Optimum Design of BLDC Motor for Cogging Torque Minimization Using Genetic Algorithm and Response Surface Method

Mun-Ho Jeon*, Dong-Hun Kim and Chang-Eob Kim†**

Abstract - This paper presents a new optimization method combining the genetic algorithm with the response surface method for the optimum design of a Brushless Direct Current motor. The method utilizes a regression function approximating an objective function and the window moving and zoom-in method so as to complement disadvantages of both the genetic algorithm and response surface method. The results verify that the proposed method is powerful and effective in reducing cogging torque by optimizing only a few decisive design factors compared with the conventional stochastic methods.

Keywords: BLDC, Cogging Torque, Genetic Algorithm, Response Surface Method

1. Introduction

Cogging torque is a fundamental phenomenon that occurs inevitably due to the interaction between permanent magnets and the slotted structure of the stator core in the Brushless Direct Current (BLDC) motor. It produces a pulsation torque ripple that results in mechanical vibration and acoustic noise. This is an obstacle in realizing constant speed and accurate position control. To reduce the cogging torque, many researches have been carried out to date [1-3]. Der-Ray [1] proposed the cogging torque reduction method in conjunction with the curvature of the salient pole. In the published article [3], the optimum stator core of an outer rotor type BLDC motor was presented by means of the genetic algorithm (GA) and an approximate field analysis was referred to as the reluctance network method (RNM). However, there is still a need for a more elaborate and practical design to develop a motor with distinctive features of quiet operation and accurate positioning at high speed. In this paper, a practical optimization method combining the GA with the response surface method (RSM) is discussed to effectively suppress the cogging torque of a surface mounted permanent BLDC motor without deteriorating the performance of the original model. For an exact torque profile, the finite element method (FEM) is used so as to take into account rotor rotation and saturation of the silicon steel. Three design methods of the GA, RSM and their hybrid method (GA+RSM) in conjunction with the FEM have been attempted and comparisons are made.

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2. Decision Of Design Variables

Fig. 1 shows only one-fourth of the design model with 4 poles, 24 stator slots, stack length of 80 mm and rated output power of 400 W.

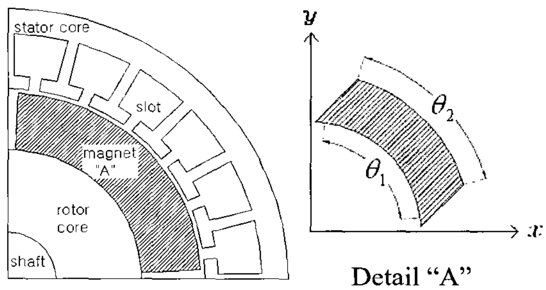


Fig. 1. Analysis model and design variables of the magnet

For the reduction of the cogging torque, the variation of the co-energy stored in a magnetic system versus the rotor position has to be minimized. To achieve the goal effectively, the effect of design factors on the variation of the stored magnetic co-energy should first be investigated and then a few predominant factors can be selected as design variables. In this paper, the continuum design sensitivity analysis combined with a standard electromagnetic software package [4] is used for this purpose. The energy sensitivity (a term used to describe the gradient information about specified dimensions of the components of the motor) can be written with respect to design variables **p** as

$$\frac{dF}{d\mathbf{p}} = 2\sum_{i=1}^{nr} W_i \cdot \int_\gamma [(\nu_1 - \nu_2)\mathbf{B}_1 \cdot \mathbf{B}_2 - \mathbf{M}_2\mathbf{B}_2)] V_n \, d\Gamma \qquad (1)$$

where nr is the number of rotor positions considered, W_i the stored co-energy computed at the i-th position, ν the magnetic reluctivity, **B** the magnetic flux density, **M** the permanent magnetization and V_n an inner product of a normal vector outward to the moveable interface γ and a directional vector imposed on each design variable. The subscripts, 1 and 2, in (1) denote either side of the interface between air and iron regions, respectively. This formula quantitatively represents the variation of the stored magnetic energy when the interface is changed. Utilizing the post-processing data and (1), the absolute values of the energy sensitivity about several major dimensions of the motor are computed as shown in Fig. 2.

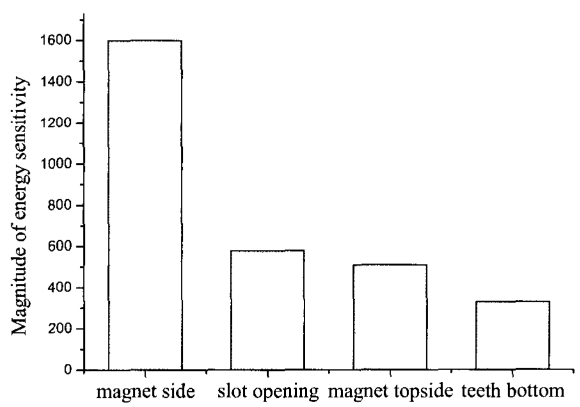


Fig. 2. Energy sensitivities about major dimensions of the motor considered

Both sides of the permanent magnet are revealed to be dominant to other design factors and the slot opening size is determined to be the second in order of the energy sensitivity. The magnetic co-energy and cogging torque are automatically calculated by the FEM at the certain angular position of the rotor angle θ as follows:

$$W_i = \frac{1}{2\mu_o}\int_V B^2 dV\,, \quad T_c = -\frac{dW_i}{d\theta} \qquad (2)$$

where $\boldsymbol{B}$ is the flux density of the air gap at each rotor position.

3. Optimum Design Methods

In order to verify the proposed hybrid method of GA and RSM, the magnet angles of the rotor, θ_1 and θ_2, are first chosen as design variables (See Fig. 1). Then three design variables including the slot opening size between stator teeth as well as the two magnet angles are optimized.

3.1. Genetic Algorithm

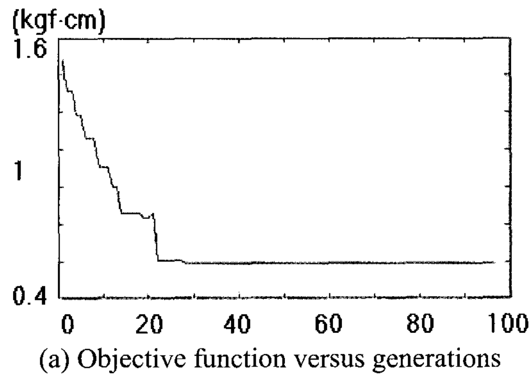
The genetic algorithm emulates the mechanism of natural selection and natural genetics. The algorithm consists of three operators: reproduction, crossover, and mutation [3, 5]. The design variables θ_1, θ_2 of magnet angles of the rotor (inner and outer angles, respectively) are allowed to move from 75 to 85 mechanical degrees. Fig. 3 shows the variations of the object function and design variables versus generations by GA. The design variables converge to the optimum values of θ_1 =80.0° and θ_2 =78.0° and the cogging torque is reduced by 35.2% compared with the initial design.

3.2. Response Surface Method

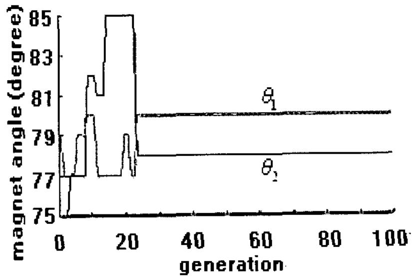
The response surface method is used to examine the relationship between one or more response variables and a set of quantitative experimental variables or factors [6] In this paper, a 2-level, 2-factor central composite design method, composed of 2^k factorial point, axial point, star point and center point, is used. The experiment number N is given as

$$\mathrm{N} = 2^k + 2k + n_c \qquad (3)$$

where 2^k is factorial, k the number of factors and n_c the duplication numbers of the experiment. A total of 13 designs of experiment are, here, executed to solve the cogging torque problem by the finite element method.

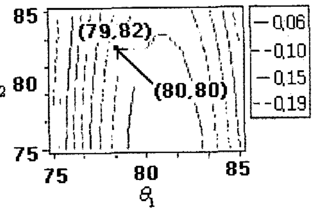


(a) Objective function versus generations

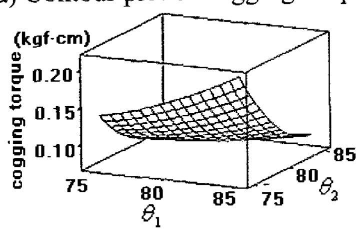


(b) Design variables versus generations

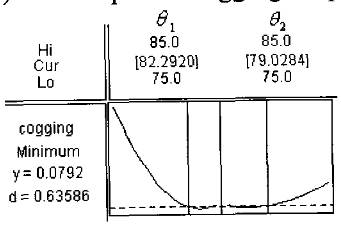
Fig. 3. Object function and design variables versus GA generations



(a) Contour plot of cogging torque



(b) Surface plot of cogging torque



(c) Optimum response

Fig. 4. Contour, surface plot and optimum response by RSM

The design variables converge to the magnet angles of θ1 =82.3o and θ2 =79o and the cogging torque is reduced by 16.5% of its original value. Fig. 4 shows the contour plot and surface plot of the approximated cogging torque profile by RSM with respect to the two magnet angles.

4. Genetic Algorithm Combined with Response Surface Method

In order to find an optimum solution by GA using FEM, it takes plenty of computing time owing to the required FE analysis of the cogging torque at every generation. On the other hand, RSM has the disadvantage of being liable to fall into the local minimum. In this paper, a new method combining GA with RSM is proposed to overcome the demerits of the two methods mentioned above. First, some designs of the experiment are selected and the corresponding responses are calculated through the FE analysis. Then, utilizing the responses values, the regression function approximating an objective function in a design space is built up in the form of a polynomial as

$$u = \beta_0 + \sum_{j=1}^{m} \beta_j x_j + \sum_{j=1}^{m} \beta_j x_j^2 + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \beta_{ij} x_i x_j \quad (4)$$

where β_0, β_i and β_{ij} are coefficients for design variables x, respectively and m denotes the number of design variables. Instead of FE analysis, the approximated function (4) is used to estimate the cogging torque at the specified design variables generated in the GA. After the convergence of (4), the initial design space explored is restricted to a smaller space around an approximated optimum solution by using the window moving and zoom-in method [7]. Hereby a new regression function corresponding to the shrunken design space is reconstructed to search for a more accurate optimum solution. In addition, for accelerating the convergence of the GA, the method exploits the approximated gradient information based on the regression function during the overall GA process. Fig. 5 presents the flow chart of the proposed optimization method.

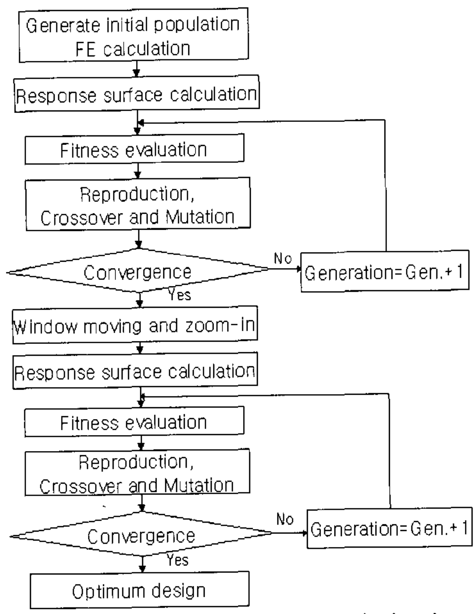
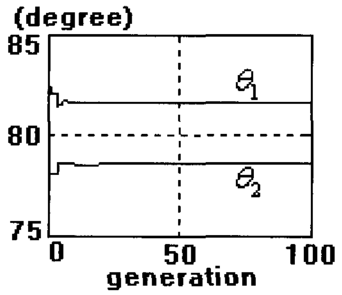
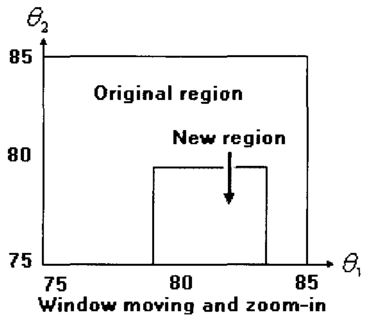


Fig. 5. Flow chart of the proposed optimization method by GA combined with RSM

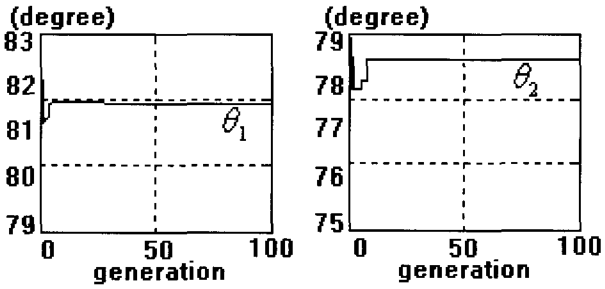
The results on optimizing two design variables using GA combined with RSM are shown in Fig. 6. In the first optimization step, the design variables converge to θ_1 =81.2° and θ_2 =78.7° and then a new design region is chosen by the window moving and zoom-in method as follows: $79 \le \theta_1 \le 83$, $75 \le \theta_2 \le 79$ (see Fig. 6). Through the second optimization step, the optimum design variables are obtained as θ_1 =81.6°, θ_2=78.6° and the cogging torque is reduced by 37.4% of its original value.



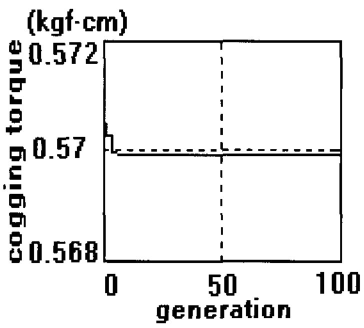
(a) Optimum response in the first optimization step



(b) New design space



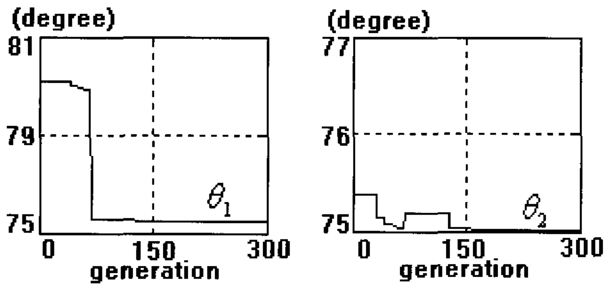
(c) Optimum response in the second optimization step



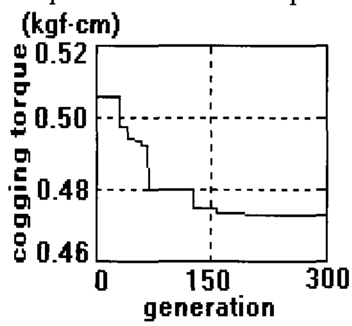
(d) Objective function versus generations

Fig. 6. Optimization process by GA combined with RSM for 2 design variables (magnet angles)

Fig. 7 shows the optimization process using GA combined with RSM for three design variables: two magnet angles and a slot opening size. The design variables converge with the generation to θ_1 =77.9°, θ_2=75.0° and a slot opening of 2 mm in the first optimization step. Using the window moving and zoom-in method and the second optimization step additionally, the optimum design variables are obtained: θ_1 =75.1°, θ_2=75.0° and a slot opening of 2 mm. The cogging torque is reduced by 48.3% compared with the initial design. From Fig. 7, it is assumed that the method can be successfully applied to more than three design variables.



(a) Optimum response in the second optimization step



(b) Objective function versus generations

Fig. 7. Optimum design by GA combined with RSM for 3 design variables (magnet angles and slot opening)

5. Comparison And Discussion

Table I reveals the comparisons between the three design methods used for minimizing the cogging torque of the BLDC motor of interest: GA, RSM, GA combined with RSM. The cogging torque waveforms are compared with one another in Fig. 8. From the results, the proposed method of GA combined with RSM is revealed to be most effective in searching for an optimum solution because of the distinctive features of fast convergence and minimum solving of a FE model necessary only to constructing the regression function.

Table 1 Initial design and optimum design with respect to two design variables

Design variables	Initial model	Optimum design		
		GA	RSM	GA+RSM
Number of FE calculations		121	13	13
Magnet angle (degree)	θ_1=80.0 θ_2=80.0	θ_1=80.0 θ_2=78.0	θ_1=82.3 θ_2=79.0	θ_1=81.6 θ_2=78.6
Cogging torque (kgf · cm)	0.91	0.59	0.76	0.57

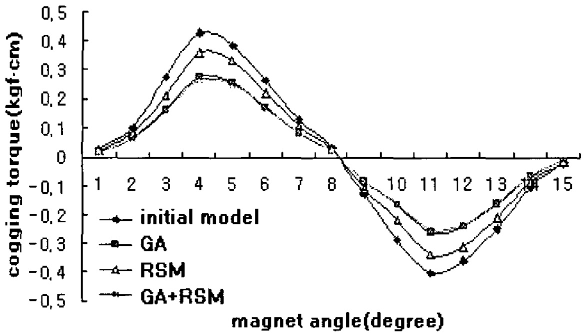


Fig. 8. Cogging torque waveforms for initial and optimum designs

6. Conclusion

To reduce the cogging torque of a BLDC motor, a new optimization technique combining the GA with RSM is proposed and compared with the conventional methods of the GA and RSM in conjunction with the finite element method. Only a few decisive design factors in the cogging torque problem are systematically chosen as the design variables, exploiting the energy sensitivity. The results show that the cogging torque is reduced by 35.2% in the GA, 16.5% in the RSM and 37.4% in the GA combined with the RSM of its original value, respectively with respect to 2 design variables when the slot opening is 3mm. In the case of selecting three design variables, the cogging is more reduced by 48.3% of its original value. It is verified that the proposed method is more powerful and effective in optimizing electrical machines compared with the stochastic methods.

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References

[1] Der-Ray Huang, Tai-Fa Ying, Shyh-Jier Wang, and Chi-mou Zhou, "Cogging Torque Reduction of a Single-phase Brushless DC Motor," *IEEE Transactions on Magnetics*, vol. 34, no. 4, pp. 2075-2077, 1998.

[2] C. Breton, J. Bartolome, J. A. Benito, G. Tassinario, I. Flotats, C. W. Lu, and B. J. Chalmers, "Influence of Machine Symmetry on Reduction of Cogging Torque in Permanent-Magnet Brushless Motors," *IEEE Transactions on Magnetics*, vol. 36, no. 5, pp. 3819-3823, 2000.

[3] Ki-Jin Han, Han-Sam Cho, Dong-hyeok Cho, and Hyun-Kyo Jung, "Optimal Core Shape Design for Cogging Torque Reduction of Brushless DC Motor Using Genetic Algorithm," *IEEE Transactions on Magnetics*, vol. 36, no. 4, pp. 1927-1931, 2000.

[4] Dong-Hun Kim, K. S. Ship and J. K. Sykulski, "Applying continuum design sensitivity analysis combined with standard EM software to shape optimisation in magnetostatic problems," *IEEE Transactions on Magnetics.*, vol. 40, pp. 1156-1159, 2004.

[5] G. Fuat Uler, Osama A. Mohammed, and Chang-Seop Koh, "Utilizing Genetic Algorithms for the Optimal Design of Electromagnetic Devices," *IEEE Transactions on Magnetics*, vol. 30, no. 6, pp. 4296-4298, 1994.

[6] Yasushi Fujishima, et al., "Multiobjective Design Optimization of Permanent Magnet Synchronous Motor for Railway Vehicle," CEFC record, p. 29, 2004.

[7] X. K. Gao, T. S, Low, Z. J. Liu, and S. X. Chen, "Robust Design for Torque Optimization Using Response Surface Methodology," *IEEE Transactions on Magnetics*, vol. 38, no. 2, pp. 1141-1144, 2002.

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