Optimization of Peltier Current Leads Cooled by Two-Stage Refrigerators

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ABSTRACT: A theoretical investigation to find thermodynamically optimum design conditions of conduction-cooled Peltier current leads is performed. A Peltier current lead (PCL) is composed of a thermoelectric element (TE), a metallic lead and a high temperature superconductor (HTS) lead in the order of decreasing temperature. Mathematical expressions for the minimum heat flow per unit current crossing the TE-metal interface and the minimum heat flow per unit current from the metal lead to the joint of the metal and the HTS leads are obtained. It is shown that the temperature at the TE-metal interface possesses a unique optimal value that minimizes the heat flow to the joint and that this optimal value depends on the material properties of the TE and the metallic lead but not the joint temperature nor electric current. It is also shown that there exists a unique optimal value for the joint temperature between the metal and the HTS leads that minimizes the sum of the power dissipated by ohmic heating in the current leads and the refrigerator power consumed to cool the lead, for a given length of the HTS.

Nomenclature

- A : cross-sectional area [m^2]
- COP : coefficient of performance
- I : electric current [A]
- J : electric current density, I/A [A/m^2]
- k : thermal conductivity [W/(m·K)]
- L : length [m]
- P : power [W]
- $\dot{Q}$: heat flow or heat transfer rate [W]
- q : heat flow per unit electric current, Q/I [W/A]
- T : temperature [K]
- V : Voltage [V]
- x : axial coordinate defined in Fig. 1 [m]

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Greek symbols

- α : Seebeck coefficient [V/K]
- ρ : electrical resistivity [Ω·m]
- τ : dummy variable

Subscripts

- 1 : first stage of two-stage refrigerator
- 2 : second stage of two-stage refrigerator
- C : cold end
- $crit$: critical
- $diss$: ohmic heating
- H : warm end
- HTS : high temperature superconductor
- I : interface between TE and metal lead
- J : joint of metal and HTS lead
- m : metal
- min : minimum
- opt : optimal
- P : Peltier (or thermoelectric) element

ref : refrigerator
tot : total

1. Introduction

Current leads, which supply power from power sources operating at room temperature to superconducting magnets operating at cryogenic temperature, represents the major source of heat leak into the magnets. The heat conduction through current leads and the heat generation due to the Joule heating in the leads are the two primary modes of such leak.[1,2]

The heat leak through conventional current leads can be substantial since they are made of normal metals such as brass or copper with high thermal conductivity and high electrical resistivity. A binary or hybrid current lead, which replaces the lower temperature part of a conventional metallic lead with a high temperature superconductor (HTS) current lead, can help reduce the heat leak since HTSs are perfect electrical conductors with much lower thermal conductivities than the normal metals.[1,3] Another configuration of the current lead proposed by Yamaguchi et al.,[4] is called the Peltier current lead (PCL) since it utilizes the Peltier effect to reduce the heat leak. The PCL has a thermoelectric element (TE) inserted into a conventional metallic lead or a hybrid current lead at the room temperature end.[2,5]

A number of studies have shown that the heat leak through current leads and the electrical power required to cool the leads could be reduced significantly by using the PCLs.[2,5-7] Xuan et al.[6] numerically obtained the optimized geometric factors, that is, the ratio of the length to cross-sectional area, of both the metal lead and the TE of a conduction-cooled PCL. Sato et al.[5] suggested the required relation between the temperature and the heat flow rate at the TE-metal interface to minimize the heat leak through PCLs cooled by boil-off helium gas. However, these works[5,6] may not provide enough understanding of the mechanism of heat leak reduction as well as information on the optimal design conditions of PCLs because the thermal behaviors of PCLs were analyzed by numerical calculations and the temperature at the metal-HTS boundary was always kept at 77 K.

In this paper, the optimal conditions of both the TEs and the metal parts of conduction-cooled PCLs were investigated analytically and the optimal temperature of the joint between the metal and the HTS leads that minimizes the power consumption of a two-stage refrigerator was obtained numerically. Also, the theoretical minimum power consumption of a Peltier current lead was compared with those of a conventional metallic lead and a hybrid current lead.

2. Analysis model

Figure 1 shows the schematic diagram of a superconducting magnet system containing PCLs. Each PCL is composed of a TE, a metallic lead and a HTS lead in the order of decreasing temperature. For a refrigerator-cooled system the joint of the metal and the HTS leads

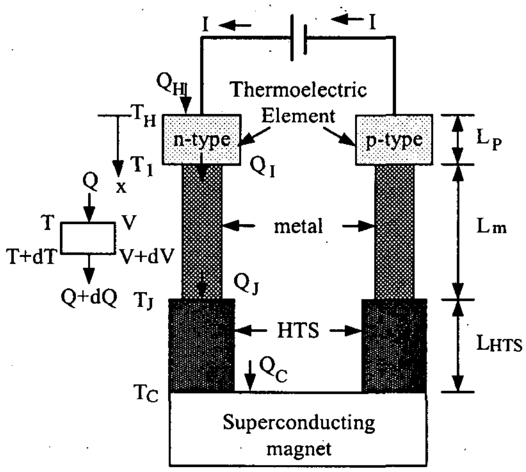

Fig. 1 Schematic diagram of a superconducting magnet system with PCLs.

is cooled by the first-stage of a two-stage refrigerator while the cold end of the HTS lead is cooled by the second-stage of the refrigerator. The temperature at the warm end, T_H, can be assumed to be held constant at room temperature, but T_J and T_C are variables dependent on the heat flow through the current lead and the first- and second-stage refrigeration capabilities of the two-stage refrigerator. The temperature at the TE-metal interface, T_I, is a variable to be determined to minimize the heat leak into the joint. The heat flow from the warm end to the cold end is defined to be positive.

Heat flow along the lead and electric current can be written as follows.[6]

$$Q = -kA\frac{dT}{dx} \pm \alpha IT \tag{1}$$

$$I = -\frac{A}{\rho}\left(\frac{dV}{dx} \pm \alpha\frac{dT}{dx}\right) \tag{2}$$

Here, α is zero for both the metal and the HTS. The sign in front of α is positive for a p-type TE and negative for an n-type TE.

Only the heat flow along the PCL with the n-type TE will be considered here. Seebeck coefficient(α) for a TE is assumed to be independent of temperature throughout this paper. For an infinitesimal length of the lead shown in Fig. 1, the one-dimensional energy balance is given as follows:

$$dQ + IdV = 0 \tag{3}$$

By combining Eqs. (2) and (3) we can obtain the following expression.

$$d(Q+\alpha IT) = \frac{\rho I^2}{A}dx \tag{4}$$

Substituting Eq. (1) into Eq. (4) gives the following relation.

$$(Q+\alpha IT)d(Q+\alpha IT) = -\rho kI^2 dT \tag{5}$$

Equation (5) reduces to Eq. (18) in Chang and Sciver[3] if α is zero for both the metal and HTS.

2.1 Optimization of thermoelectric elements

Integrating Eq. (5) over the TE gives

$$\frac{1}{2}\left[(Q_I+\alpha_P IT_I)^2 - (Q_H+\alpha_P IT_H)^2\right] = -I^2\int_{T_H}^{T_I}\rho_P k_P dT \tag{6}$$

where the variables with subscript P denote the material properties of the TE (or the Peltier element). Equation (6) can be rearranged for the heat flow from the TE to the metallic lead.

$$Q_I = \sqrt{(Q_H+\alpha_P IT_H)^2 + 2I^2\int_{T_I}^{T_H}\rho_P k_P dT} - \alpha_P IT_I \tag{7}$$

Equation (7) shows that Q_I has its minimum when $Q_H+\alpha_P IT_H$ is zero for given T_H and T_I. It can be observed from Eq. (1) that the axial temperature gradient at the warm end of the TE should be zero to minimize Q_I. Thus, the minimum heat flow per unit current crossing the TE-metal interface can be written as follows.

$$(q_I)_{\min} \equiv \left(\frac{Q_I}{I}\right)_{\min} = \sqrt{2\int_{T_I}^{T_H}\rho_P k_P dT} - \alpha_P T_I \tag{8}$$

The optimal heat flow at any axial location on the TE, where the temperature is T, can be obtained by integrating Eq. (5) from the warm end to that location and letting $Q_H+\alpha_P IT_H = 0$.

$$(Q_P)_{opt} = I\sqrt{2\int_T^{T_H}\rho_P(\tau)k_P(\tau)d\tau} - \alpha_P IT \tag{9}$$

Substituting Eq. (9) into Eq. (1) and then integrating the resultant equation over the TE gives the optimal geometric relation for the TE as follows.

$$\left(\frac{L_P I}{A_P}\right)_{opt} = \int_{T_I}^{T_H} \frac{k_P(T)}{\sqrt{2\int_T^{T_H} \rho_P(\tau) k_P(\tau) d\tau}} dT \quad (10)$$

2.2 Optimization of metallic leads

The following expression can be obtained by integrating Eq. (5) over the metal lead. The terms containing α has disappeared since α is zero for normal metals.

$$\frac{1}{2}\left(Q_J^2 - Q_I^2\right) = -I^2 \int_{T_I}^{T_J} \rho_m k_m dT \quad (11)$$

Equation (11) can be rearranged to obtain the heat flow from the metal lead to the joint of the metal and the HTS leads.

$$Q_J = \sqrt{Q_I^2 + 2I^2 \int_{T_J}^{T_I} \rho_m k_m dT} \quad (12)$$

Equation (12) shows that Q_J has a minimum when Q_I is minimized for given T_I and T_J. The expression for minimum Q_J can be obtained by substituting Eq. (8) into Eq. (12).

$$(q_J)_{\min} \equiv \left(\frac{Q_J}{I}\right)_{\min} = \sqrt{\left(\sqrt{2\int_{T_I}^{T_H} \rho_P k_P dT} - \alpha_P T_I\right)^2 + 2\int_{T_J}^{T_I} \rho_m k_m dT} \quad (13)$$

The optimal heat flow at any axial location of the metallic lead, where the temperature is T, can be obtained by integrating Eq. (5) from the TE-metal interface to that location and letting $Q_I = (Q_I)_{\min}$ and $\alpha = 0$.

$$(Q_m)_{opt} = I\sqrt{\left(\sqrt{2\int_{T_I}^{T_H} \rho_P k_P dT} - \alpha_P T_I\right)^2 + 2\int_T^{T_I} \rho_m k_m d\tau} \quad (14)$$

Substituting Eq. (14) into Eq. (1) and then integrating the resultant equation over the metallic lead gives the optimal geometric relation for the metal lead.

$$\left(\frac{L_m I}{A_m}\right)_{opt} = \int_{T_J}^{T_I} \frac{k_m(T)}{\sqrt{\left(\sqrt{2\int_{T_I}^{T_H} \rho_P k_P dT} - \alpha_P T_I\right)^2 + 2\int_T^{T_I} \rho_m k_m d\tau}} dT \quad (15)$$

2.3 Optimal temperature at TE-metal interface

In sections 2.1 and 2.2, the TE and the metallic lead have been optimized for given T_H, T_I and T_J. The temperature at the warm end, T_H, is usually the room temperature or the cooling water temperature if a water cooler is used, and the temperature at the joint of the metal and the HTS leads, T_J, is determined by the heat flow to the joint and the first-stage refrigeration capability of the two-stage refrigerator. However, the temperature at the TE-metal interface, T_I, can be readily controlled by adjusting the geometric factors of the TE and the metallic leads. Hence, the next step to further optimize PCLs is to determine the optimal T_I which minimizes the heat flow from the metallic lead to the joint, Q_J.

Equation (13) can be rewritten as:

$$(q_J)_{\min}^2 = \left(\sqrt{2\int_{T_I}^{T_H} \rho_P k_P dT} - \alpha_P T_I\right)^2 + 2\int_{4K}^{T_I} \rho_m k_m dT - 2\int_{4K}^{T_J} \rho_m k_m dT \quad (16)$$

It can be observed that minimizing $(q_J)_{\min}$ is equivalent to minimizing the sum of the first and the second terms on the right-hand side of Eq. (16) since the last term is constant for given T_J. Hence, $(T_I)_{opt}$ which minimizes $(q_J)_{\min}$ does not depend on electric current I and T_J, but depends on the thermal conductivities and electrical resistivities of the TE and metallic parts. But $(q_J)_{min}$ increases as T_J is lowered. Equation (10) shows that the optimal ratio of the length to the cross-sectional area for a TE does not depend on T_J but is inversely proportional to the electric current I.

2.4 Heat leak through a HTS

Heat leak through a HTS is constant in axial direction since the heat generation due to the Joule heating is zero in a HTS. Heat flow through a HTS can be obtained by integrating Eq. (1) over the HTS and using $\alpha = 0$ for a HTS.

$$Q_C = \frac{A_{HTS}}{L_{HTS}} \int_{T_C}^{T_J} k_{HTS}\, dT \qquad (17)$$

Heat flow per unit current can be written as follows.[3]

$$q_C \equiv \frac{Q_C}{I} = \frac{1}{J_{HTS} L_{HTS}} \int_{T_C}^{T_J} k_{HTS}\, dT \qquad (18)$$

Equation (18) shows that q_C gets smaller as the current density J_{HTS} increases. The current density J_{HTS} should not be larger than the critical current density J_{crit} for the HTS not to lose its superconducting property. The critical current density of Bi-2223 can be represented reasonably well by the following relation.

$$J_{crit} = J_{C0}\left(1 - \frac{T}{T_{crit}}\right) \qquad (19)$$

where $J_{C0} = 10{,}000$ A/cm^2 and $T_{crit} = 104$ K.[3]

2.5 Power consumption

The electrical power consumed by a current lead in a superconducting system is the sum of the power dissipated by ohmic heating in the TE and metallic parts and that consumed by a refrigerator which cools the current leads. For a system using hybrid current lead or Peltier current lead a two-stage refrigerator is used to cool both the cold end of a HTS lead and the joint of the metal and the HTS leads.

The power consumed by ohmic heating can be obtained by integrating Eq. (3) over the TE and metallic leads.

$$P_{diss} = -\int_H^J I\, dV = \int_H^J dQ = Q_J - Q_H \qquad (20)$$

As shown in section 2.1 $Q_H = -\alpha_P I T_H$ for optimally designed Peltier current leads and $Q_H = 0$ for optimally designed hybrid current leads. The minimum value of Q_J can be obtained by substituting $(T_J)_{opt}$ into T_J of Eq. (13).

The first stage of a two-stage refrigerator cools the joint of the metal and HTS leads and the second stage cools the cold end of the HTS lead. The refrigeration powers of the first stage and the second stage can be expressed as follows.

$$P_{ref,1} = \frac{(Q_J - Q_C)}{COP_1} \qquad (21)$$

$$P_{ref,2} = \frac{Q_C}{COP_2} \qquad (22)$$

The COPs of the first and the second stage of a two-stage refrigerator are obtained from the relation suggested by Chang and Sciver.[8]

$$COP_1 = \frac{T_J(0.4198\, T_J + 2.740)}{(T_H - T_J)(T_J + 1115)} \qquad (23)$$

$$COP_2 = \frac{T_C(0.1202\, T_C + 1.316)}{(T_H - T_C)(T_C + 84.81)} \qquad (24)$$

3. Results and discussion

The material properties of the TE used in this work are $\alpha_P = 0.19 \times 10^{-3}$ V/K, $\rho_P = 0.85 \times 10^{-5}$ Ω·m and $k_P = 1.90$ W/(m·K).[9] Pure copper with residual resistance ratio (RRR) of 100 is used as the metal lead, and the electrical resistivity and the thermal conductivity of the copper as functions of temperature have been obtained from Maehata et al.[10] and NIST,[11] respectively. The thermal conductivity of Bi-2223 was obtained from Herrmann et al.[12] The length of the HTS lead is 0.2 m. The warm end temperature, T_H, and the cold end temperature, T_C, are 300 K and 4.2 K, respectively.

Figure 2 shows the effect of the temperature at the TE-metal interface, T_I, on the minimum heat flow to the joint of the metal and the HTS leads per unit current, $(Q_J/I)_{\min}$, when T_J=77 K. It can be clearly seen that there exists an optimal T_I which minimizes $(Q_J/I)_{\min}$ since the first term on the right-hand side of Eq. (16) increases but the second term decreases as T_I increases. Figure 2 shows that the optimal value of T_I is about 219 K, which does not depend on T_J as described in section 2.3. It can be seen that the TE is thermally decoupled from the metal lead when T_J is about 237 K since $(Q_I/I)_{\min}$ is zero at this temperature. It is obvious from Fig. 2 that the optimal condition for a PCL is not the condition which thermally decouples the TE from the metallic lead, suggested by Gehring et al.,[7] but the condition which minimizes Eq. (13) or (16).

The optimized geometric factor, that is, the ratio of the length to cross-sectional area of the metallic lead of the PCL, is compared with that of the conventional metallic lead without a TE in Fig. 3. It shows that attaching a TE at the warm end of the metallic lead reduces $(L_m I/A_m)_{opt}$ significantly since the warm end temperature of the metal lead of the PCL, $(T_I)_{opt}$ =219 K, is far lower than that of the metal lead without a TE which is T_H =300 K. The ratio of the length to cross-sectional area of the TE, $(L_P I/A_P)_{opt}$, is far smaller than that of the metal lead and it does not vary with T_J since the temperature difference between the warm end and the cold end of the TE is kept constant. $(L_P I/A_P)_{opt}$ calculated from Eq. (10) using the material properties of the TE used in this work is 6018. For a TE with A_P =225×10^{-6} m^2 the optimal length is 0.0135 m when current I=100 A and the optimal length is 0.0027 m when I=500 A.

Figure 4 shows the effect of the temperature at the joint of the metallic lead and the HTS lead, T_J, on the power consumed by a PCL. The power dissipated in the TE and the metallic lead due to ohmic heating is negligible compared with the power consumed by the first stage of the refrigerator and does not vary with T_J. The power consumed by the second stage of the refrigerator is far smaller than that consumed by the first stage when T_J is below 100 K, but it increases rapidly when T_J is

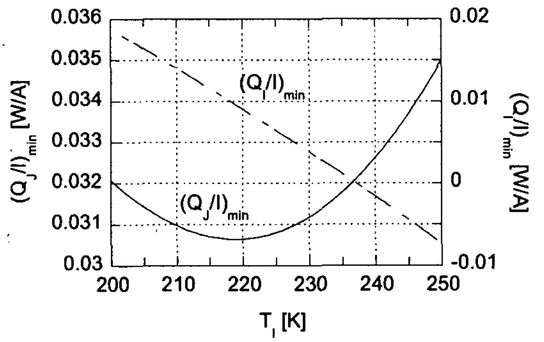

Fig. 2 Effect of the TE-metal interface temperature on the minimum heat flow to the joint (T_J=77 K).

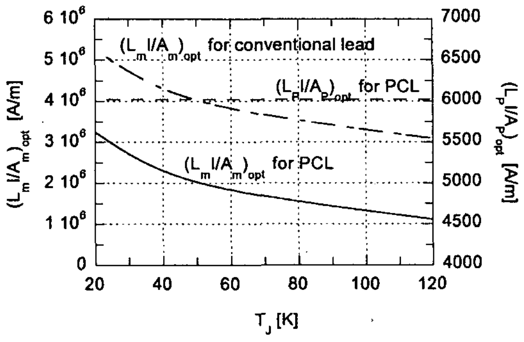

Fig. 3 The optimal geometric factors for a PCL and a conventional metallic lead as a function of the joint temperature.

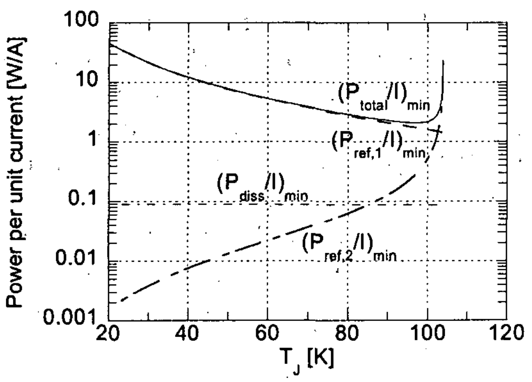

Fig. 4 Minimum power per unit current of Peltier current leads.

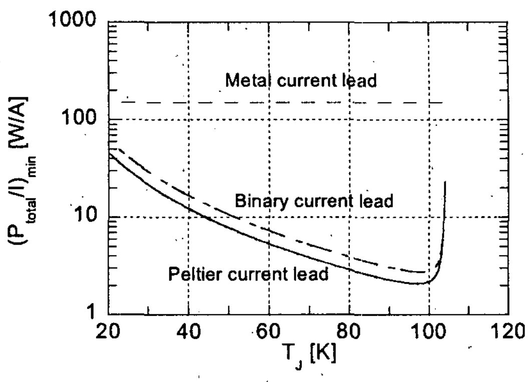

Fig. 5 Comparison of minimum total power of metal, hybrid and Peltier current leads.

above 100 K. Equation (18) shows that the critical current density decreases rapidly as T_J increases beyond 100 K. Hence, the cross-sectional area of the HTS should increase for the current I to be kept constant and the heat flow per unit current increases. There exists an optimal value of T_J at which the total power consumption has a minimum since the refrigeration power consumed by the first stage decreases and that by the second stage increases as T_J increases. The optimal value of T_J is about 97 K and the minimum total power consumption per unit current is about 2.10 W/A.

Figure 5 compares the minimum total power consumption of the PCL with those of the conventional metallic lead without a TE and the hybrid current lead. The electrical power consumption of a current lead is defined as the sum of the power dissipation due to the ohmic heating in the lead and the refrigeration power needed to remove the heat leak through the lead as described in section 2.5. The TE and the metallic part of the lead satisfy the optimal geometric relations given in Eqs. (10) and (15), respectively. It can be seen that the use of a hybrid current lead or a PCL instead of a conventional metallic lead reduces the total power consumption significantly. The minimum total power consumption per unit current of the binary current lead is 2.75 W/A when T_J is 98 K and that of the PCL is 2.10 W/A when T_J is 97 K.

The effects of the HTS length on the optimum junction temperature, T_J, and the minimum total power are shown in Fig. 6. As can be seen in Eq. (18) the magnitude of the axial temperature gradient in the HTS increases as L_{HTS} decreases. Hence, the heat leak through the HTS and the total power consumption increase as L_{HTS} decreases. It can be seen that the optimum junction temperature of the PCL is about 1 K lower than that of the hybrid lead and that the optimum junction temperature increases as L_{HTS} increases. The minimum total power decreases as L_{HTS} increases, but it does

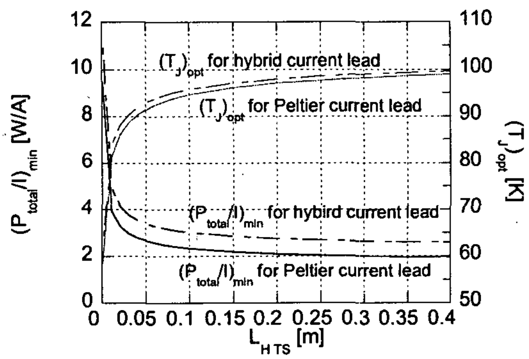

Fig. 6 Effects of the HTS length on the minimum total power and the optimum junction temperature.

not decrease significantly as L_{HTS} increases beyond about 0.1 m.

4. Conclusions

In this paper an optimization method to minimize the total electrical power consumption of a Peltier current lead, which is conduction-cooled by a two-stage refrigerator, has been proposed. Mathematical expressions for the optimal geometric relation, that is, the ratio of the length to cross-sectional area, were obtained for both the TE and the metal part of a Peltier current lead. It was shown that the TE-metal interface temperature has a unique optimal value which minimizes the heat flow from the metal lead to the joint and it does not depend on the electrical current or the joint temperature but depends on the material properties of the TE and the metal lead. There exists an optimal value of T_J at which the total power consumption is minimum. It was shown that the use of a hybrid current lead or a PCL instead of a conventional metallic lead could reduce the total power consumption significantly. The minimum total power consumption per unit current of the Peltier current lead was lower than that of the binary current lead by about 23.6%.

Acknowledgement

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