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# CMOS OTA를 이용한 펄스폭 변조회로

## ( Pulse Width Modulation Circuits Using CMOS OTAs )

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### 요 약

본 논문은 CMOS OTA를 이용한 3 개의 PWM 변조기를 제안한다. 이들은 램프 적분기와 전류로 임계전압은 조정할 수 있는 슈미트 트리거로 구성되었다. 개별 소자를 사용해 만든 원형회로는 제안된 PWM 회로들의 시비율이 선형적으로 제어될 수 있음을 보여준다. 제안된 변조기들은 구조가 간단하기 때문에 모노리식 IC로 쉽게 제작할 수 있다.

### Abstract

This paper presents three PWM modulators using CMOS OTAs. They consist of a ramp integrator and current-tunable Schmitt triggers. Prototype circuits built using discrete components exhibited that their duty cycles are linearly controllable. The proposed modulators can be easily fabricated in a monolithic IC, because they have a simple structure.

**Keywords:** 펄스폭 변조, operational transconductance amplifiers, 아날로그 집적회로

## I. Introduction

Pulse-width modulation (PWM) is being used for digital audio, communication, and power conversion systems<sup>[1-3]</sup>. A PWM signal can be generated by comparing an information signal with a saw-tooth or triangular waveform. In general, this circuit is readily implemented using an operational amplifier (OP-amp) RC integrator and a Schmitt trigger<sup>[4]</sup>, but can also be obtained using a current conveyor instead of the OP-amp<sup>[5]</sup>. A derivative PWM signal generator using

operational transconductance amplifiers (OTAs) has also been reported<sup>[6]</sup>. There are other methods of producing a PWM pulse train, such as, with the use of voltage- or current-controlled delay cells<sup>[7, 8]</sup> and a voltage-controlled phase shifter<sup>[9]</sup>.

This paper presents three PWM modulators using CMOS OTAs and compares their performances. Since the identical OTAs are used as circuit-building blocks, the advantage of the proposed circuits is their integration. Their operating principles are described in section II. In section III, the pulse periods of the PWM modulators that are considered non-ideal effects and their temperature stabilities are discussed. The experimental results using discrete elements are presented in section IV.

## II. Circuit Description

### 1. Adjustable Schmitt Triggers and Their Second-order Effects

Schmitt triggers with adjustable threshold voltages

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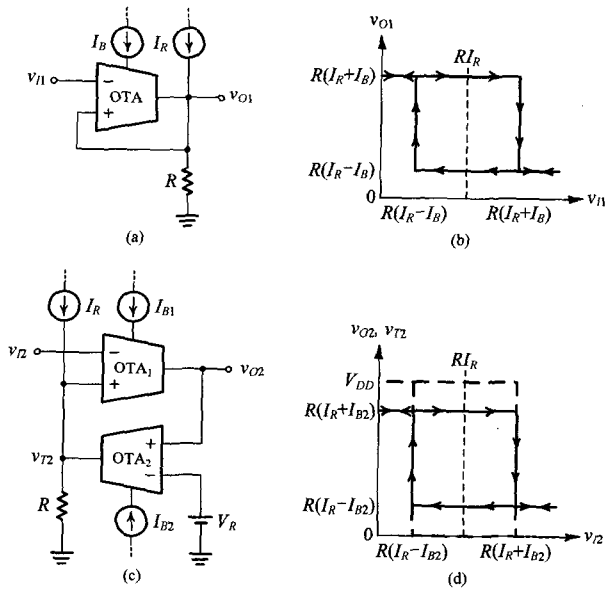


그림 1. (a) 직접 정귀환된 조정 가능한 슈미트 트리거 회로도, (b) 그림 (a) 회로의 이상적인 전달특성, (c) OTA-R 정귀환된 조정 가능한 슈미트 트리거 회로도, (d) 그림 (c) 회로의 이상적인 전달특성.

Fig. 1. (a) Circuit diagram of a direct positive feedback adjustable Schmitt trigger, (b) Ideal transfer characteristics of the circuit in (a), (c) Circuit diagram of an OTA-R positive-feedback adjustable Schmitt trigger, and (d) Ideal transfer characteristics of the circuit in (c).

can be obtained by using OTAs. Circuit diagrams of two OTA Schmitt triggers for a single supply and their transfer characteristics are shown in Fig. 1<sup>[10]</sup>. The direct positive-feedback Schmitt trigger shown in Fig. 1(a) consists of an OTA and a positive-feedback resistor  $R$ . Ideal high and low threshold voltages of this Schmitt trigger are as follows:

$$V_{TH1} = R(I_R + I_B) \quad (1a)$$

$$V_{TL1} = R(I_R - I_B) \quad (1b)$$

These threshold voltages are linearly adjustable with the dc bias current of the OTA, but are identical to the output voltage.

The Schmitt trigger shown in Fig. 1(c) has an OTA-R amplifier in a positive-feedback path. This results in the independent of the output voltage from the threshold one. The reference voltage  $V_R$  in Fig. 1(c) guarantees the saturation of the OTA<sub>2</sub> at a low

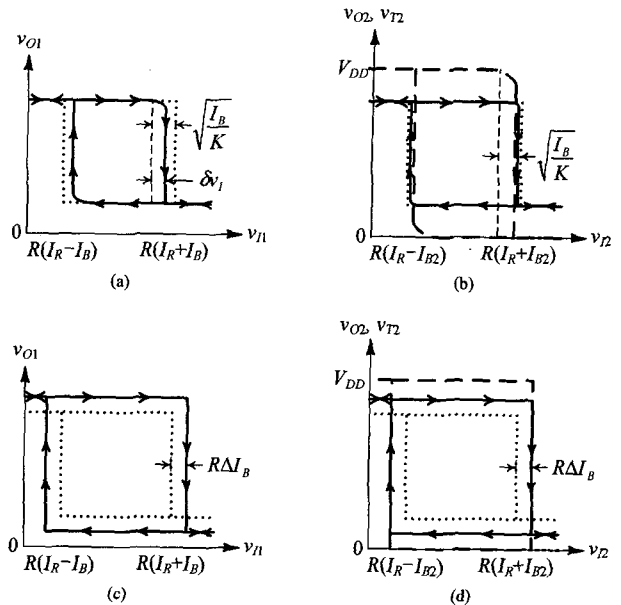


그림 2. OTA 슈미트 트리거의 비이상적인 전달특성: (a) 와 (b) 그림 1(a) 와 (c) 회로의 유한한 입력 선형범위에 대한 영향, (c) 와 (d) 포화전류 증가에 대한 영향.

Fig. 2. Non-ideal transfer characteristics of the OTA Schmitt triggers: (a) and (b) effect on the finite input linear range of the circuit in Fig. 1(a) and (c), and (c) and (d) effect on the saturation current increase.

saturation level when the output voltage  $v_{O2}$  is zero. The short dashed line in Fig. 1(d) represents the transfer characteristics of  $v_{O2}$ , and the solid line represents those of the threshold voltage. Ideal threshold voltages are given by

$$V_{TH2} = R(I_R + I_{B2}) \quad (2a)$$

$$V_{TL2} = R(I_R - I_{B2}) \quad (2b)$$

In practice, the threshold voltages deviate from the ideal ones by the finite input linear range, and the saturation current increases over the bias current of an OTA. Fig. 2 shows the non-ideal transfer characteristics of the two Schmitt triggers when the OTAs shown in Fig. 3 were used: Fig. 2(a) and (b) show the effect on the finite input linear range of the circuit in Fig. 1(a) and (c), and Fig. 2(c) and (d) show the effect on the saturation current increase.

In Fig. 2(a), the dotted line shows the ideal transfer characteristics of the circuit in Fig. 1(a) and

the solid line shows the transfer characteristics affected by the finite input linear range of the OTA. To estimate such threshold voltage variations, assume that the output voltage  $v_{O1}$  is saturated at its high saturation level and the input voltage  $v_{I1}$  is increasing into the high threshold voltage level. When the differential input voltage of the OTA reaches the boundary voltage of the input linear range,  $\sqrt{I_B/K}$  ( $K = (1/2)\mu C_{OX}W/L$ ), the OTA begins to operate in the linear region and the threshold voltages decrease. The threshold voltage variation is given by the following equation, using linear approximation:

$$\begin{aligned} v_{O1} &= R(I_R + I_B) - \frac{dv_{O1}}{dv_{I1}} \delta v_{I1} \\ &= R(I_R + I_B) - \frac{G_m R}{G_m R - 1} v_{I1} \\ &\quad - \frac{G_m R}{G_m R - 1} \left\{ \sqrt{\frac{I_B}{K}} - R(I_R + I_B) \right\} \end{aligned} \quad (3)$$

where  $G_m$  is the transconductance gain and  $\delta v_{I1}$  is the input voltage variation from the boundary voltage of the input linear range. As  $v_{I1}$  increases,  $v_{O1}$  falls in terms of (3) until the differential input voltage again reaches  $\sqrt{I_B/K}$ , after it passes the point at which  $v_{I1}$  and  $v_{O1}$  are equal. This, in turn, causes the Schmitt trigger to switch to the low saturation level. The deviation of  $v_{I1}$  during the OTA operation in the linear region is twice that from the starting point of the linear range to the point at which  $v_{I1}$  and  $v_{O2}$  are equal. The transconductance gain of the CMOS OTAs shown in Fig. 3 is  $\sqrt{2KI_B}$ , when its input voltage is zero. Substituting  $v_{I1} = v_{O1}$  in (3) and deriving the variation of  $v_{O1}$  in the linear region, the high threshold voltage that is affected by the finite input linear range is given by

$$\begin{aligned} V_{TH} &= R(I_R + I_B) \\ &\quad - \sqrt{\frac{I_B}{K}} \left( \frac{2R\sqrt{2KI_B}}{2R\sqrt{2KI_B} - 1} - 1 \right) \end{aligned} \quad (4)$$

Fig. 2(b) shows the effect of the finite input linear

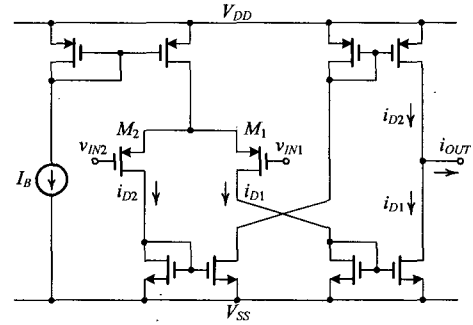


그림 3. CMOS OTA 회로도.

Fig. 3. Circuit diagram of a CMOS OTA.

range of the circuit in Fig. 1(c). The short dashed line shows the transfer characteristics of the output voltage  $v_{O2}$ , and the solid line shows those of the threshold voltage  $v_{T2}$ . Assume that the output voltage  $v_O$  is saturated at its high saturation level and that the input voltage  $v_{I2}$  is increasing into the high threshold voltage level. The OTA<sub>1</sub> begins to operate in the linear region when its differential input voltage reaches the boundary voltage of the input linear range, but the OTA<sub>2</sub> is still saturated and thus, the threshold voltage does not change. The output voltage variation in this region is given by, the following equation, using linear approximation:

$$\begin{aligned} v_{O2} &= V_{DD} - G_{m1} R_o v_{I2} \\ &\quad + G_{m1} R_o \left\{ \sqrt{\frac{I_{B1}}{K}} - R(I_R + I_{B2}) \right\} \end{aligned} \quad (5)$$

where  $G_{m1}$  is the transconductance gain of OTA<sub>1</sub>. When the decreasing  $v_{O2}$  reaches the boundary voltage of the input linear range of OTA<sub>2</sub>,  $V_R + \sqrt{I_{B2}/K}$ , OTA<sub>2</sub> will begin to operate in the linear region. The value of  $v_{I2}$  at this point is given by

$$\begin{aligned} v_{I2} &= R(I_R + I_{B2}) \\ &\quad - \sqrt{\frac{I_{B1}}{K}} + \frac{V_{DD} - V_R - \sqrt{I_{B2}/K}}{G_{m1} R_o} \end{aligned} \quad (6)$$

OTA<sub>1</sub> and OTA<sub>2</sub> operate in the linear region until the decreasing  $v_{T2}$  reaches the boundary voltage,  $v_{I2} - \sqrt{I_{B1}/K}$ , that makes OTA<sub>1</sub> again goes to the saturation region. The variation of  $v_{T2}$  in this region

is given by

$$v_{T2} = R(I_R + I_{B2}) - \frac{G_{m1}G_{m2}R_oR}{G_{m1}G_{m2}R_oR - 1}(v_{I2}) - \frac{G_{m1}G_{m2}R_oR}{G_{m1}G_{m2}R_oR - 1}(v_{I2}|_{v_{oz} = V_R + \sqrt{I_{B2}/K}}) \quad (7)$$

Substituting  $v_{T2}$  with  $v_{D2} - \sqrt{I_{B1}/K}$  in (7), the variation of the high-threshold voltage is given by

$$V_{TH2} = R(I_R + I_{B2}) - \frac{\sqrt{\frac{I_{B1}}{K}} - \sqrt{2}RI_{B2}\left(\frac{V_{DD} - V_R}{\sqrt{I_{B2}/K}} - 1\right)}{8K\sqrt{I_{B1}I_{B2}R_oR} - 1} \quad (8)$$

As a result, the OTA- $R$  positive-feedback Schmitt trigger is insensitive to the finite input linear range of an OTA, compared with the direct positive-feedback Schmitt trigger.

The effect of the OTA's saturation current increase, which consisted of the circuit in Fig. 1(a) and (c), is illustrated in Fig. 2(c) and (d), respectively. The ideal saturation current of the OTA should be equal to the current value of its dc bias. The non-ideal saturation current slightly increased, however, with the channel length modulation of the MOSFETs, which consisted of the current mirrors of the OTA. Assuming that the simple current mirror was employed and the MOSFETs were perfectly matched, the non-ideal saturation current of the OTA  $I_O$  would be as follows:

$$I_O = \left\{ 1 + \frac{\lambda(V_{DS2} - V_{DS1})}{1 + \lambda V_{DS1}} \right\} I_B \approx \{ 1 + \lambda(V_{DS2} - V_{DS1}) \} I_B \quad (9)$$

where  $\lambda$  is the channel length modulation parameter,  $V_{DS1}$  is the drain-to-source voltage of the diode connect MOSFET,  $V_{DS2}$  is that of the other one, and  $I_B$  is the bias current of the OTA. The equation (9) shows that  $\Delta I_B$  in Fig. 2(c) and (d) is  $\lambda(V_{DS2} - V_{DS1})I_B$ .

## 2. Saw-tooth Carrier Based PWM Modulator

Fig. 4 shows a circuit diagram of the saw-tooth carrier based PWM modulator operating on a single supply. It consists of the direct positive-feedback Schmitt trigger, an integrator composed of the current source  $I_T$  and the capacitor  $C_1$ , the inverter, the n-MOSFET switch  $M_1$ , and a comparator composed of OTA<sub>2</sub> and its output resistance. The waveforms associated with this modulator are shown in Fig. 5. To see how the modulator operates, assume that the output voltage of the Schmitt trigger  $v_T$  is at its high saturation level,  $R_1(I_R + I_B)$ , thus opening the n-MOSFET switch  $M_1$  and charging the capacitor  $C_1$  with a slope of  $I_T/C_1$ . The input voltage of the Schmitt trigger  $v_I$  continues to increase until  $v_I$  reaches its high-threshold voltage,  $V_{TH} = R_1(I_R + I_B)$ . During the interval  $T_1$ , the following equation is obtained, from Fig. 5,

$$\frac{R_1(I_R + I_B) - R_1(I_R - I_B)}{T_1} = \frac{I_T}{C_1} \quad (10)$$

Rearranging the equation gives

$$T_1 = 2C_1R_1\frac{I_B}{I_T} \quad (11)$$

When  $v_I$  reaches  $V_{TH}$ ,  $v_T$  will switch to the low saturation level,  $R_1(I_R - I_B)$ . At this instant, switch  $M_1$  shall close. This, in turn, will exponentially discharge capacitor  $C_1$  via switch  $M_1$ . Since the on-resistance of  $M_1$  is very small, interval  $T_2$  can be neglected compared to  $T_1$ . The period of the saw-tooth waveform  $T_{s1}$  is

$$T_{s1} \approx 2C_1R_1\frac{I_B}{I_T} \quad (12)$$

Note that the period of the saw-tooth waveform is directly proportional to the currents ratio,  $I_B$ , so their temperature-dependent terms are cancelled out. Neglecting the temperature coefficients of  $C_1$  and

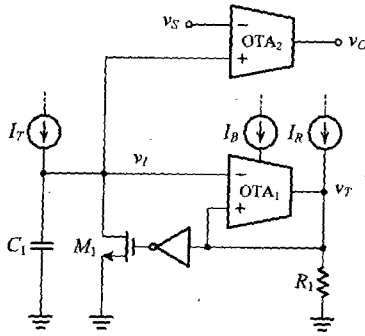


그림 4. 톱니파 캐리어 PWM 회로도.

Fig. 4. Circuit diagram of the saw-tooth carrier PWM.

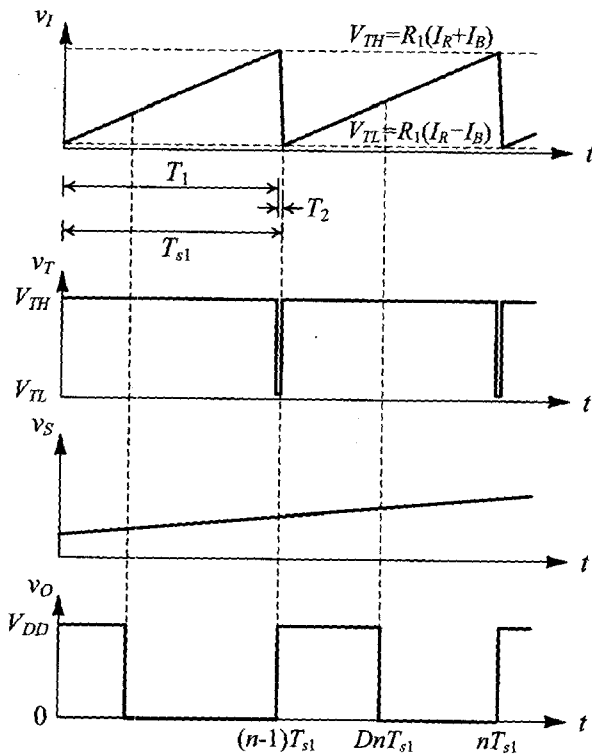


그림 5. 그림 4 회로의 다양한 마디에서 이상적인 전압 파형.

Fig. 5. Ideal voltage waveforms at various nodes of the circuit in Fig. 4.

$R_1$ , the period is stable against temperature variations.

The PWM signal is periodically generated by comparing the signal voltage  $v_S$  with the saw-tooth wave voltage  $v_I$ . From the waveform of the PWM signal  $v_O$  shown in Fig. 5, the duty cycle  $D$  is given by

$$D = \frac{v_S(DnT_{s1})}{2R_1I_B} + \frac{1}{2} \left( 1 - \frac{I_R}{I_B} \right) \quad (13)$$

### 3. Triangular Carrier Based PWM Modulator

A circuit diagram of the triangular carrier based PWM modulator is shown in Fig. 6. It consists of the OTA-R positive-feedback Schmitt trigger, an OTA-C integrator, and a comparator composed of OTA<sub>4</sub> and its output resistance. The waveforms associated with this modulator are shown in Fig. 7. To see how the modulator operates, assume that the output voltage of Schmitt trigger  $v_{O1}$  is at its high saturation level,  $V_{DD}$ , thus saturating OTA<sub>3</sub> and charging capacitor  $C_2$  with a slope of  $I_{B3}/C_2$ . The input voltage of Schmitt trigger  $v_I$  continues to increase until  $v_I$  reaches the high-threshold voltage,  $V_{TH} = R_2(I_R + I_{B2})$ . Interval  $T_1$  is given by the following equation, from Fig. 7:

$$T_1 = 2C_2R_2 \frac{I_{B2}}{I_{B3}} \quad (14)$$

When  $v_I$  reaches  $V_{TH}$ ,  $v_T$  will switch to the low saturation level,  $R_2(I_R - I_{B2})$ , which is much lower than  $V_R$  and this, in turn, will reverse the output current of the OTA<sub>3</sub>. Thus,  $v_I$  starts to decrease with a slope of  $-I_{B3}/C_2$ . Interval  $T_2$  is given by

$$T_2 = 2C_2R_2 \frac{I_{B2}}{I_{B3}} \quad (15)$$

Combining (14) and (15), the period of the triangular waveform  $T_{s2}$  is

$$T_{s2} = 4C_2R_2 \frac{I_{B2}}{I_{B3}} \quad (16)$$

Similarly to the equation (12), the temperature-dependent terms of the bias currents,  $I_{B2}$  and  $I_{B3}$ , are cancelled out, and the period is stable against temperature variations.

The PWM signal is periodically generated by comparing the signal voltage  $v_S$  with the triangular wave voltage  $v_I$ . During interval  $D_1T_{s2}$ , the following equation is obtained from the waveform of the PWM signal  $v_O$  in Fig. 6:

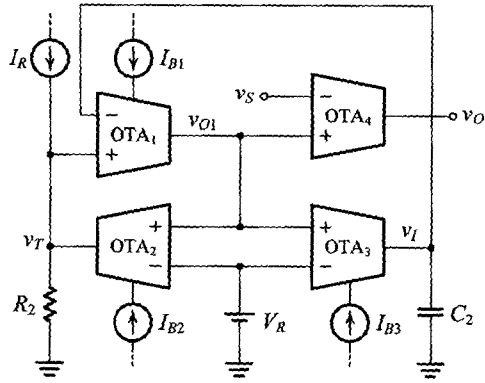


그림 6. 삼각파 캐리어 PWM 회로도.  
Fig. 6. Circuit diagram of the triangular carrier PWM.

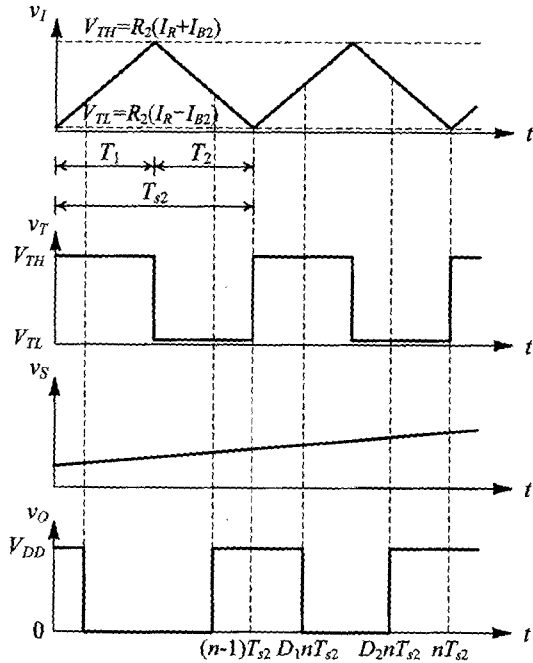


그림 7. 그림 6 회로의 다양한 마디에서 이상적인 전압 파형.  
Fig. 7. Ideal voltage waveforms at various nodes of the circuit in Fig. 6.

$$\frac{v_S(D_1 n T_{s2}) - R_2(I_R - I_B)}{D_1 n T_{s2}} = \frac{I_{B3}}{C_2} \quad (17)$$

from which is obtained:

$$D_1 n T_{s2} = \frac{C_2}{I_{B3}} \{v_S(D_1 n T_{s2}) - R_2(I_R - I_B)\} \quad (18)$$

Similarly, during  $(1 - D_2)n T_{s2}$  the following is obtained:

$$\frac{v_S(D_2 n T_{s2}) - R_2(I_R - I_B)}{(1 - D_2)n T_{s2}} = \frac{I_{B3}}{C_2} \quad (19)$$

from which is obtained:

$$(1 - D_2)n T_{s2} = \frac{C_2}{I_{B3}} \{v_S(D_2 n T_{s2}) - R_2(I_R - I_B)\} \quad (20)$$

From (18) and (20), the duty cycle  $D$  is given by the following equation, assuming that  $v_S(D_1 n T_{s2}) = v_S(D_2 n T_{s2}) = v_S(D n T_{s2}/2)$ :

$$D = \frac{v_S(D n T_{s2}/2)}{2R_2 I_{B2}} + \frac{1}{2} \left(1 - \frac{I_R}{I_{B2}}\right) \quad (21)$$

#### 4. Current-sensing PWM Modulator

Fig. 8 shows a circuit diagram of a current sensing PWM modulator. It consists of the two OTA-R positive-feedback Schmitt triggers, an integrator composed of the current source  $I_T$  and a capacitor  $C_3$ , and the n-MOSFET switch  $M_3$ . The waveforms associated with this modulator are shown in Fig. 9. The generation principle of the saw-tooth wave voltage  $v_I$  is similar to that of the circuit in Fig. 4. The period of the saw-tooth waveform  $T_{s3}$  is given by

$$T_{s3} = 4C_3 R_3 \frac{I_B}{I_T} \quad (22)$$

The duty cycle is given by

$$D = \frac{1}{2} \frac{i_A}{I_B} \quad (23)$$

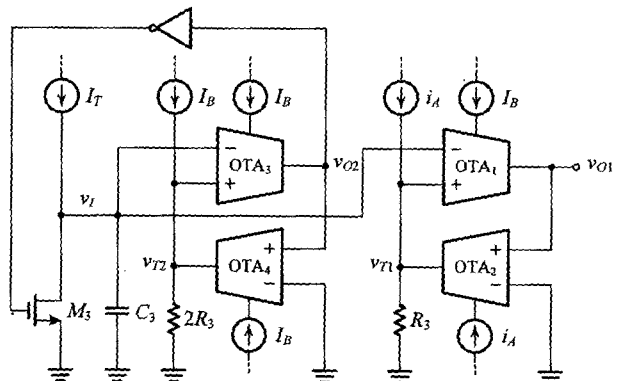


그림 8. 전류-센싱 PWM 회로도.  
Fig. 8. Circuit diagram of a current-sensing PWM.

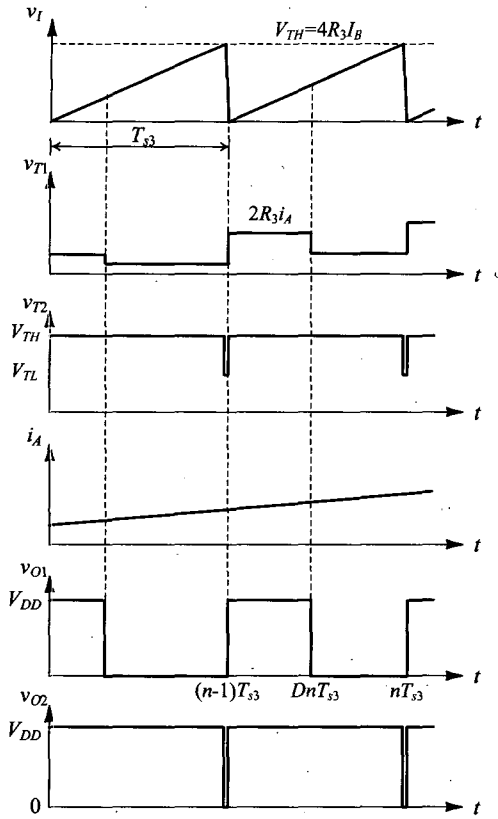


그림 9. 그림 8 회로의 다양한 노드에서 이상적인 전압 파형.

Fig. 9. Ideal voltage waveforms at various nodes of the circuit in Fig. 8.

Note that the period and the duty cycle are directly proportional to the currents ratio, so their temperature-dependent terms are balanced out.

Comparing the theoretical periods, the triangular carrier based PWM modulator needs two times the integration slope required by the other modulators, with the same passive components. In the third PWM modulator, letting  $v_{T1}$  be fixed and changing  $v_{T2}$ , the pulse frequency modulation (PFM) can be realized.

### III. Accuracy and Stability

#### 1. Accuracy of the Period and the Duty Cycle

The sources of error are the threshold voltage variations of the Schmitt triggers due to the finite input linear range, the saturation current increase, and the finite switching delay due to the slew rate of the amplifiers. The non-ideal waveforms of the

circuit in Fig. 4 are shown in Fig. 10. Assume that the errors of the high- and low-threshold voltages are identical and that the current source  $I_T$  is constant. The main source of error at interval  $T_1$  is the variation of the threshold voltage. Thus, the non-ideal  $T_1$  is given by the following equation, with reference to (4) and (9):

$$T_1 = 2C_1 R_1 \frac{I_B}{I_T} \left\{ 1 + \frac{\Delta I_B}{I_B} \right\} + 2C_1 R_1 \frac{I_B}{I_T} \left\{ \sqrt{\frac{I_B}{K} - 2R_1 I_B a} \right\} \quad (26)$$

where

$$a = \frac{\sqrt{2}}{2R_1 \sqrt{2KI_B} - 1}$$

At interval  $T_2$ , the time delay is caused by the slew rate of the Schmitt trigger and the finite discharging time of the capacitor  $C_1$ . Accounting for these factors,  $T_2$  is given by

$$T_2 = 2 \frac{V_{TH} - V_{TL}}{S_R} + C_1 R_{on} \ln \frac{V_{TH}}{V_{TL}} \quad (27)$$

$$\approx 2 \frac{2I_B}{S_R} + C_1 R_{on} \ln \frac{I_R + I_B}{I_R - I_B}$$

where  $S_R$  is the slew rate of the Schmitt trigger and  $R_{on}$  is the on-resistance of the n-MOSFET switch. Considering the above errors, period  $T_{s1}$  is expressed as:

$$T_{s1} = 2C_1 R_1 \frac{I_B}{I_T} \left( 1 + \frac{\Delta I_B}{I_B} + \sqrt{\frac{I_B}{K}} \right) - 2C_1 R_1 \frac{I_B}{I_T} (2R_1 I_B a) + 2C_1 R_1 \frac{I_B}{I_T} \frac{2I_T}{C_1 S_R} \left( 1 + \frac{\Delta I_B}{I_B} \right) + 2C_1 R_1 \frac{I_B}{I_T} \frac{R_{on} I_T}{2R_1 I_B} \ln \frac{I_R + I_B}{I_R - I_B} \quad (28)$$

If  $I_R = I_B$  or  $R_{on}$  is significantly small, the last term in (28) may be expressed as  $(5R_{on} I_T) / (2R_1 I_B)$ . The equation (28) shows that  $T_{s1}$  increases over the theoretical value due to the slew rate and  $R_{on}$  and decreases with the amplifier gain.

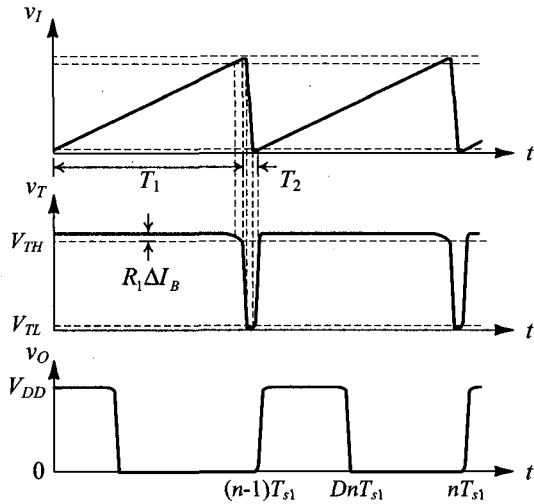


그림 10. 그림 4 회로의 비이상적인 전압 파형.  
Fig. 10. Non-ideal voltage waveforms of the circuit in Fig. 4.

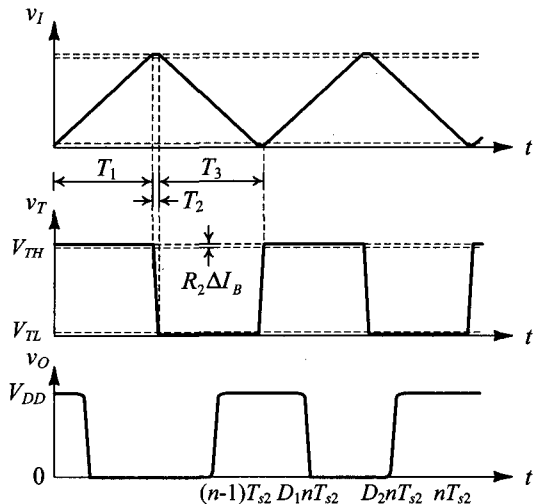


그림 11. 그림 6 회로의 비이상적인 전압 파형.  
Fig. 11. Non-ideal voltage waveforms of the circuit in Fig. 6.

The duty cycle affected by the slew rate of OTA<sub>2</sub> is given by the following equation, neglecting the error of T<sub>s1</sub>:

$$D = \frac{v_s(DnT_s)}{2R_1I_B} + \frac{1}{2} \left( 1 - \frac{I_R}{I_B} \right) + \frac{V_{DD}I_T}{2C_1R_1I_B} \frac{C_{o2}}{I_{B2}} \quad (29)$$

where C<sub>o2</sub> is the parasitic capacitance of the output of OTA<sub>2</sub>, and I<sub>B2</sub> is the bias current of OTA<sub>2</sub>.

Fig. 11 shows the non-ideal waveforms of the

circuit in Fig. 6. The source of error during interval T<sub>1</sub> is the saturation current increase in the OTAs and the variation of the threshold voltage, and the source of error during interval T<sub>2</sub> is the slew rate of the Schmitt trigger. Interval T<sub>3</sub> equals the sum of T<sub>1</sub> and T<sub>2</sub>. Considering these effects, the period T<sub>s2</sub> is as follows:

$$T_{s2} = 4C_2R_2 \frac{I_{B2}}{I_{B3}} \left( \frac{1 + \Delta I_{B2}/I_{B2}}{1 + \Delta I_{B3}/I_{B3}} \right) - 4C_2R_2 \frac{I_{B2}}{I_{B3}} (\sqrt{I_{B1}/K} - \sqrt{2}b)d + 4C_2R_2 \frac{I_{B2}}{I_{B3}} \left\{ \frac{I_{B3}}{C_2S_R} \left( 1 + \frac{\Delta I_{B2}}{I_{B2}} \right) \right\} \quad (30)$$

where

$$b = R_2I_{B2} \left( \frac{V_{DD} - V_R}{\sqrt{I_{B2}/K}} - 1 \right)$$

$$c = 8K\sqrt{I_{B1}I_{B2}}R_oR_2$$

$$d = \frac{1}{c - 1}$$

S<sub>R</sub> = the slew rate of the Schmitt trigger.

The equation (30) shows that T<sub>s2</sub> increases with the slew rate and decreases with the gain of the amplifier and V<sub>R</sub>. The non-ideal duty cycle which affected by the slew rate of OTA<sub>4</sub> is given by

$$D = \frac{v_s(DnT_s/2)}{2R_2I_{B2}} + \frac{1}{2} \left( 1 - \frac{I_R}{I_{B2}} \right) + \frac{V_{DD}I_{B3}}{4C_2R_2I_{B2}} \frac{C_{o4}}{I_{B4}} \quad (31)$$

where C<sub>o4</sub> is the parasitic capacitance of the output of OTA<sub>4</sub> and I<sub>B4</sub> is the bias current of OTA<sub>4</sub>.

In the circuit in Fig. 8, the possible minimum v<sub>O</sub>, V<sub>DD</sub> - √2R<sub>o</sub>I<sub>B1</sub>, is always greater than √I<sub>B2</sub>/K. I<sub>B1</sub> is the bias current of OTA<sub>1</sub> and I<sub>B2</sub> is that of OTA<sub>2</sub>, because the possible maximum v<sub>I</sub> is R<sub>3</sub>(I<sub>R</sub> + I<sub>B2</sub>). Thus, OTA<sub>2</sub> maintains its high saturation level until OTA<sub>1</sub> switches to the low saturation level. As soon as OTA<sub>1</sub> switches, OTA<sub>2</sub> also switches to the low saturation level. Similar to



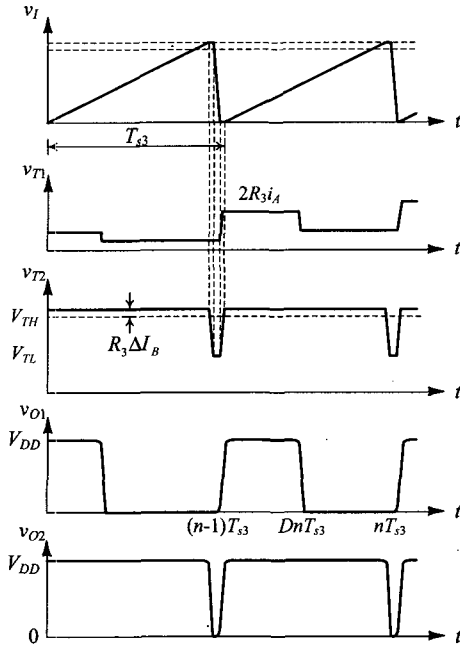


그림 12. 그림 8 회로의 비이상적인 전압 파형.  
Fig. 12. Non-ideal voltage waveforms of the circuit in Fig. 8.

the behavior of the saw-tooth carrier based PWM modulator, except for the effect of the input linear range of the OTA, the non-ideal period of the circuit in Fig. 8 is given by the following equation, from Fig. 12:

$$T_{s3} = 4C_3R_3\frac{I_B}{I_T}\left(1 + \frac{\Delta I_B}{I_B} + \frac{5R_{on}I_T}{4R_3I_B}\right) + 4C_3R_3\frac{I_B}{I_T}\frac{2I_T}{C_3S_R}\left(1 + \frac{\Delta I_B}{2I_B}\right) \quad (32)$$

where  $S_R$  is the slew rate of the Schmitt trigger and  $R_{on}$  is the on-resistance of the n-MOSFET switch. The equation (32) shows that  $T_{s3}$  increases with the slew rate and  $R_{on}$ . The non-ideal duty cycle which changed by the slew rate of the Schmitt trigger is given by

$$D = \frac{1}{2}\frac{i_A}{I_B}\left(1 + \frac{V_{DD}I_T}{2C_3R_3S_Ri_A}\right) \quad (33)$$

## 2. Temperature Stability

The temperature stability of the signal-generating period of the modulators depends on the temperature coefficient (TC) of the charge and discharge currents,

switching delays, and Schmitt trigger thresholds, as well as on the passive components. To simplify this discussion, assume that the current sources and the passive components are ideal, and neglect the TC of the slew rate. Thus, the temperature-dependent terms shown in (28), (30), and (32), are the conductivity parameter  $K$  and the on-resistance of MOSFETs. The TC of the on-resistance is given by

$$\frac{1}{R_{on}}\frac{\partial R_{on}}{\partial T} = -\frac{1}{K}\frac{\partial K}{\partial T} - \frac{1}{V_{GS} - V_{th}}\frac{\partial V_{th}}{\partial T} \quad (34)$$

Differentiating (28), (30) and (32), the temperature stabilities of the proposed modulators are as follows:

$$\begin{aligned} \frac{1}{T_{s1}}\frac{\partial T_{s1}}{\partial T} &= (R_1I_B\alpha + R_1I_B\alpha^2)\frac{1}{K}\frac{\partial K}{\partial T} \\ &- \left(\frac{1}{2}\sqrt{\frac{I_B}{K}} + \frac{R_{on}I_T}{2R_1I_B}\right)\frac{1}{K}\frac{\partial K}{\partial T} \\ &- \frac{R_{on}I_T}{2R_1I_B}\frac{1}{V_{GS} - V_{th}}\frac{\partial V_{th}}{\partial T} \end{aligned} \quad (35a)$$

$$\begin{aligned} \frac{1}{T_{s2}}\frac{\partial T_{s2}}{\partial T} &= \left(\sqrt{\frac{I_{B1}}{K}} + bd\frac{R_2I_{B2}}{\sqrt{2}}\right)\frac{d}{2K}\frac{\partial K}{\partial T} \\ &+ \left(\sqrt{\frac{I_{B1}}{K}} + \sqrt{2}b\right)\frac{c \cdot d^2}{K}\frac{\partial K}{\partial T} \end{aligned} \quad (35b)$$

$$\begin{aligned} \frac{1}{T_{s3}}\frac{\partial T_{s3}}{\partial T} &= -\frac{5R_{on}I_T}{4R_3I_B}\frac{1}{K}\frac{\partial K}{\partial T} \\ &- \frac{5R_{on}I_T}{4R_3I_B}\frac{1}{V_{GS} - V_{th}}\frac{\partial V_{th}}{\partial T} \end{aligned} \quad (35c)$$

To compare the temperature stability of the modulators, let all the values in (35) be equal to one except that of  $V_R = 0.5$ ,  $R_1 = R_2 = R_3 = 10$ , and let  $R_o = 100$ . Then, their TCs will be 70 ppm/°C, 17 ppm/°C, and 687 ppm/°C, respectively. This reveals that the second PWM modulator is the most stable of the proposed modulators against temperature drift. In (35c), there is no term related to the finite input linear range and only  $R_{on}$  terms exist. In general,  $R_{on}$  is very small. Thus, the TC of the third PWM modulator may be much less than the estimated value.

#### IV. Experiment Results and Discussion

The prototype PWM modulators shown in Fig. 4, 6 and 8 were built using discrete components. The MOSFET arrays MC14007 were used for the CMOS OTAs shown in Fig. 3. The passive component values with 5% tolerance were adopted as follows:  $C_1 = C_3 = 2.3 \text{ nF}$ ,  $C_2 = 2 \text{ nF}$ ,  $R_1 = 3 \text{ k}\Omega$ , and  $R_2 = R_3 = 1 \text{ k}\Omega$ . The values of the current sources in Fig. 4 and 8 were set at  $0.5 \text{ mA}$ , and those in Fig. 6, at  $1 \text{ mA}$ , to make the oscillation frequency  $100 \text{ kHz}$ . The signal current  $i_A$  in Fig. 8 was produced by a voltage-to-current converter using LM324, a  $1\text{-k}\Omega$  resistor, and current mirrors. The supply voltage  $V_{DD}$  was  $+5 \text{ V}$  and the reference voltage  $V_R$  was  $+2.5 \text{ V}$ .

Fig. 13 to 15 show the measured voltage waveforms of the circuit in Fig 4, 6 and 8, in turn: from the top trace, the output voltage of the integrator, the threshold voltage of the Schmitt trigger, the PWM voltage, and the waveform-shaped PWM voltage. Since the parasitic capacitances and the input linear range of the OTAs built using MC14007 were relatively large and wide, the rising and falling time of the PWM waveforms generated were large, similar to those in the figures. Thus, they had to be shaped into a sharp rectangular wave with the two inverters connected in cascade. Observing the threshold voltage waveforms, one can see the non-ideal effects due to the finite input linear range of the OTAs such as those mentioned in subsection A of section II.

Fig. 16 shows the pulse-width modulated waveforms of the proposed modulators when their input signals were  $10 \text{ kHz}$  sine wave. Fig. 16 shows that the proposed modulators give the changed pulse width as the magnitude of the input signal.

Fig. 17 shows the measured duty cycle  $D$  of the modulators with varying the input signals. This figure shows that the duty cycles of the proposed modulator were linearly controlled by the input signal variations. The duty cycle change was nonlinear,

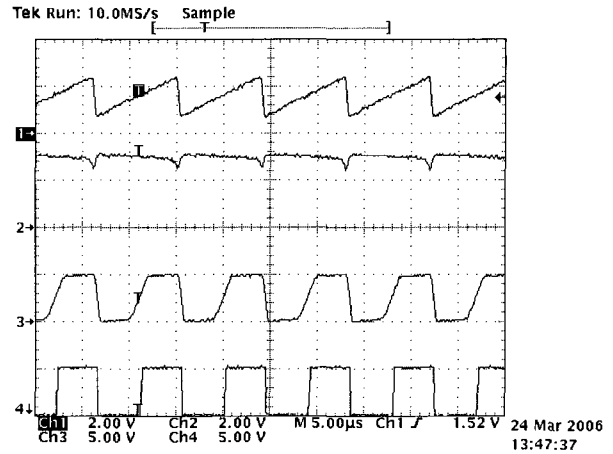


그림 13. 그림 4 회로의 측정된 전압 파형.  
Fig. 13. Measured voltage waveforms of the circuit in Fig. 4.

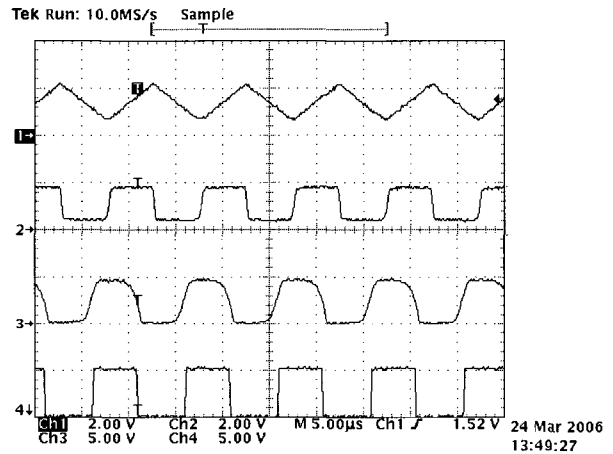


그림 14. 그림 6 회로의 측정된 전압 파형.  
Fig. 14. Measured voltage waveforms of the circuit in Fig. 6.

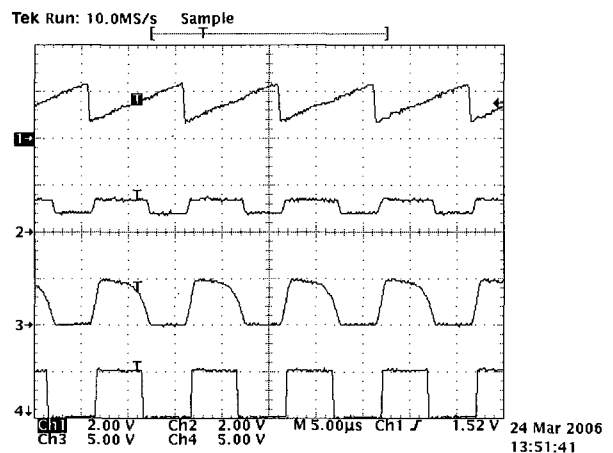
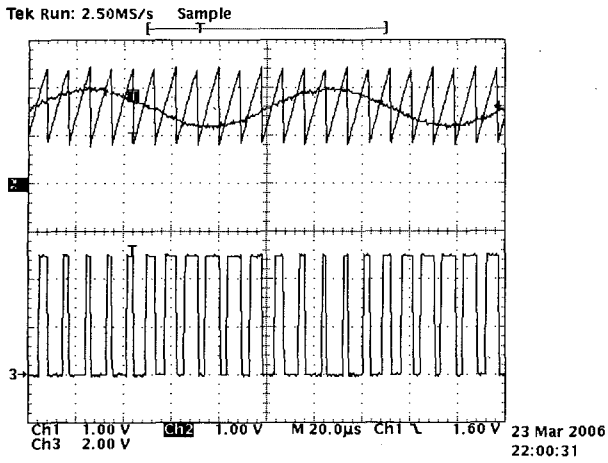
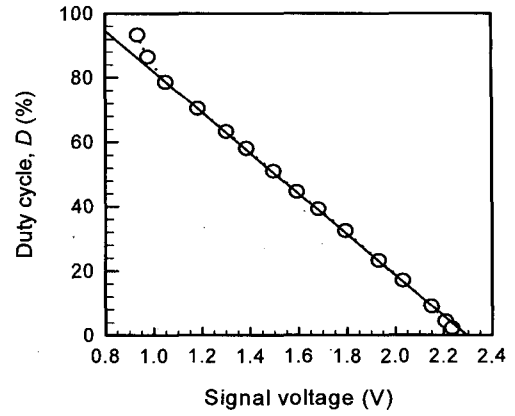


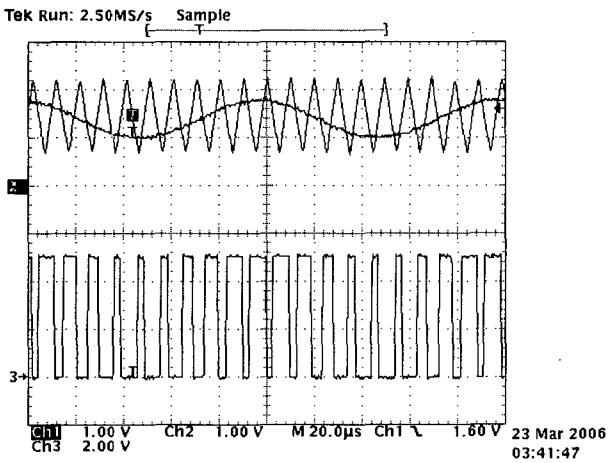
그림 15. 그림 8 회로의 측정된 전압 파형.  
Fig. 15. Measured voltage waveforms of the circuit in Fig. 8.



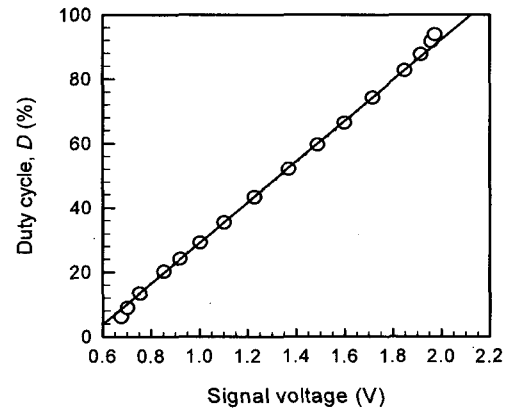
(a)



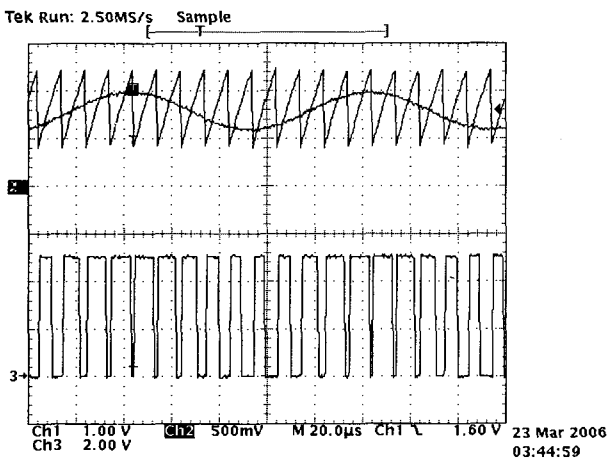
(a)



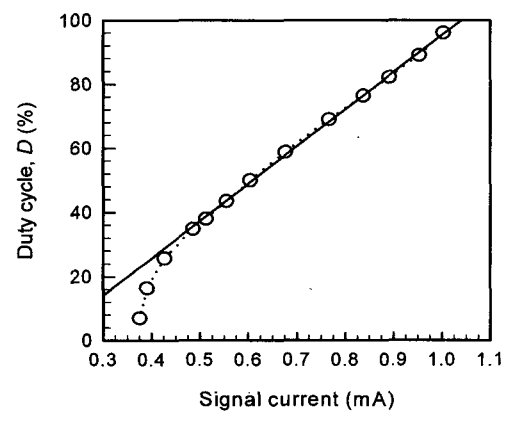
(b)



(b)



(c)



(c)

그림 16. 정현파 입력에 대해 측정된 PWM 파형들.  
Fig. 16. Measured PWM waveforms for a sinusoidal input.

however, at the vicinity of the duty cycle, which was 0 and 100%, because the edge of the PWM waveform

그림 17. 측정된 시비율  $D$  대 입력 신호.  
Fig. 17. Measured duty cycle  $D$  versus the input signals.

generated was not sharp, similar to the third trace in Fig. 13-15.

## V. Conclusion

Three PWM modulators using CMOS OTAs were described in this paper. Their design principle and circuit configurations are simple and their estimated temperature stabilities are relatively low. They can be fabricated into a monolithic IC form by scaling their component values. The experiment results presented demonstrate that the duty cycles of the modulators linearly change with their input signals. In addition, the current-sensing modulator can be used as a PFM circuit. The proposed modulators can be applied to analog-to-digital and power conversion systems.

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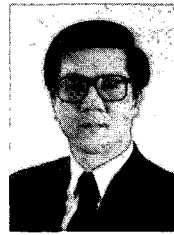
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