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Experimental Verification of the Unified Formula for Electromechanical Coupling Coefficient of Piezoelectric Resonators

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Abstract

In a previous theoretical paper, we have derived a unified formula by considering 2-D coupled mode vibrations. The unified formula for electromechanical coupling coefficient of piezoelectric resonator was verified experimentally. The capacitance change near the resonant frequency was investigated to estimate the effective coupling coefficient of the resonator instead of the conventional method based on 1-D model. The susceptance spectra were measured for the seven samples of piezoelectric resonator with different aspect ratio. Excellent agreement between theoretical and experimental results was obtained.

Keywords: *Electromechanical coupling coefficient, Coupled vibration, Aspect ratio, Piezoelectric ceramic resonator, Ultrasonic array transducer.*

I. Introduction

The electromechanical coupling coefficient k is the most important parameter for the characterization of a piezoelectric material and is very important for the design of electromechanical devices. The effective k- value appears to depend on the aspect ratio of the resonator. The physical definition of the electromechanical coupling coefficient is given by an energy ratio: $k=U_m/\sqrt{U_eU_d}$, where U_m is the electromechanical coupling energy, U_e and U_d are the elastic and dielectric energies, respectively. [1] However, experimentally measured k-values appear to depend strongly on the aspect ratio of the piezoelectric resonator. The physical origin of this aspect ratio dependence comes from the fact that the electromechanical energy conversion is strongly influenced by the boundary conditions and also by the mode coupling effect when the frequencies of two or more modes are close to each other. As can be found in the literature, the difference of the coupling coefficient values for different aspect ratios is large. For example, in the case of $Pb(Zr,Ti)O_3$ [PZT] ceramic resonators, k_{33} (a long bar along poling direction) is as high as 70%, while k_t (thin plate) is only about 48% and k_{33}' (rectangular slender bar) is about 65%. These multiple k-values are not only inconvenient but also confusing in physical concept since they all correspond to the longitudinal resonance along the poling direction.

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Recently, based on mode coupling theory and the original energy ratio definition of the electromechanical coupling coefficient, we have derived a unified formula [2] that can be used to calculate the electromechanical coupling coefficient for any given aspect ratio for a rectangular resonator, such as those used in ultrasonic array transducers. However, it is hard to verify the formula experimentally by the conventional method using the resonance and anti-resonance frequencies because the method was derived based on 1-D approximation. Moreover the resonance and anti-resonance frequencies cannot be determined correctly in a coupled vibrating resonator. In this paper, we verify the formula experimentally by the capacitance change between before and after resonant frequency in susceptance spectra of the resonator. As the samples, the seven piezoelectric resonators with different aspect ratio are fabricated by PZT ceramics.

II. Unified Formula for Coupling Coefficient

The resonator under analysis is shown in Fig. 1 together with its dimensions. As described in detail in Ref. [2], the unified formula is given by

$$k=\frac{\left[\frac{1+\sigma}{1-\sigma}\left(g^{2}(G)-1\right)\frac{s_{13}^{E}}{s_{11}^{E}}d_{31}+\left(d_{33}-\frac{s_{13}^{E}}{s_{11}^{E}}d_{31}\right)\right]}{\sqrt{\left[\frac{1+\sigma}{1-\sigma}\left(g^{2}(G)-1\right)\frac{\left(s_{13}^{E}\right)^{2}}{s_{11}^{E}}+\left(s_{33}^{E}-\frac{\left(s_{13}^{E}\right)^{2}}{s_{11}^{E}}\right)\right]\left[\frac{1+\sigma}{1-\sigma}\left(g^{2}(G)-1\right)\frac{d_{31}^{2}}{s_{11}^{E}}+\left(\varepsilon_{33}^{T}-\frac{d_{31}^{2}}{s_{11}^{E}}\right)\right]}} \tag{1}$$

where the aspect ratio is defined as $G = l_3/l_2$ and the dimension l_1 is kept long so that the boundary condition along x_1 can be treated a constant strain, s_{ij}^{E} are components of the elastic compliance, d_{ij} are the piezoelectric coefficients and the function $g(G)$ is given by [2]

$$g(G)=\frac{G}{2}\frac{\pi}{X_t}\sqrt{\frac{c_{11}^{E}}{c_{33}^{D}\left(1-\Gamma^{2}\right)}}\frac{f_1}{f_2} \tag{2}$$

and the two frequencies are defined as

$$l_3 f_2=\sqrt{\frac{G^{2}c_{11}^{E}}{8\rho}+\frac{c_{33}^{D}X_t^{2}}{2\pi^{2}\rho}+\frac{\sqrt{-16\pi^{2}c_{11}^{E}c_{33}^{D}X_t^{2}\left(1-\Gamma^{2}\right)G^{2}+\left(G^{2}c_{11}^{E}\pi^{2}+4c_{33}^{D}X_t^{2}\right)^{2}}}{8\pi^{2}\rho}} \tag{3a}$$

$$l_3 f_2=\sqrt{\frac{G^{2}c_{11}^{E}}{8\rho}+\frac{c_{33}^{D}X_t^{2}}{2\pi^{2}\rho}+\frac{\sqrt{-16\pi^{2}c_{11}^{E}c_{33}^{D}X_t^{2}\left(1-\Gamma^{2}\right)G^{2}+\left(c_{11}^{E}\pi^{2}G^{2}+4c_{33}^{D}X_t^{2}\right)^{2}}}{8\pi^{2}\rho}} \tag{3b}$$

where X_t is the first root of the transcendental equation.

$$1-k_t^{2}\frac{\tan X}{X}=0 \tag{4a}$$

Here, k_t is electromechanical coupling coefficient of thickness mode for thin plate, and the mode coupling coefficient Γ is given by

$$\Gamma=\frac{c_{13}^{E}}{\sqrt{c_{33}^{E}c_{11}^{E}}} \tag{4b}$$

Near the resonance, the solution of Eq. (4a) can be approximated as

$$X_t\approx\frac{1}{2}\pi\left(1-\frac{4k_t^{2}}{\pi^{2}}\right) \tag{4c}$$

The novel feature of Eq. (1) is that it can predict the electromechanical coupling coefficient of rectangular resonators of any aspect ratio. As expected, k is a kink type monotonic function of G and changes from k_t to k_{33}' as G changes from a very small value to a very large value.

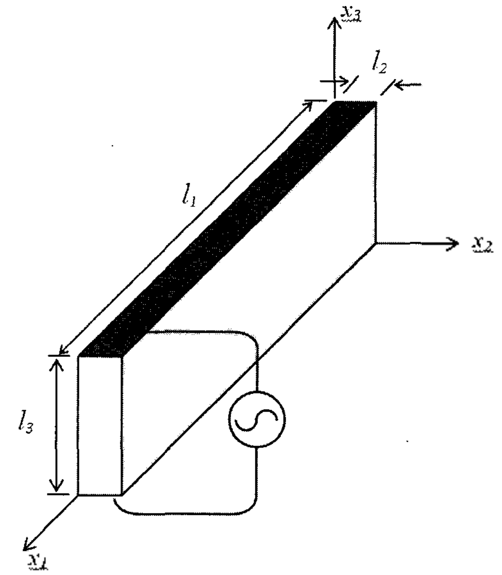

Fig. 1. Illustration of the resonator and its dimensions.

III. Experiments and Results

A) Specimens

In the experiments, seven resonators were fabricated using Motorola PZT3203HD material as shown in Fig. 2. In order to minimize the sample to sample variation cause by the fabrication process, two of the 3-dimensions in each sample are fixed, i.e., l_1 = 10mm and l_3 = 1mm while l_2 is the only variable. These samples were all cut from the same uniform plate to guarantee consistency. All material constants listed in Table I were actually measured by us using the combination of resonance method and ultrasonic method [5-7]. Noticeably, the dielectric and piezoelectric coefficients of these samples are larger compared to other published data on PZT ceramics and some data are different

Table 1. Measured material constants of Motorola PZT3203HD.

Elastic properties $c_{\lambda\mu}$, $s_{\lambda\mu}$	Piezoelectric constant $e_{i\lambda}$, $d_{i\lambda}$	Dielectric constant	Density (kg/m^3)
c^E_{11}=141.3×10^9	e_{31}=-3.69	ε^S_{33}=1461.93	ρ=7813.7
c^E_{12}=84.8×10^9	e_{33}=26.84	e_{33}=3876.33	
c^E_{13}=95.0×10^9			
c^E_{33}=122.0×10^9			
s^E_{11}=15.25×10^{-12}	d_{31}=-314.62×10^{-}		
s^E_{12}=-2.45×10^{-12}	d_{33}=710.00×10^{-1}		
s^E_{13}=-9.96×10^{-12}			
s^E_{33}=23.71×10^{-12}			

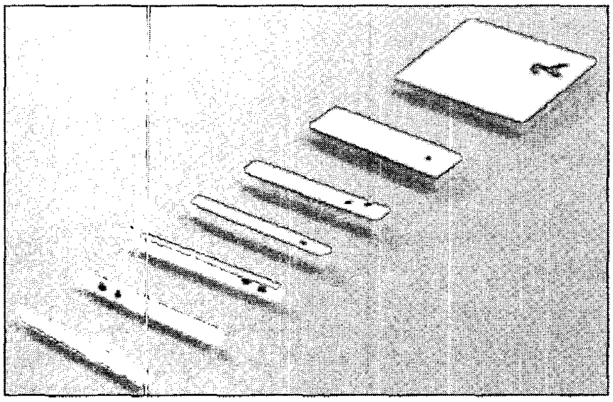

Fig. 2. Samples of seven resonators.

from the data sheet provided by the material manufacture. For accuracy, four identical resonators were prepared for each specified aspect ratio. Direct material property characterization showed that the property variation from sample to sample is less than 0.2%.

B) Procedures and results

Experimentally, it is easy to check the correctness of the limiting values of Eq. (1) since the k values can be calculated from the impedance spectrum based on the 1-D formula [1, 3]

$$k = \sqrt{\left(\frac{\pi}{2}\frac{f_r}{f_a}\right)\cot\left(\frac{\pi}{2}\frac{f_r}{f_a}\right)} \tag{5}$$

where f_r and f_a are the resonance and anti-resonance frequencies. However, we can not use Eq. (5) for resonators with arbitrary aspect ratios because the formula was derived based on 1-D approximation, which is valid only for the case of very large G and very small G. In addition, the resonance and anti-resonance frequencies cannot be determined correctly when there are mode couplings.

In order to experimentally measure the k values of resonators with intermediate aspect ratios, we adopt an averaging scheme to deal with the mode coupling effect. Based on the equivalent circuit theory for a piezoelectric material, near a resonance, the piezoelectric resonator can be modeled by an LRC resonance circuit as shown in Fig. 3(a), and the effective electromechanical coupling coefficient k_{eff} may be calculated from the following equation. [4]

$$k^2_{eff} = \frac{C^T - C_0}{C^T} = 1 - \frac{C_0}{C^T} \tag{6}$$

where, C_0 is the clamped capacitance and $C^T = C_0 + C_l$ is the low frequency limit of the capacitance, and C_l is the capacitance in series with the effective inductance. Capacitance C_0 and C^T are related to the slopes of the linear part before and after the resonance in the frequency spectrum of the susceptance. The C_0 and C^T values for each case can be derived from the slopes of the susceptance spectra as shown in Fig. 3(b). The asymptotic straight lines of ωC_0 (dotted line) and ωC^T (dashed line) are derived using the susceptance data below and above the resonance frequency. When mode coupling occurs, multiple peaks will appear and the peak positions will be shifted, but as will be demonstrated below that the average slope does not change significantly, therefore, the slopes of the spectra may be used to estimate the C_0 and C^T values.

The admittance spectra of these samples are measured using HP4294A precision impedance analyzer. Typical susceptance spectra, i.e., the imaginary part of the admittance spectra, are shown in Fig. 4(a-d). The average straight lines used for estimating the C_0 and C^T values for each case are also shown on

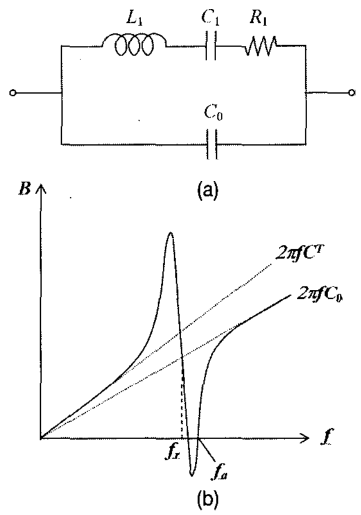

Fig. 3. (a) Equivalent circuit of a piezoelectric resonator near a resonance (b) Capacitance change between before and after the resonance in the frequency spectrum of the susceptance.

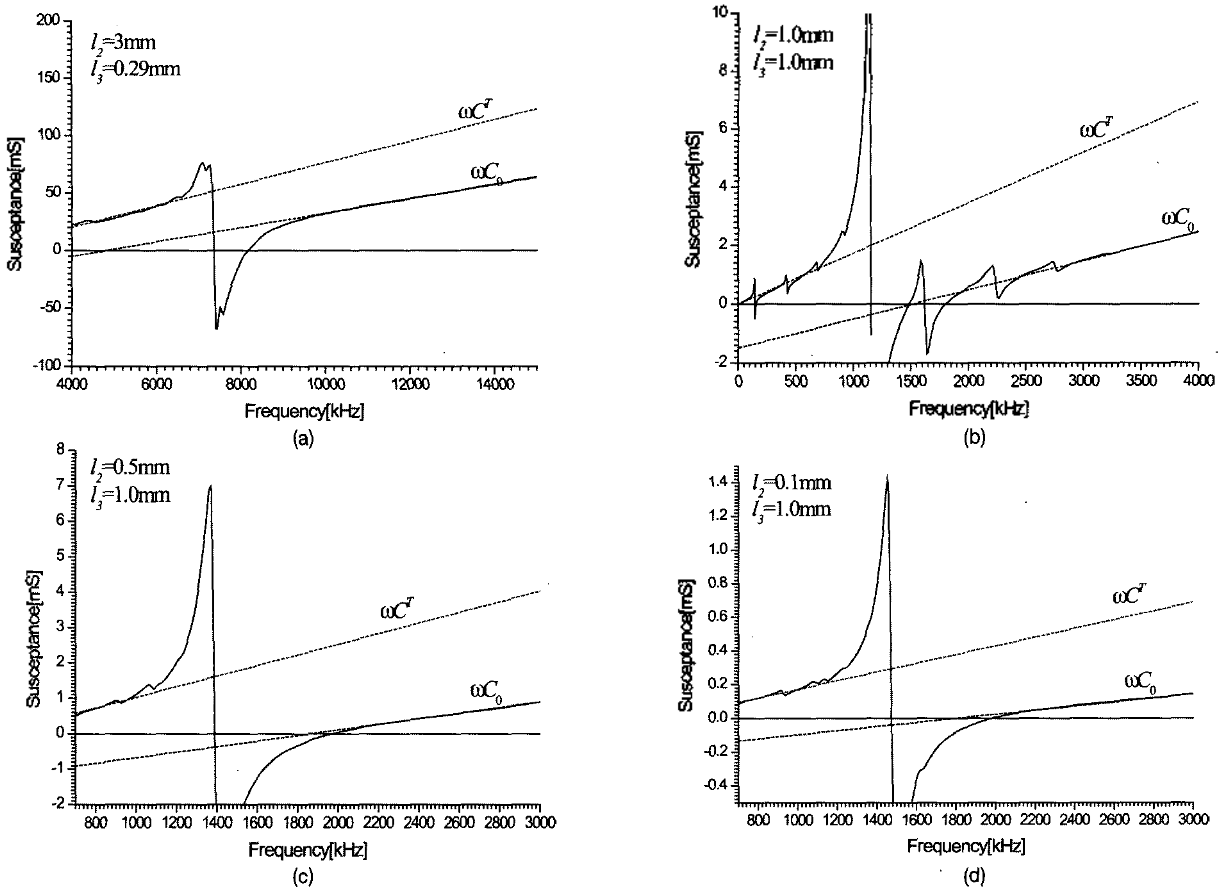

Fig. 4. Typical measured susceptance spectra. The average straight lines used for the calculations of the effective electromechanical coupling coefficients are also shown for each case.

these figures. As expected, for very large and very small aspect ratios, the spectra are "clean" and one can easily identify the resonance and anti-resonance frequencies. But when the aspect ratio is near 1, strong mode coupling occurs which invalidates the 1-D formula Eq. (5). On the other hand, for the cases of G=1 and G=0.67, we can clearly identify the two coupled modes, hence, both the longitudinal and transverse resonant frequencies can be obtained. As shown in Fig. 5, the measured resonance frequencies, including the transverse modes, match well with the theoretical predictions calculated using Eq. (3) for all cases [2].

Based on the slopes of these spectra, the electromechanical coupling coefficient for each case is calculated using Eq. (6) and the final results are shown in Fig. 6 together with the theoretical

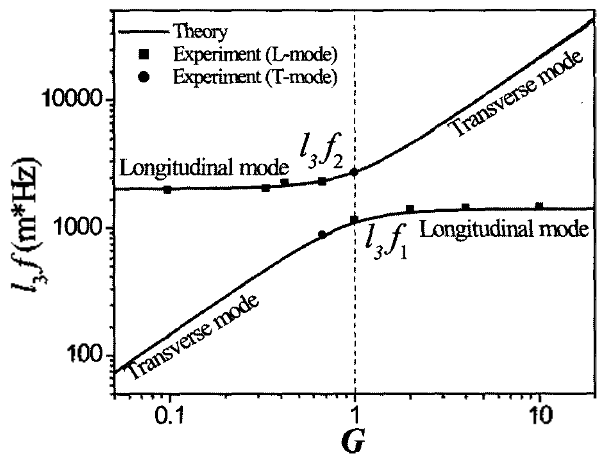

Fig. 5. Comparison of theoretical and measured resonant frequencies for different aspect ratio cases.

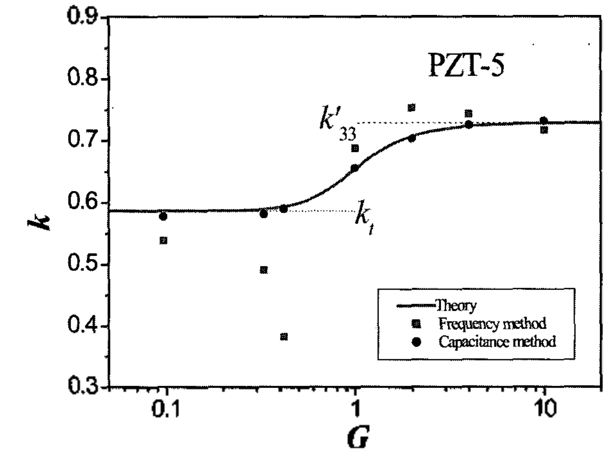

Fig. 6. Electromechanical coupling coefficient as a function of the aspect ratio. The line is from the unified formula Eq. (1), the squares are results calculated using Eq. (5) and the dots are results from the capacitance formula Eq. (6).

predicted curve and the k-values calculated using the traditional 1-D formula Eq. (5). Obviously, the 1-D formula does not work when the aspect ratio is close to 1. The results make no sense and are unphysical. On the other hand, the measured effective k-values using the capacitance scheme described above are in excellent agreement with the theoretical predictions from Eq. (1).

IV. Conclusion

We have measured the electromechanical coupling coefficients of seven different aspect ratio resonators. The measured k-values were calculated based on the capacitance formula with the capacitance values C_0 and C^T estimated from the low and high frequency slopes of the susceptance spectra. These measured electromechanical coupling coefficients show excellent agreement with theoretical predictions from Eq. (1), while the electromechanical coupling coefficient calculated using the classical formula Eq. (5) produced very unphysical results. In conclusion, our experimental results confirmed the validity of the unified formula.

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