A Practical Method of Balancing a Rigid Rotor

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Diagnosis and repair tasks of an unbalanced rigid rotor reduce the chances of unexpected failure and the consequent losses in production, time, and money. This paper presents investigation of a balancing system for equilibration of rigid rotor unbalance. A practical vibration signal based method is developed for unbalance diagnosis using wavelet technology and a Lissajous diagram. This paper shows that a mass unbalance can be efficiently estimated through an appropriate narrow-band filter used to extract the required spectra component. The wavelet technology is used to design specified narrow filter bank. A modified Lissajous diagram is also introduced with statistical analysis to compute the phase position. Several experimental tests demonstrate the effectiveness in balancing the mass unbalance of a rigid rotor.

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1. Introduction

A rotating machine is an excellent means for kinetic energy storage for electromagnetic launch applications due to high energy density available in a spinning rotor and flywheels. A rigid rotor is a core part of such a rotating machine and the unbalance of the rotor considered does not only cause vibration, but it also transmits rotational force to the machine and to the supporting structure. These transmitted forces may damage the system and shorten its working life[1-4]. Very often, a mass unbalance results very dangerous damage in a rotating machine, which can lead to a catastrophe. Actually, there have been many reports on this disasters[5-7]. Thus, it is necessary to develop a practical method to estimate the mass unbalance and its phase position, as efficient as possible, to keep the stability of a system, to guarantee the safety for men and to save the running cost.

Over the years, many studies have commonly been employed. The modal balancing technique was developed and further investigated by several researchers[8-10], but in this method, the critical speed mode shapes of the rotor must be known in advance and mass distribution determined from the geometry of the rotor is not accurate. Although the finite element method (FEM) also played an important role in the analysis of a rotating machine because of its usefulness in vibration diagnosis, there still remain some difficulties in the computational aspect of unbalance analysis due to its inconvenient classical modal and complicated dynamics properties[11]. Furthermore, various unbalance diagnosis methods have also been developed and discussed with spectrum analysis in the frequency domain, such as the method recently proposed by the authors using Fourier-based signal processing[12]. However, the spectra component given by Fourier analysis usually cover an extremely wide frequency domain compared with the very narrow-band characteristics of the critical frequency. This makes the conventional Fourier analysis difficult to estimate the mass unbalance accurately.

In this paper, a practical vibration signal based method for balancing the rigid rotor using wavelet and a Lissajous diagram is proposed. The above-mentioned difficulties experienced by conventional Fourier analysis are addressed. A wavelet based signal processing is applied to extract a narrow-band vibration spectrum that contains the unbalance features at the critical frequency (rotating speed). This enables the mass unbalance to be estimated effectively by analyzing the unbalance spectrum component at the critical frequency extracted. Also the phase position can then be obtained by computation of the modified Lissajous diagram with combined uses of the vibration signal and reference time signal after statistical signal processing. In this paper, given the fact that the vibration signal and reference time signal are obtained through the sensors on the rotor, and the rotating speed is measurable by the sensors.

In the following, the theory used for mass unbalance and its phase position estimation is briefly described. Then the proposed method for balancing the rigid rotor is presented. Finally, several experimental tests are also conducted to validate the efficiency and applicability of the proposed method.

2. Diagnosis of rigid rotor unbalance

2.1 Mass unbalance analysis

The diagnosis of the mass unbalance of the rigid rotor can be viewed as a problem of vibration signal analysis. Theoretically only the characteristic frequency on rotating speed corresponds to the unbalance[13]. A mass unbalance causes a high intensity radical vibration at its rotating speed f_r and the effect on the spectrum is a remarkable increase in the amplitude of the tone at f_r. In this paper, the revolution per second is employed as a measure of the rotating speed of the rotor, and denoted by f_r in hertz. For the practical unbalance distribution of a rotor, convergence conditions are always satisfied and would not cause any convergence problem s[14]. So the unbalance distribution should converge at some point of the rotor. In this paper, the work focuses on the detection of the equivalent mass unbalance and its phase position of such point.

It is required to analyze the vibration signal in spectra with different rotating speeds. The conventional Fourier based analysis usually covers an extremely wide frequency domain compared with the very narrow-band characteristics of the rotating frequency and therefore makes it difficult to estimate the mass unbalance accurately. The windowed Fourier transform may be used for separating frequency components but at the expense of resolution. It will not be able to satisfy the variety of signal components of different duration. Clearly, unbalanced rotating system diagnosis is a typical example where the required resolution varies with cases. Unlike the Fourier transform, in which the signal is mapped to a sinusoid function basis, and the windowed Fourier transforms, in which it has a fixed scale and thus cannot follow the instantaneous frequency of rapidly varying events, the wavelet transform uses a more general function as the basis. The wavelet transform, an extension of the Fourier transform, projects the original signal down onto wavelet basis functions and provides a mapping from the time domain to the time scale plane[15]. Thus by changing the time-scale factor, the proposed wavelet analysis method exhibits a very useful property of zooming in the frequency domain and estimates the mass unbalance effectively.

In order to extract features about unbalance, a wavelet function $\psi(t)$ called the Gaussian oscillation is used and given by

$$\psi(t) = c\exp(-\sigma^2 t^2)\cos(\omega_0 t) \tag{1}$$

where c, σ and ω_0 are positive. The parameter σ determines the energy bandwidth of the wavelet in the time and frequency domain, ω_0 is the center frequency of the band, and c can usually be unity. The selection of the wavelet function is made from the expectation that it can be appropriately matched with the characteristic frequency associated with the unbalance.

The Fourier transform of Eq. (1) is given by

$$\hat{\psi}(\omega) = \frac{c\sqrt{\pi}}{2\sigma}\left[\exp\left[-(\frac{\omega-\omega_0}{2\sigma})^2\right] + \exp\left[-(\frac{\omega+\omega_0}{2\sigma})^2\right]\right] \tag{2}$$

For (2), as long as $\omega_0 / 2\sigma$ is large enough, $\hat{\psi}(0)$ will be approach ing zero and the results of $\omega_c = \omega_0$ and $\sigma_\omega = \sigma$ can be obtained by,

$$\omega_c = \frac{1}{2\pi E}\int_0^\infty \omega\left|\hat{\psi}(\omega)\right|^2 d\omega \tag{3}$$

$$\sigma_\omega^2 = \frac{1}{2\pi E}\int_0^\infty (\omega - \omega_0)^2\left|\hat{\psi}(\omega)\right|^2 d\omega \tag{4}$$

where $E = (1/2\pi)\int_0^\infty |\hat{\psi}(\omega)|^2 d\omega$. Applying the convolution theorem, the wavelet transform of $f(t) \in L^2(R)$ at the scale a and position b can be written as,

$$Wf(a,b) = \frac{1}{\sqrt{a}} f(b) * \psi^*(-\frac{b}{a}) = \sqrt{a}F^{-1}\left[\hat{f}(\omega)\hat{\psi}^*(a\omega)\right] \tag{5}$$

where the symbol $*$ is the convolution operation and F^{-1} represents the inverse Fourier transform. By taking the Fourier transform on both sides of (5) on b, the Fourier transform of $Wf(a,b)$ is given by

$$Wf(a,\omega) = F[Wf(a,b)] = \sqrt{a}\,\hat{f}(\omega)\hat{\psi}^*(a\omega) \tag{6}$$

In (6), $Wf(a,\omega)$ is the result of a frequency filtering operation that applies the filter banks, $\sqrt{a}\,\psi^*(a\omega)$, on $f(\omega)$. Inserting Eq. (2) into Eq. (6) yields,

$$Wf(a,\omega) = \frac{c\sqrt{a\pi}}{2\sigma}\left\{\hat{f}(\omega)\left[\exp(-\frac{(\omega-\frac{\omega_0}{a})^2}{(\frac{2\sigma}{a})^2}) + \exp(-\frac{(\omega+\frac{\omega_0}{a})}{(\frac{2\sigma}{a})^2})\right]\right\} \tag{7}$$

In this unbalance application, we take the part of $Wf(a,\omega)$ over $\omega \geq 0$ for mass unbalance estimation. Then (7) can be defined by,

$$Wf(a,\omega)_+ = \frac{c\sqrt{a\pi}}{2\sigma}\hat{f}(\omega)\exp(-\frac{(\omega-\frac{\omega_0}{a})^2}{(\frac{2\sigma}{a})^2}),\quad \omega \geq 0 \tag{8}$$

From (8), it can be seen that the unbalance component in spectra contained in a signal $f(t)$ can be picked up by the filter bank. The Gaussian distributed filter bank is with energy bandwidth σ / a around the center frequency ω_0 / a, but maintains the constant fractional bandwidth σ / ω_0. The calculation of $WF(a,\omega)_+$ in (8) can be obtained by using an efficient FFT algorithm. In order to match ω_0 / a with the rotating frequency f_r, the relationship between ω_0 / a and f_r is,

$$\frac{\omega_0}{a} = f_r \tag{9}$$

Let $a \geq 1$ and the energy bandwidth of the filter bank be equal to 10% of the center frequency. The center frequency and the energy bandwidth can be given by

$$\begin{cases}\omega_0 = a \times f_r \\ \sigma_f = 0.1 \times a \times f_r\end{cases} \tag{10}$$

Using the filter bank, the spectrum component associated with the unbalance can be efficiently extracted. Estimation of the mass unbalance can be performed by comparing the maximum amplitude M_f in the filter bank with a benchmark, then,

$$M_f = \max\{Wf(a,\omega) \mid \omega \in B_f\} \tag{11}$$

where B_f is the energy band $[\omega_0 - \sigma_f, \omega_0 + \sigma_f]$. In comparison with the benchmark amplitude M_{UNIT} which is defined in advance by experiment, the mass unbalance of the rotating system can be estimated as,

$$W = M_f / M_{UNIT} \tag{12}$$

2.2 Phase Position Analysis

Only with the vertical vibration signal, it is impossible to find the position of the mass unbalance easily. So the time reference signal is also used. After low pass filtering and down-sampling, the two signals are sine-wave signals with the same frequency, which is equal to the rotor rotating speed. There is a phase lag between the vibration signal and time signal according to the position of the unbalance on the rotor. By calculating the phase lag, it is possible to find the phase position of the mass unbalance with the reference point. A modified Lissajous diagram algorithm[13] is implemented to computer the phase lag.

The Lissajous diagram is a basic approach to determine the relative characteristics of two sources, primarily their frequency and phase relations. By applying two signals as vertical axis and horizontal axis inputs, an ellipse trace can be obtained, except for the phase lag that is the multiple of $\pi/2$. From the ellipse curve, the phase lag between two inputs can be obtained as,

$$\sin\phi = \frac{y-intercept}{y-amplitude} = \frac{y_1}{y_2} \tag{13}$$

where ϕ is the phase lag between two signals ($0<\phi<\pi/2$), y_1 is the value where x is 0 and y_2 is the maximum value of magnitude. The time signal is set as x input while vibration signal is y input. The mean value of the time signal b_0 is removed to make time signal a sine wave signal on x axis. The intercept value and amplitude value of the periodic vibration signal are obtained statistically. The y axis intercept value is obtained as,

$$y_1 = \frac{1}{N}\sum_{n=1}^{N} V_{intercept} \tag{14}$$

where N is the number of periods of the vibration signal, $V_{intercept}$ is the magnitude value of each period where the time signal magnitude is 0. The y axis amplitude value is also obtained as,

$$y_2 = \frac{1}{N}\sum_{n=1}^{N} V_{amplitude} \tag{15}$$

where N is the number of period of the vibration signal, $V_{amplitude}$ is the maximum value of the magnitude in each period. So the phase lag ϕ can be obtained from y_1 and y_2.

As the original Lissajous diagram only can indicate the phase lag less than π, the modified algorithm divides

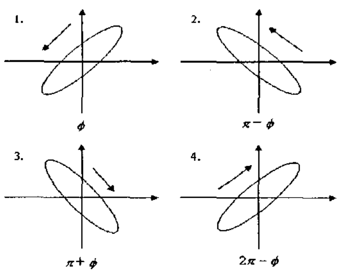

Fig. 1 Modified Lissajous diagram algorithm

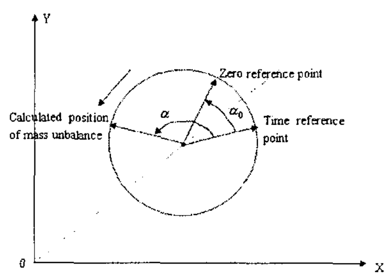

Fig. 2 Geometries of a rigid rotator

the Lissajous diagram into four forms according to clockwise rotation or anti-clockwise rotation of the diagram. By distinguishing the diagram form first, the phase lag can be calculated in 2π range. The algorithm is illustrated in Fig. 1. So the phase lag α of the mass unbalance on the rotor is defined as,

$$\alpha = \begin{cases} \phi & 0 \le \alpha < \pi/2 \\ \pi-\phi & \pi/2 \le \alpha < \pi \\ \pi+\phi & \pi \le \alpha < 3\pi/2 \\ 2\pi-\phi & 3\pi/2 \le \alpha < 2\pi \end{cases} \tag{16}$$

where α is the phase lag on the rotor, and ϕ is obtained in (13).

Therefore the phase position of the mass unbalance on the rotor can be computed according to the phase lag. The geometries of a rotor with unbalance mass are shown in Fig. 2. First, a zero reference point of the mass unbalance is defined on the rotor. The α_0 is the phase lag between the zero reference point and time reference point obtained, which is obtained in advance by experiment. Then the calculated position of mass unbalance would be,

$$\alpha_{MASS} = \alpha - \alpha_0 \tag{17}$$

With the reference of zero reference point, the position of unbalance can be located efficiently and accurately through this method.

3. Proposed method of balancing rigid rotor

A block diagram of the proposed system for unbalance diagno sis is shown in Fig. 3, which consists of a test rig, the measuring transducers, an ADC model and a host computer.

The test rig here is a motor and clutch arrangement, where the rotor is driven by a motor through a belt and brought up to a predetermined rpm value for analysis. A pair of piezoelectric sensors are mounted in the pedestal adjacent to and spaced axially along the rotor and an optical sensor above the rotor, in order to assure an optimum coupling. The structure of the test rig is shown in Fig. 4. The force transducers are coupled mechanically to the shaft and provide periodic electrical output signals indicative of unbalance forces transmitted through the shaft when the rotor is driven rotationally. The reference position of the rotor is monitored with respect to the optical sensor during every rotation. Fig. 5 shows a picture of the test rig used in this research. The vertical force exhibited by the mass unbalance and rotation time data are converted to digital values through the use of NI 6070E device for further processing.

The processing part can be subdivided into three main procedures corresponding to the three steps needed to achieve unbalance diagnosis: i) signal processing for denoising, ii) mass unbalance estimation, and iii) phase position estimation. The diagnosis system is schematized in Fig. 6.

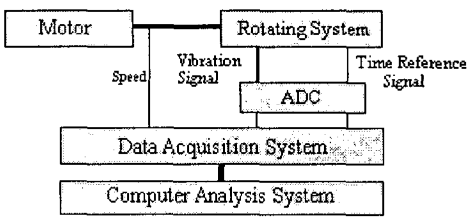

Fig. 3 Structure of the balancing system

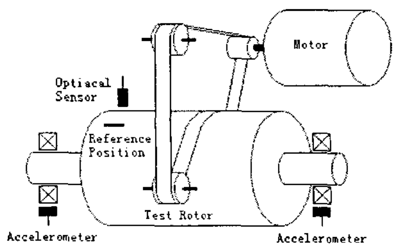

Fig. 4 Structure of the test rig

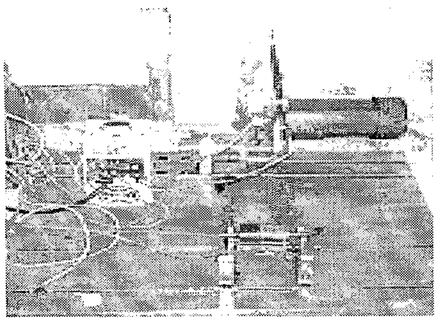

Fig. 5 Experimental test rig

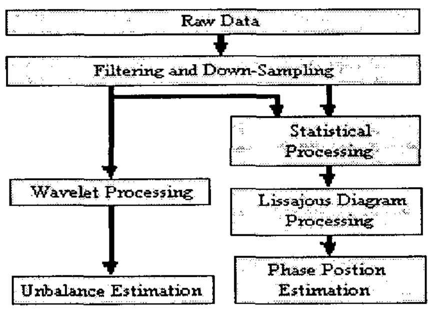

Fig. 6 Procedures of balancing system

The signal processing procedure analyzes the vibration signal i n time domain and denoises the signal for proper analysis. The sampling frequency is set to be 10800 Hz to avoid aliasing.

To improve the frequency resolution and eliminate the noise, the raw signal is filtered according to the test rotation speed and decimated with a decimation factor equal to 20. The so-obtained sample sequence is windowed in 540 points.

Each time the signal processing procedure is completed, the narrow-bank vibration spectrum is extracted through wavelet based processing. The chosen characteristic amplitude associated with the unbalance is compared with the benchmark unit amplitude, with the aim of estimating the mass unbalance of the rotating system. The high variability in successive measurements, carried out in different test speeds (360-1800 rpm), suggests the use of wavelet transform to suit variety of signal components of different rotating speeds.

An efficient phase position estimation for the unbalance on the rotor is realized through the combined use of vibration signal and time reference signal, which are also obtained from the signal processing step.

The modified Lissajous diagram is directly implemented in order to obtain better time performance. First the checks on the vibration signal and time signal are performed, to find the $V_{intercept}$ and $V_{amplitude}$ in each period of the vibration signal as well as the Lissajous diagram form it belongs to.

Then the mean intercept and amplitude are

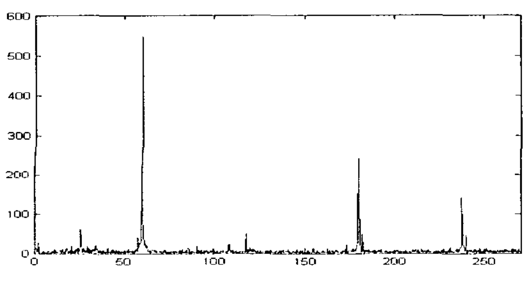

Fig. 7(a) Magnitude spectrum of balance rotor

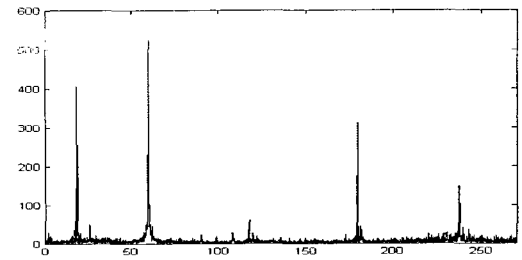

Fig. 7(b) Magnitude spectra of unbalance rotor

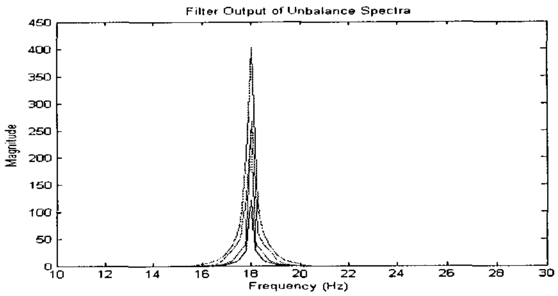

Fig. 8 Magnitude spectra of the filter output at 1080 rpm

generated with the statistical processing to calculate the phase lag between vibration signal and time reference signal. The phase position of the unbalance is finally estimated by comparing the phase lag with the reference point.

The diagnosis result of the proposed system is the exact mass unbalance and its phase position.

4. Experimental Evaluation

As for the performance evaluation of the system, several experimental tests using the test rig are carried out. In order to provi de a higher resolution in the sampled signal, forty groups of time-independent data are used in this study with a sampling rate of 10 800 Hz.

The four test unbalance conditions are set to 1g, 2.1g, 3.2g in one position and two separate 1g in adjacent positions, respectivel y. The phase between the separate 1g unbalance is 45 degrees. A summary of the test cases used to analyze the performance of the proposed system

Table 1 Summary of analyzed unbalance experiments

Test Condition	Number of Cases	Detailed Description
Balance	15	Balance rig
Unbalance	60	Unbalance mass: 1, 2.1, 3.2, 2×1g 15 test speed 360 rpm 15 test speed 1080 rpm 15 test speed 1800 rpm
Total cases	75	

Table 2 Results of the balancing system

Theoretical Value	1g	2.1g	3.2g	1g×2
Test speed 360 rpm	1g	2.0693g	3.0427g	1.9479g
Test speed 1080 rpm	1g	2.1776g	3.3205g	1.9556g
Test speed 1800 rpm	1g	2.2671g	3.3887g	1.9637g
Average error %	0	3.39	1.58	2.21
Effectiveness %			98.2	

Table 3 Numerical result of phase position

Test speed 1080 rpm	Position	Result	Error %
	0°	0°	0
	45°	44.1204°	0.24
	90°	89.7250°	0.07
Test unbalance 3.2g	135°	131.8349°	0.87
	180°	180.8455°	0.23
	225°	225.0710°	0.01
	270°	272.5329°	0.70
	315°	319.3762°	1.21

is given in Table 1. The vibration signals are down-sampled to 540 Hz in the balance and four unbalance cases at rotating speed of 1080 rpm ($f_r = 18Hz$) are illustrated. The corresponding magnitude spectra in the frequency range of 0~270 Hz before being filtered are given by Fig. 7. After these vibration signals pass through the narrow-band filter around the rotating frequency, the magnitude spectra of the filter output for the unbalance cases are given in Fig. 8. In Fig. 8, it is shown that the specific peaks at its rotating frequency can be efficiently extracted from the vibration signal with the narrow-band filter. The estimation results of unbalance through test speed of 60-1800 rpm are listed in Table 2. Checked with theoretical values, it can be found that the method has a reliable performance for estimating the unbalance. The convergence weight of the two separate 1g mass unbalance is also estimated accurately, which proves that the convergence conditions are always satisfied and this method can be used to balance the unbalance rotating system practically.

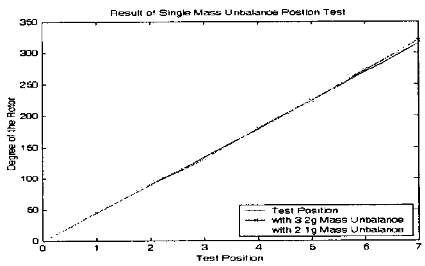

Fig. 9 Comparison of single position unbalance

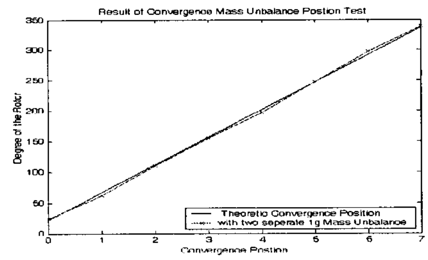

Fig. 10 Comparison of convergence position unbalance

Also the phase position is estimated and analyzed with three examples in the proposed method. The outputs of the numerical results are shown in Table 3, and compared with test position in Figs. 9 and 10. The difference of the first two cases is the unbalance distribution on the rotor and the two separate unbalance masses of 1g apart from 45 degrees is tested in the last case, which the convergence position is calculated. The comparison between the test position and the calculated result shows accurate estimation of the proposed metho d. The proposed method is more advanced as the exact phase position is told instead of the indication of such subsystem or boundary by the former methods.

5. Conclusions

In the present paper, a vibration signal based practical method is proposed for balancing the rigid rotor. Based on wavelet and a Lissajous diagram, the mass unbalance and phase position are estimated effectively and accurately. This paper outlined the use of a narrow-bandwidth filter applied to the estimation of unbalance operating at different speeds. It has been shown that the filter bank with narrow bandwidth based on wavelet transform can be employed to extract the important spectrum component from vibration signals. Also the modified Lissajous diagram is applied and show that this new method can estimate the phase position more accurately. Theoretical results have been given. Several examples are illustrated and compared to verify the applicability of the proposed method.

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