \\ -\frac{\sqrt{3}}{4}c_1^2+\frac{3}{2}c_1s_1+\frac{\sqrt{3}}{4}s_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{29}$$

There does not exist ϕ_1 satisfying (27) unless $\mu=1$, which implies that there is no isotropic configuration.

D. Summary

The existence of the isotropic configurations of a COMR is dependent on the relationship of the ratio μ and the steering link offset d:

$$\mu = 1 \tag{30}$$

$$d=\frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \tag{31}$$

$$d=\frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \tag{32}$$

Since μ is an auxiliary parameter introduced to expedite the systematic isotropy analysis, it is better to cast the relationships of μ and d into the relationships of the wheel radius r and d.

First, from (15) and (30), it is trivial that

$$d_{iso} = r \tag{33}$$

Next, plugging (15) into (31), we have

$$\sqrt{3}d^2 + 4r^2 d - \sqrt{3}r^2 = 0 \tag{34}$$

thus it follows that

$$d_{iso} = \sqrt{r^2 + \left(\frac{2}{\sqrt{3}}r^2\right)^2} - \frac{2}{\sqrt{3}}r^2 < r \tag{35}$$

subject to $0 < d_{iso} < \frac{\sqrt{3}}{4}$. Similarly, it can be shown that (32) is equivalent to

$$d_{iso} = \sqrt{r^2 + \left(\frac{2}{\sqrt{3}}r^2\right)^2} + \frac{2}{\sqrt{3}}r^2 > r \tag{36}$$

subject to $d_{iso} > 0$. For a given wheel radius r, the specific value of the steering link offset d, which is required for the isotropy of a COMR, is called the *isotropic steering link offset*, d_{iso}.

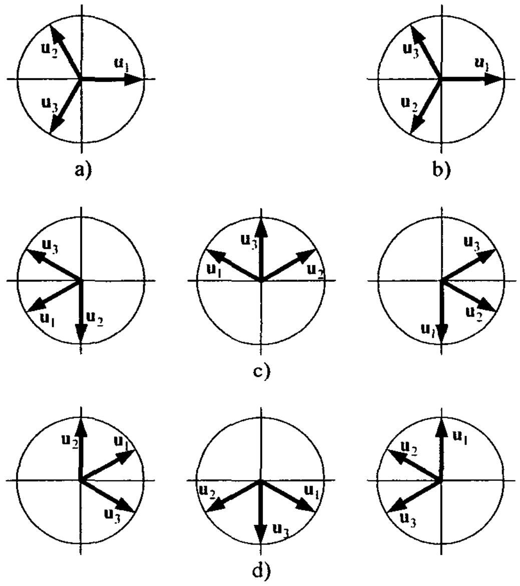

Fig. 3. Four different sets of the isotropic configurations: a) S1 with $\phi_1 = 0$, b) S2 with $\phi_1 = 0$, c) S3 and d) S4.

Summarizing the results obtained so far, all possible isotropic configurations of a COMR can be categorized into four different sets according to the relationships of the wheel radius r and the isotropic steering link offset d_{iso}. Attached at the end of this paper, Table 2 lists four different sets of the isotropic configurations, denoted by S1, S2, S3, and S4, and the corresponding value of d_{iso}. It should be noticed that S1 places no restriction on d_{iso} unlike the other three sets, S2, S3, and S4. Fig. 3 illustrates four different sets of the isotropic configurations, which are characterized by the tuple of ($\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$). It is interesting to observe that there exist certain geometrical symmetries among four sets: the symmetry between S1 and S2, shown in Fig. 3a) and 3b), and the symmetry between S3 and S4, shown in Fig. 3c) and 3d).

V. Isotropic Characteristic Lengths

Once the isotropic configuration has been identified under the constraints of (12) and (13), the isotropic characteristic length L_{iso} can be determined under the constraint of (14):

$$L_{iso} = \sqrt{\frac{2}{3}\frac{\sum_{i=1}^{3}\left[\mu(\mathbf{v}_i^t\mathbf{p}_i)^2 + (\mathbf{u}_i^t\mathbf{p}_i)^2\right]}{\mu+1}} \tag{37}$$

The expression of (37) can be further elaborated for four different sets of the isotropic configurations listed in Table 2. Note that the isotropy of a COMR cannot be achieved unless the characteristic length is chosen as the isotropic characteristic length, that is, $L = L_{iso}$.

First, in the case of S1 where the isotropic configurations are given by $(\phi_1, \phi_1 + \frac{2\pi}{3}, \phi_1 - \frac{2\pi}{3})$, it can be shown that (37) becomes

$$L_{iso} = \sqrt{\frac{2}{\mu+1}\left\{\left(\frac{1}{\sqrt{3}}\sin(\phi_1 - \frac{\pi}{6})\right)^2\mu + \left(\frac{1}{\sqrt{3}}\cos(\phi_1 - \frac{\pi}{6}) - d\right)^2\right\}} \tag{38}$$

Geometrically, (38) represents a kind of weighted norm of the vector $\mathbf{p}_1$, as illustrated in Fig. 4a). Especially when $\mu = 1$, (38) reduces to

$$L_{iso} = \sqrt{d^2 - \frac{2}{\sqrt{3}}\cos(\phi_1 - \frac{\pi}{6})d + \left(\frac{1}{\sqrt{3}}\right)^2} \tag{39}$$

which is simply the Euclidean norm of $\mathbf{p}_1$. Note that

$$L_{iso} = (\|\mathbf{p}_1\| = \|\mathbf{p}_2\| = \|\mathbf{p}_3\|) \tag{40}$$

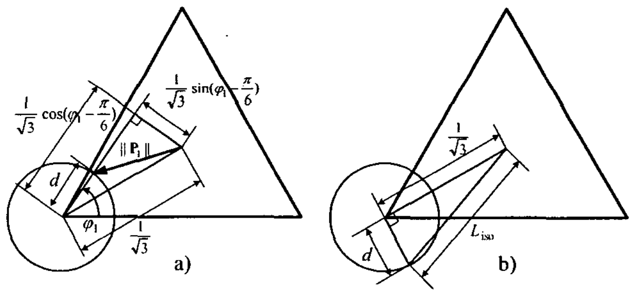

Fig. 4 The geometrical representation of the isotropic characteristic length.

Next, in the case of S2 where $\mu = 1$ and the isotropic configurations are given by

Table 2. Four different sets of all isotropic configurations

Set	Isotropic configurations	d_{iso}	L_{iso}
S1	$(\phi_1, \phi_1+\frac{2\pi}{3}, \phi_1-\frac{2\pi}{3})$	No restriction	$\sqrt{\frac{2}{\mu+1}\left\{\left(\frac{1}{\sqrt{3}}\sin(\phi_1-\frac{\pi}{6})\right)^2\mu + \left(d-\frac{1}{\sqrt{3}}\cos(\phi_1-\frac{\pi}{6})\right)^2\right\}}$
S2	$(\phi_1, \phi_1-\frac{2\pi}{3}, \phi_1+\frac{2\pi}{3})$	r	$L_{iso} = \sqrt{d_{iso}^2+\frac{1}{3}}$
S3	$(\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6})$, $(-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2})$, $(\frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6})$	$\sqrt{r^2+\frac{4}{3}r^4}-\frac{2}{\sqrt{3}}r^2$	$L_{iso} = \frac{1}{\sqrt{1-\frac{2}{\sqrt{3}}d_{iso}}}\left(\frac{1}{\sqrt{3}}-d_{iso}\right)$
S4	$(-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{5\pi}{6})$, $(\frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2})$, $(-\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6})$	$\sqrt{r^2+\frac{4}{3}r^4}+\frac{2}{\sqrt{3}}r^2$	$L_{iso} = \frac{1}{\sqrt{1+\frac{2}{\sqrt{3}}d_{iso}}}\left(\frac{1}{\sqrt{3}}+d_{iso}\right)$

$(\phi_1, \phi_1-\frac{2\pi}{3}, \phi_1+\frac{2\pi}{3})$, it can be shown that (37) becomes

$$L_{iso} = \sqrt{\frac{1}{3}\sum_{i=1}^{3}\| \mathbf{p}_i \|^2} = \sqrt{d_{iso}^2+\left(\frac{1}{\sqrt{3}}\right)^2} \quad (41)$$

which is illustrated in Fig. 4b). Note that (41) can be considered as a special case of (39), seen from Fig 4a) and 4b).

Next, in the case of the isotropic configuration $(\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6})$ belonging to S3, it can be shown that (37) becomes

$$L_{iso} = \frac{1}{\sqrt{1-\frac{2}{\sqrt{3}}d_{iso}}}\left(\frac{1}{\sqrt{3}}-d_{iso}\right) \quad (42)$$

subject to $0 < d_{iso} < \frac{\sqrt{3}}{4}$. It can be shown that the expression of (42) is also valid for the other two isotropic configurations belonging to S3, $(-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6})$.

Similar analysis to the above can be made for all three isotropic configurations belonging to S4, which results in

$$L_{iso} = \frac{1}{\sqrt{1+\frac{2}{\sqrt{3}}d_{iso}}}\left(\frac{1}{\sqrt{3}}+d_{iso}\right) \quad (43)$$

subject to $d_{iso} > 0$.

For four different sets of the isotropic configurations, Table 2 lists the expressions of the isotropic characteristic length L_{iso} required for the isotropy of a COMR.

Now, it is interesting to investigate the possible relationships existing among four different sets listed in Table 2. It is obvious that 1) S1 and S2 are disjoint, 2) S3 and S4 are disjoint, and 3) both S3 and S4 are disjoint with S2 unless $r=0$. To check the relationship between S3 and S1, we impose the restriction of S3, given by (31), onto S1, given by (38). It can be shown that for a given the isotropic steering link offset d_{iso}, both S1 and S3 have the same value of the isotropic characteristic length L_{iso}, given by (42).

For a given wheel radius r, the relationship between S3 and S1 can be stated as follows: With the values of d_{iso} and L_{iso} chosen as (35) and (42), respectively, there exist six isotropic configurations: three belonging to S3, $(\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6})$, $(-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2})$, and $(\frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6})$, and three belong to S1, $(\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{2})$, $(-\frac{\pi}{6}, \frac{\pi}{2}, -\frac{5\pi}{6})$, and $(\frac{\pi}{2}, -\frac{5\pi}{6}, -\frac{\pi}{6})$.

Similar analysis to the above can be made for the relationship between S4 and S1: For a given wheel radius r, with the values of d_{iso} and L_{iso} are chosen as (36) and (43), respectively, there exist six isotropic configurations: three belonging to S4, $(-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{5\pi}{6})$, $(\frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2})$, and $(-\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6})$, and three belong to S1, $(-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2})$, $(\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6})$, and $(-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6})$.

VI. Conclusion

In this paper, we presented the systematic isotropy analysis of a fully actuated caster wheeled omnidirectional mobile robot (COMR) with the steering link offset different from the wheel radius, which can be considered as the generalization of the previous analysis. First, with the characteristic length introduced, we obtained the kinematic model of a COMR based on the orthogonal decomposition of the wheel velocities. Second, we derived and examined the necessary and sufficient conditions for the isotropy of a

COMR to categorize three different groups, each of which can be handled in a similar way. Third, for each group, we further explored the isotropy conditions so as to identify four different sets of all possible isotropic configurations. Fourth, for each set, we elaborated the expressions of the isotropic characteristic length required for the isotropy of a COMR.

The key of the systematic isotropy analysis of a COMR with variable steering link offset was the incorporation of the ratio of the steering link offset to the wheel radius into the development of the isotropy conditions. Although the isotropy analysis was made for a COMR having three caster wheels, the analytic framework can be readily extended for a COMR having multiple caster wheels attached to a regular polygonal mobile platform. We hope that the results of this paper help the optimal design and control of a COMR with variable steering link for improved tipover stability. For example, given a task trajectory of a COMR over some time interval, the prior knowledge of the isotropic wheel configurations can be useful to generate the optimal wheel trajectory that is close to the isotropy but away from the singularity as much as possible [15].

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Sung-bok Kim

received the B.S. degree in Electronics Engineering from Seoul National University, Korea, in 1980, the M.S. degree in Electrical and Electronics Engineering from KAIST, Korea, in 1982, and the Ph.D. degree in Electrical Engineering from the University of Southern California, USA, in 1993. Since 1994, he has been with the School of Electronics and Information Engineering, Hankuk University of Foreign Studies, Korea, where he is currently a Professor. His recentresearch interests include the analysis, design and control of mobile robots and humanoids.