

An Application of Fuzzy Logic with Desirability Functions to Multi-response Optimization in the Taguchi Method

Seong-Jun Kim

Department of Industrial and Systems Engineering, Kangnung National University
120 University Road, Kangnung, KW 210-702, KOREA REPUBLIC

Abstract

Although it is widely used to find an optimum setting of manufacturing process parameters in a variety of engineering fields, the Taguchi method has a difficulty in dealing with multi-response situations in which several response variables should be considered at the same time. For example, electrode wear, surface roughness, and material removal rate are important process response variables in an electrical discharge machining (EDM) process. A simultaneous optimization should be accomplished. Many researches from various disciplines have been conducted for such multi-response optimizations. One of them is a fuzzy logic approach presented by Lin et al. [1]. They showed that two response characteristics are converted into a single performance index based upon fuzzy logic. However, it is pointed out that information regarding relative importance of response variables is not considered in that method. In order to overcome this problem, a desirability function can be adopted, which frequently appears in the statistical literature. In this paper, we propose a novel approach for the multi-response optimization by incorporating fuzzy logic into desirability function. The present method is illustrated by an EDM data of Lin and Lin [2].

Key words : Fuzzy Logic, Desirability Function, Taguchi Method, Multi-response Optimization

1. Introduction

The Taguchi method is one of the most popular techniques for finding an optimum operating condition of manufacturing process parameters. In order to evaluate process performances, Dr. Taguchi proposed to use a signal-to-noise ratio, which is obtained from realizations of process performance variable. In the Taguchi method, orthogonal array designs of experiments are employed for comparing performances over various process conditions and an optimum process setting is determined by maximizing the signal-to-noise ratio. In spite of widespread use in a variety of industrial fields, the Taguchi method has some limitations. One of them is that multi-response variables are poorly supported in the Taguchi method. As described in the example of polysilicon deposition process design [3], the final recommendation of the optimum condition with multiple performance characteristics is left to engineering judgement.

It is not very difficult to find that a manufacturing process in general has several response variables. Consider an electrical discharge machining (EDM) process in which a metal cutting is conducted. Main response variables in this process include electrode wear weight, machined surface roughness, material removal rate, and so on. These process response variables are affected by process parameters such as polarity of workpiece, pulse-on time, duty factor, open discharge voltage, discharge current, dielectric fluid, and so on. At this point, a multi-response optimization problem is established because an optimum parameter setting should be determined by simulta-

neously considering electrode wear weight, machined surface roughness, and material removal rate. For more technical explanations on EDM process, the reader is referred to DeGarmo et al. [4].

In order to solve such multi-response problems, Taguchi recommended individually conducting single response optimizations and then compromising the results. However, his procedure is apt to suffer from uncertainty involved in decision making process. To overcome this difficulty, some alternatives have been studied. They are broadly divided into response modelling based mathematical programming approach and simultaneous optimization approach. Among them, this paper is concerned with the latter. For the former, the reader is referred to Myers and Montgomery [5].

A key step of simultaneous optimization is to define an overall performance index from multi-response variables. One such simplest form would be a weighted average. It is of course assumed that weights are known *a priori*. A utility function, one form of weighted averages, is employed by Grabiec & Piasta [6] in order to deal with four cement paste properties. The notion of utility function can be generalized by the desirability function enhanced by Derringer & Suich [7]. Because it is useful to describe satisfaction levels increased as the response variable approaches to its target, desirability function frequently appears in the statistical literature [5]. For example, Tong et al. [8] recently used desirability functions to conduct the Taguchi method with two biological process variables. On the other hand, Antony [9] proposed using principal component analysis to obtain a composite performance index without predetermined weights. More precisely speaking, in that method, weights are as good as directly computed from experimental data. It is noted that, in using such data-de-

pendent scores, there is a risk of producing results far from designer's expectation.

How to consider the preference between multi-response variables is very crucial in establishing an overall performance index and, however, it has a fuzzy aspect in nature. In order to tackle this nature, a fuzzy logic approach is presented by Lin et al. [1]. Although still not incorporated with relative importance, their approach has a contribution that modelling the vagueness in multi-response performance evaluation is attempted by fuzzy logic.

This paper presents a generalization of works done by Lin et al. [1] for dealing with both variable importance and evaluation vagueness. This is accomplished by embedding desirability functions onto fuzzy logic. To illustrate the present method, an experimental data of Lin and Lin [2] is used.

2. Fuzzy Logic with Desirability

2.1 Desirability Function

The desirability function technique popularized by Derringer & Suich [4] is one of useful approaches for the simultaneous multi-response optimization. By this function, a response variable y_i is converted into an individual desirability d_i . The desirability takes a value between 0 and 1 and it represents the closeness of a response to its target value. The optimum setting is finally chosen by maximizing the composite desirability defined as a geometric mean of the individual desirability, i.e.,

$$D = (d_1 \times d_2 \times \dots \times d_m)^{1/m} \tag{1}$$

where m denotes the number of response variables.

The individual desirability function for the larger-the-better response is defined as

$$d = \left(\frac{y - y_{\min}}{y_{\max} - y_{\min}} \right)^r, \quad y_{\min} \leq y \leq y_{\max}$$

where y_{\min} is a minimum value of y , y_{\max} is a maximum value of y , and r is the weight specified for y . If $r=1$, the desirability function becomes linear. Choosing $r>1$ places more emphasis on being close to the target value, and taking $0<r<1$ makes this less important. In the similar manner, the desirability is obtained by

$$d = \left(\frac{y_{\max} - y}{y_{\max} - y_{\min}} \right)^r, \quad y_{\min} \leq y \leq y_{\max}$$

for the smaller-the-better response variable. If y is the nominal-the-best, the two-sided desirability function is given by

$$d = \begin{cases} \left(\frac{y - y_{\min}}{T - y_{\min}} \right)^r, & y_{\min} \leq y \leq T \\ \left(\frac{y_{\max} - y}{y_{\max} - T} \right)^s, & T \leq y \leq y_{\max} \end{cases}$$

where T denotes the target value and r and s are weights. For more descriptions on desirability functions, the reader is referred to Myers and Montgomery [5].

If the relative importance between response variables is available, the overall desirability function (1) is generalized into the following [10]:

$$D = (d_1^{w_1} \times d_2^{w_2} \times \dots \times d_m^{w_m})^{1/(w_1 + w_2 + \dots + w_m)} \tag{2}$$

where w_i is an importance of y_i . In this paper, we use (2) to obtain the composite desirability assuming all of individual desirability functions are linear, i.e., $r=s=1$.

2.2 Fuzzy Logic Approach

As done by Lin et al. [1], the fuzzy logic can be considered as an alternative method for the simultaneous optimization with multi-response variables. A fuzzy logic system consists of fuzzifier, computational unit, rule base, and defuzzifier as shown in Figure 1. Each input is fuzzified by its membership functions. Computational unit then conducts a fuzzy reasoning using rule base and max-min compositional operations. The resulting fuzzy value is finally defuzzified into a single output, which is employed as an overall performance index in Lin et al. [1].

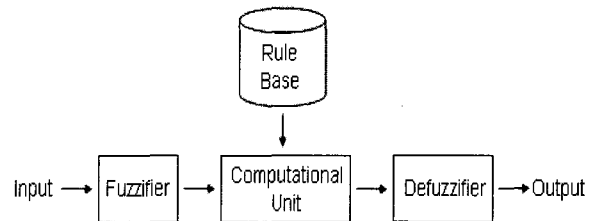


Figure 1. A Schematic Diagram of Fuzzy Logic

However, as pointed out earlier, variable preference is not fully considered within such fuzzy logic approach. Accordingly, we introduce a notion of desirability to fuzzy logic system in this paper. This is accomplished by using the desirability function (2) to construct the fuzzy rule base. This will be described in Section 3.2.

First consider a three-input-and-one-output problem. Let x_1 , x_2 and x_3 denote three inputs respectively and let y denote output. Rules in the rule base can be stated as:

- If x_1 is A_1 , x_2 is B_1 , x_3 is C_1 , then y is D_1 else;
- if x_1 is A_2 , x_2 is B_2 , x_3 is C_2 , then y is D_2 else;
- ...
- if x_1 is A_k , x_2 is B_k , x_3 is C_k , then y is D_k .

where k is the number of rules in the rule base and A_i , B_i , C_i and D_i are fuzzy subsets defined by corresponding membership functions of x_1 , x_2 and x_3 and y , i.e., $\mu_{A_i}(x_1)$, $\mu_{B_i}(x_2)$, $\mu_{C_i}(x_3)$ and $\mu_{D_i}(y)$. In this paper, as done in Lin et al. [7], simple linear functions are used in order to represent fuzzy set memberships as illustrated by the following Figures 2 and 3.

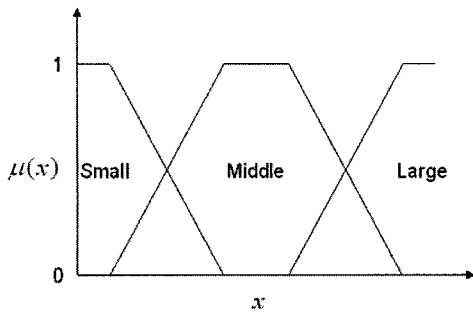


Figure 2. Membership Functions for Input Variable

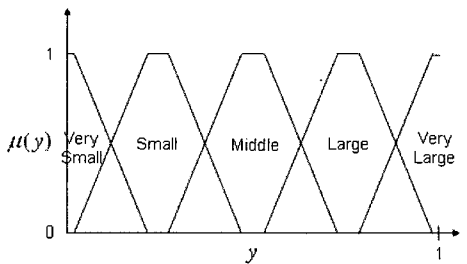


Figure 3. Membership Functions for Output Variable

In this paper, as shown by the above figures, three fuzzy subsets are assigned for each of input variables and five fuzzy subsets are assigned for output variable.

3. Multi-response Optimization in the Taguchi Method

3.1 Experimental Design and Data

In order to illustrate our proposed method, we use electrical discharge machining (EDM) data reported by Lin and Lin [2]. EDM process removes workpiece using an electrical spark erosion. Its operational performance is in general evaluated by

electrode wear rate (EWR), machined surface roughness (MSR), and workpiece removal rate (WRR). In order to find the optimum operating condition, the Taguchi design of experiment is conducted. As in other machining processes, many parameters are involved with EDM. Among them, six parameters are selected for the experiment as shown by Table 1.

Table 1. Parameters and Levels [2]

Parameter	Unit	Level		
		1	2	3
A. Workpiece Polarity	-	(+)	(-)	N/A
B. Pulse-on Time	μs	20	150	300
C. Duty Factor	-	0.3	0.5	0.7
D. Open Discharge Voltage	V	100	120	150
E. Discharge Current	A	1.5	4.0	6.0
F. Dielectric Fluid	g/l	2	4	8

An orthogonal array $L_{18}(2^1 \times 3^7)$ is adopted for the experimental design and three samples are machined at each experimental setting. The experimental design and data are given in Table 2.

Orthogonal array designs are widely used for industrial experiments because they reduce the number of experimental runs. In our case, there are $21 \times 35 = 486$ runs for the full experiment. Table 2 shows only 18 conditions among them.

In the Taguchi method, signal-to-noise ratios are proposed for performance criteria to compare experimental settings. These are computed as:

$$S/N_{EWR} = 10 \log \frac{EWR_1^2 + EWR_2^2 + EWR_3^2}{3}$$

$$S/N_{MSR} = 10 \log \frac{MSR_1^2 + MSR_2^2 + MSR_3^2}{3}$$

and

Table 2. Experimental Design and Data of EDM Process [2]

No.	A	B	C	D	E	F	EWR (%)			MSR (mm)			WRR (g/min)		
							1	2	3	1	2	3	1	2	3
1	1	1	1	1	1	1	32.21	38.57	25.24	0.00149	0.00117	0.00139	3.02	3.17	2.07
2	1	1	2	2	2	2	23.96	25.02	27.29	0.00362	0.00408	0.00276	2.67	3.46	3.47
3	1	1	3	3	3	3	29.19	25.19	23.66	0.00324	0.00356	0.00361	5.61	3.99	4.35
4	1	2	1	1	2	2	7.16	8.72	6.31	0.00452	0.00413	0.00322	3.62	3.73	3.65
5	1	2	2	2	3	3	3.55	3.62	3.04	0.00451	0.00461	0.00384	3.50	4.57	4.27
6	1	2	3	3	1	1	33.06	57.18	53.62	0.00323	0.00128	0.00134	2.50	3.43	3.32
7	1	3	1	2	1	3	44.95	61.10	38.39	0.00125	0.00128	0.00141	4.87	2.58	3.15
8	1	3	2	3	2	1	10.79	3.74	5.57	0.00321	0.00303	0.00323	3.20	2.91	2.18
9	1	3	3	1	3	2	5.80	2.49	8.40	0.00316	0.00348	0.00345	2.23	2.45	2.80
10	2	1	1	3	3	2	24.71	7.92	21.28	0.00028	0.00034	0.00031	1.78	2.00	2.06
11	2	1	2	1	1	3	179.17	5.02	52.54	0.00016	0.00166	0.00039	1.65	1.71	1.74
12	2	1	3	2	2	1	28.23	37.80	30.56	0.00041	0.00027	0.00024	5.62	3.98	5.96
13	2	2	1	2	3	1	17.31	20.97	16.18	0.00017	0.00021	0.00023	2.65	1.53	3.50
14	2	2	2	3	1	2	99.21	76.35	74.34	0.00042	0.00049	0.00051	2.36	1.50	2.32
15	2	2	3	1	2	3	39.62	66.67	21.74	0.00018	0.00011	0.00023	1.63	2.33	2.30
16	2	3	1	3	2	3	47.89	21.54	101.72	0.00024	0.00043	0.00019	4.45	4.06	4.43
17	2	3	2	1	3	1	30.00	57.32	10.53	0.00023	0.00027	0.00032	3.43	2.08	3.67
18	2	3	3	2	1	2	94.66	99.36	111.36	0.00044	0.00052	0.00044	1.89	1.82	2.17

$$S/N_{WRR} = 10 \log \frac{WRR_1^{-2} + WRR_2^{-2} + WRR_3^{-2}}{3}$$

Recall that EWR and MSR are the-smaller-the-better characteristics and WRR is the-larger-the-better one. Because the signal-to-noise ratio is a measure of variation, the larger variation results in the smaller signal-to-noise ratio. Thus an optimum EDM parameter setting is chosen by maximizing the three signal-to-noise ratios. For more details on the Taguchi's signal-to-noise ratio, the reader is referred to Phadke [3]. In this paper, the above signal-to-noise ratios are used as input variables of the fuzzy logic system.

3.2 Analysis and Results

For the purpose of illustrations, the relative importance of EWR, MSR, and WRR are assumed as $w_1=5$, $w_2=3$, and $w_3=2$ respectively. First, the desirability computed from the three signal-to-noise ratios is analyzed. The analysis of variance (ANOVA) is given in Table 3. This was obtained by using MINITAB [12], a well-known commercial software for statistical analysis.

Table 3. ANOVA on Composite Desirability

Source of Variation	Degrees of Freedom	Sum of Squares	Mean of Squares	F	P
A	1	0.7454	0.7454	67.53	0.001
B	2	0.0335	0.0168	1.52	0.323
C	2	0.0655	0.0328	2.97	0.162
D	2	0.0086	0.0043	0.39	0.700
E	2	0.1952	0.0976	8.84	0.034
F	2	0.1093	0.0546	4.96	0.083
A*B	2	0.0179	0.0089	0.81	0.507
error	4	0.0442	0.0110		
Total	17	1.2197			

ANOVA is one of the most popular methods to analyze experimental data provided as the form of Table 2. An important objective of ANOVA is to show the relative importance of parameters. This is accomplished by decomposing total sum of squares into two parts: sums of squares due to parameters and sum of squares due to error. In the present case study, parameter A (i.e., workpiece polarity) has the largest sum of squares (SS), which indicates that it is the most dominant parameter. Mean of squares (MS) is obtained by

$$MS = SS / (\text{degrees of freedom}).$$

An F-value represents the relative magnitude of the parameter effect compared with the error effect. For example,

$$F_A = MS_A / MS_{error} = 0.7524 / 0.0110 = 67.53$$

indicates that the effect of workpiece polarity is 67.53 times larger than the error. P-value is defined as the probability that F-statistic is larger than the F-value in the statistical F-test. Therefore, as the F-value increases, the corresponding P-value approaches to zero. The smaller P-value means the higher significance of the corresponding parameter effect. Other than main effects of parameters, an interaction between parameters

A and B is included in Table 3. But its effect looks insignificant. More details on ANOVA and F-test, the reader is referred to MINITAB [12]. MINITAB also provides main effect plots of parameters in Figure 4.

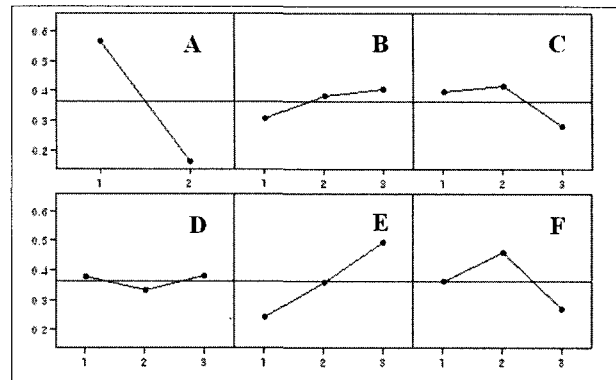


Figure 4. Main Effect Plots on Composite Desirability

By these plots, we can see that parameters A, E and F have relatively more significant effects on composite desirability, which is same as mentioned in the previous ANOVA table. Moreover, the optimum EDM operation setting which maximizes the overall desirability is chosen as $A_1B_3C_2D_3E_3F_2$ from the plots.

Now the fuzzy logic approach with desirability proposed in this paper is illustrated. First, signal-to-noise ratios are converted into fuzzy membership values. Next, fuzzy reasoning should be subsequently conducted by using the fuzzy rule base, which is given by Table 4.

Table 4. Fuzzy Rule Base for EDM Variables

EWR	MSR	WRR		
		Small	Middle	Large
Small	Small	VS	VS	VS
	Middle	VS	S	S
	Large	S	S	S
Middle	Small	S	S	S
	Middle	S	M	M
	Large	M	M	L
Large	Small	S	M	M
	Middle	M	L	L
	Large	L	VL	VL

Constructing this table can be described as follows. Because we assigned three subsets (Small, Middle, and Large) to input variable, individual desirability is also divided into three areas as shown in Figure 5. In this figure, medians are respectively given as $d_{small}=0.17$, $d_{middle}=0.50$, and $d_{large}=0.83$. For example, if (EWR, MSR, WRR) corresponds to (Small, Large, Large), the overall desirability is calculated by

$$D = (d_{small}^{w_1} \times d_{large}^{w_2} \times d_{large}^{w_3})^{1/(w_1+w_2+w_3)}$$

$$= (0.17^5 \times 0.83^3 \times 0.83^2)^{1/10}$$

$$= 0.376$$

from Equation (2). We can see that this value is corresponding to the subset 'Small' in Figure 3. All of the rules stated in Table 4 can be obtained in the same way. This is how desirability is embedded onto the fuzzy logic system in this research.

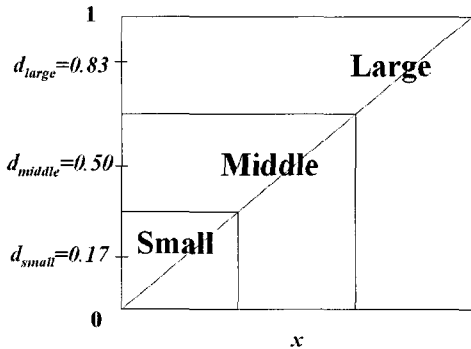


Figure 5. Three Divisions and Medians of Individual Desirability

Conducting max-min operations is needed to produce fuzzy output values [11]. Accordingly, membership function of output of the fuzzy reasoning can be written by

$$\mu_{OUT}(y) = \max_i \min_j [\mu_{EWR_i}(\eta_1), \mu_{MSR_i}(\eta_2), \mu_{WRR_i}(\eta_3), \mu_{OUT_i}(y)]$$

where $i=1,2,\dots,27$ and $\eta_1, \eta_2,$ and η_3 are signal-to-noise ratios of EWR, MSR, and WRR respectively. Using center-of-gravity method [11], we can finally obtain a defuzzified output as

$$y^* = \frac{\sum y \mu_{OUT}(y)}{\sum \mu_{OUT}(y)}$$

This is employed as the overall performance criterion for the present multi-response EDM process. In this study, y^* is supposed to fall between 0 and 1 and it is the larger-the-better characteristic as is the desirability.

Analysis of variance on y^* is provided in Table 5. More strong significant effects are observed for parameters A and E. Main effect plots are depicted in Figure 6 and they recommend $A_1B_2C_2D_1E_3F_2$ as an optimum condition of the EDM process.

Table 5. Analysis of Variance on y^*

Source of Variation	Degrees of Freedom	Sum of Squares	Mean of Squares	F	P
A	1	0.4278	0.4278	33.06	0.005
B	2	0.0110	0.0055	0.43	0.679
C	2	0.0131	0.0066	0.51	0.637
D	2	0.0033	0.0017	0.13	0.883
E	2	0.1079	0.0540	4.17	0.105
F	2	0.0278	0.0139	1.08	0.423
A*B	2	0.0136	0.0068	0.52	0.628
error	4	0.0518	0.0129		
Total	17	0.6564			

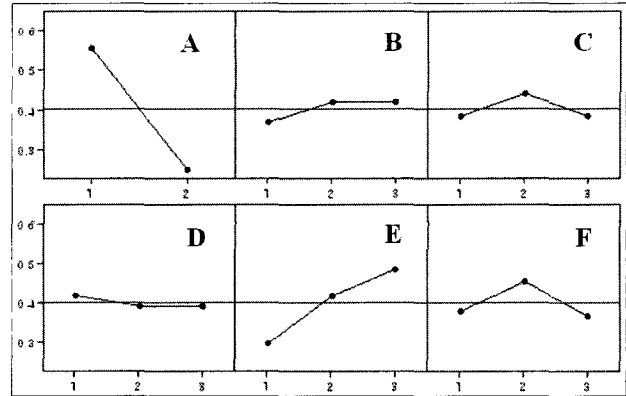


Figure 6. Main Effect Plots on y^*

4. Conclusion

In this paper, a fuzzy logic approach incorporated with desirability functions is proposed for a multi-response optimization problem in the Taguchi method. The proposed approach has a distinction that it is capable of accommodating relative importance between response variables. EDM experimental data by Lin and Lin [2] is used for illustrating the present method. Whereas the desirability function technique recommends $A_1B_3C_2D_3E_3F_2$ as an optimum setting, our proposed approach recommends $A_1B_2(3)C_2D_1E_3F_2$. As far as the optimum settings are concerned, no disagreement is found for significant parameters. However, parameter D shows a small but definite difference, which indicates that the proposed method has a capability of accommodating variable importance for multi-response optimization in the Taguchi method.

As stated earlier, for the present procedure to be operational, relative importance should be known *a priori*. The influence of incorrect information upon analysis results deserves further investigating. Although not dealt with in this paper, how to design membership functions embedded with desirability is also fruitful for the future research.

References

- [1] J.L. Lin, K.S. Wang, B.H. Yan, and Y.S. Tarn, "Optimization of the Electrical Discharge Machining Process Based on the Taguchi Method with Fuzzy Logics," *Journal of Materials Processing Technology*, Vol. 102, No. 2, pp. 48-54, 2000.
- [2] J.L. Lin and C.L. Lin, "The Use of Orthogonal Array with Grey Relational Analysis to Optimize the Electrical Discharge Machining Process with Multiple Performance Characteristics," *International Journal of Machine Tools and Manufacture*, Vol. 42, pp. 237-244, 2002.
- [3] M.S. Phadke, *Quality Engineering Using Robust Design*, Eaglewood Cliffs, NJ: Prentice-Hall, 1989.
- [4] E.P. DeGarmo, J.T. Black, and R. A. Kohser, *Materials and Processes in Manufacturing*, New York, NY: Wiley, 2003.

- [5] R.H. Myers and D.C. Montgomery, Response surface methodology, New York, NY: Wiley, 2002.
- [6] A.M. Grabiec and Z. Piasta, "Study on Compatibility of Cement-superplasticiser Assisted by Multicriteria Statistical Optimization," Journal of Materials Processing Technology, Vol. 152, pp. 197-203, 2004.
- [7] G. Derringer and R. Suich, "Simultaneous Optimization of Several Response Variables," Journal of Quality Technology, Vol. 12, No. 4, pp. 214-219, 1980.
- [8] L.I. Tong, C.H. Wang, J.Y. Hong, and J.Y.Chen, "Optimizing Dynamic Multiresponse Problems Using the Dual-response Surface Method," Quality Engineering, Vol. 14, No. 1, pp. 115-125, 2000.
- [9] J. Antony, "Multi-response Optimization in Industrial Experiments Using Taguchi's Quality Loss Function and Principal Component Analysis," Quality and Reliability Engineering International, Vol. 16, No. 1, pp. 3-8, 2000.
- [10] G. Derringer, "A Balancing Act: Optimizing a Product's Properties," Quality Progress, Vol. 27, pp. 51-58, 1994.
- [11] H.J. Zimmermann, Fuzzy Set Theory and Its Applications, Boston, MA: Kluwer Academic Publishers, 2001.
- [12] MINITAB® Release 14, Minitab Inc., 2004.



Seong-Jun Kim

He is an associate professor of industrial engineering, Kangnung National University, Kangnung, Korea. He received a B. S. in applied statistics from Yonsei University, Seoul, Korea, in 1989 and an M. S. and a Ph. D. in industrial engineering from KAIST (Korea Advanced Institute of Science and Technology), Taejon, Korea in 1991 and 1995 respectively. He was a visiting assistant professor of industrial engineering, Texas A&M University, College Station, Texas, United States in 1999. His research interests include multivariate process control with artificial intelligences, design of experiments, and quality management in business and industrial fields. Dr. Kim has also worked as a quality consultant for Samsung Electronics and LG Chemistry.

Phone : +82-33-640-2375

Fax : +82-33-640-2244

E-mail : sjkim@kangnung.ac.kr