IEMS Vol. 4, No. 1, pp. 75-81, June 2005.

Exponentially Weighted Moving Average Chart for High-Yield Processes

Takayuki Kotani
Department of Industrial Engineering
Tel: 81-72-254-9349, E-mail: takayuki@eis.osakafu-u.ac.jp

Etsuko Kusukawa†
Department of Industrial Engineering
Tel: +81-72-254-9349, E-mail: kusukawa@eis.osakafu-u.ac.jp

Hiroshi Ohta
Department of Industrial Engineering
Tel: +81-72-254-9349, E-mail: ohta@eis.osakafu-u.ac.jp

Abstract. Borror *et al*. discussed the EWMA(Exponentially Weighted Moving Average) chart to monitor the count of defects which follows the Poisson distribution, referred to the EWMA_c chart, as an alternative Shewhart *c* chart. In the EWMA_c chart, the Markov chain approach is used to calculate the ARL (Average Run Length). On the other hand, in order to monitor the process fraction defectives P in high-yield processes, Xie *et al*. presented the CCC(Cumulative Count of Conforming)-*r* chart of which quality characteristic is the cumulative count of conforming item inspected until observing $r(\geq 2)$ nonconforming items. Furthermore, Ohta and Kusukawa presented the CS(Confirmation Sample)$_{\text{CCC-}r}$ chart as an alternative of the CCC-*r* chart. As a more superior chart in high-yield processes, in this paper we present an $\text{EWMA}_{\text{CCC-}r}$ chart to detect more sensitively small or moderate shifts in P than the $\text{CS}_{\text{CCC-}r}$ chart. The proposed $\text{EWMA}_{\text{CCC-}r}$ chart can be constructed by applying the designing method of the EWMA_c chart to the CCC-*r* chart. ANOS(Average Number of Observations to Signal) of the proposed chart is compared with that of the $\text{CS}_{\text{CCC-}r}$ chart through computer simulation. It is demonstrated from numerical examples that the performance of proposed chart is more superior to the $\text{CS}_{\text{CCC-}r}$ chart.

Keywords: High-yield process, CCC (Cumulative Count of Confirming)-*r* chart, CS (Confirmation Sample) $_{\text{CCC-}r}$ chart, EWMA(Exponentially Weighted Moving Average) chart, Markov chain approach, ANOS (Average Number of Observations to Signal)

1. INTRODUCTION

Recently, it is well known that the traditional Shewhart-type charts do not respond sensitively to small or moderate shifts in quality characteristics obtained from the process (See Montgomery, (2001)). Roberts (1959) (1966) presented the EWMA (Exponentially Weighted Moving Average) chart to detect small shifts in the process mean as an alternative of the Shewhart $\overline{x}$ chart. Gan (2002) and Borror *et al*. (2002) presented the EWMA chart to monitor the count of defects which follows the Poisson distribution, referred to the EWMA*c* chart, as an alternative Shewhart *c* chart. The Markov chain approach is used to calculate the Average Run Length (ARL).

On the other hand, in high-yield processes such as the manufacturing process of today, the process fraction defectives P is substantially less than one per cent. Few or even no items are nonconforming even for a fairly large sample. Xie *et al*. (1998) presented an alternative chart to monitor the process fraction defectives P for high-yield processes, referred to the CCC (Cumulative Count of Conforming)-*r* chart. The quality characteristic is the cumulative count of item inspected until observing $r(\geq 2)$ nonconforming ones.

Furthermore, as an alternative of the CCC-*r* chart, Ohta and Kusukawa (2004) presented the CS (Confirma-

† : Corresponding Author

tion Sample)$_{CCC\text{-}r}$ chart by applying the CS charting procedure proposed by Steiner(1999) to the CCC-r chart. It is illustrated that the CS$_{CCC\text{-}r}$ chart is more sensitive than the CCC-r chart to detect small or moderate shifts in P for toward both upward and downward directions, which indicate the process deterioration and the process improvement, respectively.

As a more superior chart in high-yield processes, in this paper we present a EWMA$_{CCC\text{-}r}$ chart to detect more sensitively small or moderate shifts in P than the CS$_{CCC\text{-}r}$ chart. The proposed EWMA$_{CCC\text{-}r}$ chart can be constructed by applying the designing method of the EWMAc chart to the CCC-r chart. According to the designing method of the EWMAc chart, the Markov chain approach is used to calculate ANOS(Average Number of Observations to Signal) for any shifts in P. ANOS of the proposed chart is compared with that of the CS$_{CCC\text{-}r}$ chart through computer simulation. It is demonstrated from numerical examples that the performance of proposed chart is more superior to the CS$_{CCC\text{-}r}$ chart.

2. OUTLINE OF THE CCC-*r* CHART

The quality characteristic is the cumulative count of item inspected until observing $r(\geq 2)$ nonconforming ones. The CCC-r chart can be designed by using the probability limit method based on the negative binomial distribution. The lower and upper control limits of the CCC-r chart denoted as $LCL_{CCC\text{-}r}$ and $UCL_{CCC\text{-}r}$ are given respectively as the solutions of the following equations:

$$\sum_{i=r}^{LCL_{CCC-r}} \binom{i-1}{r-1} P_0^r (1-P_0)^{i-r} = \alpha/2, \quad (1)$$

$$\sum_{i=r}^{UCL_{CCC-r}} \binom{i-1}{r-1} P_0^r (1-P_0)^{i-r} = 1-\alpha/2, \quad (2)$$

where α is the type I error and P_0 is the in-control fraction defectives. As r is getting larger, the CCC-r chart is more and more sensitive to detect small or moderate shifts in P toward the upward direction, while the CCC-r chart needs more and more observations to obtain a potting point on the chart, then the cost is fairly high. The recommended value for r is about 2-5 depending on the fraction nonconforming level and type of process being monitored. The decision procedure in the CCC-r chart is made as follows: in the CCC-r charts, if P decreases as like $P< P_0$, then the quality characteristics of the CCC-r chart is expected to be larger, while if P increases as like $P > P_0$, then it is expected to be smaller. Therefore, according to the CCC-r chart, if a plotted point is above the upper control limit, the process is likely to have improved, while if a plotted point is below the lower control limit, the process has probably deteriorated.

3. OUTLINE OF THE CS$_{CCC\text{-}r}$ CHART

The following notation is used in this section:

y_A: the first quality characteristic based on the cumulative count of item inspected until observing r noncon-forming ones from the negative binomial distribu-tion $NB(r, P)$;

y_B: the second quality characteristic for confirmation based on the cumulative count of item inspected until observing r nonconforming ones from the negative binomial distribution $NB(r, P)$;

In the CS$_{CCC\text{-}r}$ chart, y_A is initially taken from the process. Then y_A is judged whether the process is in-control or out-of-control by using the upper and lower confirmation control limits of the CS$_{CCC\text{-}r}$ chart, $UCSL_{CCC\text{-}r}$ and $LCSL_{CCC\text{-}r}$, respectively. If y_A is satisfied the following condition:

$$LCSL_{CCC-r} < y_A < UCSL_{CCC-r}, \quad (3)$$

it would be judged that the process is in-control. On the other hand, if y_A satisfies the following condition:

$$y_A < LCSL_{CCC-r} \quad \text{OR} \quad UCSL_{CCC-r} < y_A, \quad (4)$$

the second quality characteristic for confirmation, y_B, would be independently taken from the same process. Further, the initial quality characteristic y_A and the con-firmation quality characteristic y_B would be judged whether the process is in-control or out-of-control by using the same confirmation control limits. If both y_A and y_B, denoted as y_A & y_B, satisfy the following condition:

$$y_A \,\&\, y_B < LCSL_{CCC-r} \text{ OR } UCSL_{CCC-r} < y_A \,\&\, y_B, \quad (5)$$

it would be judged that the process is out-of-control. On the other hand, if both y_A and y_B do not satisfy equation (5), it would be judged that the process is in-control. Let α be the type I error of the CS$_{CCC\text{-}r}$ chart. The probability of the type I error assuming fixed values for the confirmation control limits can be given as follows:

$$\alpha = P(y_A \,\&\, y_B > UCSL_{CCC-r})^2 + P(y_A \,\&\, y_B < LCSL_{CCC-r})^2. \quad (6)$$

Here, assume that the type I error in either direction is equally undesirable. From equation (6), the upper and lower con-firmation control limits of the CS$_{CCC\text{-}r}$ chart, $UCSL_{CCC\text{-}r}$ and $LCSL_{CCC\text{-}r}$, can be given as solutions of the following equations:

$$\sum_{i=r}^{LCSL_{CCC-r}} \binom{i-1}{r-1} P_0^r (1-P_0)^{i-r} = \sqrt{\alpha/2}, \quad (7)$$

$$\sum_{i=r}^{UCSL_{CCC-r}} \binom{i-1}{r-1} P_0^r (1-P_0)^{i-r} = 1-\sqrt{\alpha/2}. \quad (8)$$

4. OUTLINE OF THE EWMA CHART

A plotting point of the EWMA chart is given by

$$Z_t = \lambda x_t + (1-\lambda) Z_{t-1} \ (0 < \lambda \le 1 \ , t = 1,2\cdots), \tag{9}$$

where λ is a smoothing constant such that $0 < \lambda \le 1$, and x_t is the sample average observed from the process at time t, assumed to be normally distributed. Z_0 is the initial plotting point assigned and Z_t is a plotting point of the EWMA chart after the t^{th} observation.

When the process is in control $E[x] = \mu_0$ and $V[x] = \sigma_0^2$. Here, define $Z_0 = E[x] = \mu_0$. Let L be width of the control limits. The control limits of the EWMA chart are as follows:

$$UCL^{EWMA} = \mu_0 + L\,\sigma_0\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{10}$$

$$LCL^{EWMA} = \mu_0 - L\,\sigma_0\sqrt{\frac{\lambda}{(2-\lambda)}}. \tag{11}$$

5. OUTLINE OF THE EWMA$_{CCC\text{-}r}$ CHART

In this section, we present an EWMA$_{CCC\text{-}r}$ chart to detect more sensitively small or moderate shifts in P toward both upward and downward directions in high-yield processes. The EWMA$_{CCC\text{-}r}$ chart can be constructed by applying the designing method of the EWMAc chart to the CCC-r chart. A plotting point of the EWMA$_{CCC\text{-}r}$ chart is given by

$$Z_t = \lambda x_t + (1-\lambda) Z_{t-1} \ (0 < \lambda \le 1, \ t = 1,2\cdots), \tag{12}$$

where λ is a smoothing constant such that $0 < \lambda \le 1$, and x_t is the cumulative count of item inspected until observing r nonconforming ones, assumed to obey the negative binomial distribution. When the process is in control

$$E[x] = \frac{r}{P_0} \text{ and } V[x] = \frac{r(1-P_0)}{P_0^2}. \tag{13}$$

Define the initial plotting point Z_0 as

$$Z_0 = E[x] = \frac{r}{P_0}, \tag{14}$$

where P_0 is the in-control fraction defectives. The control limits for the EWMA$_{CCC\text{-}r}$ chart are as follows:

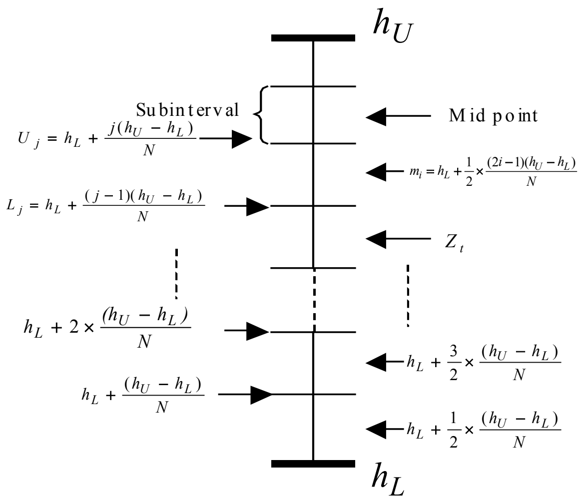
$$h_U = \frac{r}{P_0} + L\frac{\sqrt{r(1-P_0)}}{P_0}\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{15}$$

$$h_L = \frac{r}{P_0} - L\frac{\sqrt{r(1-P_0)}}{P_0}\sqrt{\frac{\lambda}{(2-\lambda)}}. \tag{16}$$

6. CALCULATION OF ANOS USING MARKOV CHAIN APPROACH

According to the designing method of the EWMA$_C$ chart, the Markov chain approach is used to calculate ANOSs for any shifts in P. The interval (h_U, h_L) is divided into N subintervals, as shown in Figure 1. The j^{th} subinterval (L_j, U_j) are given as

$$L_j = h_L + \frac{(j-1)(h_U - h_L)}{N} \tag{17}$$



and

Figure 1. In-control region divided into N subintervals

$$U_j = h_L + \frac{j(h_U - h_L)}{N}. \tag{18}$$

The midpoint m_i of the i^{th} subinterval (L_j, U_j) is given as

$$m_i = h_L + \frac{(2i-1)(h_U - h_L)}{2N}. \tag{19}$$

The $(N+1)^{th}$ state is absorbing and represents the out-of-control region above and below the control limits. This region is considered absorbing because the process is stopped when an out-of-control signal is raised. ANOS is thus the expected time to absorption of the Markov chain.

The transition probability P_{ij} is the probability of moving from state i to state j in one step and is given by

$$P_{ij} = P(L_j < Z_t < U_j \mid L_i < Z_{t-1} < U_i). \tag{20}$$

This is the probability that Z_t is within the boundaries of state j, conditioned on Z_{t-1} being equal to the midpoint of state i. This transition probability is given as

$$\begin{aligned} P_{ij} &= P(L_j < Z_t < U_j \mid Z_{t-1} = m_i) \\ &= P(L_j < \lambda x_t + (1-\lambda) Z_{t-1} < U_j \mid Z_{t-1} = m_i) \\ &= P(L_j < \lambda x_t + (1-\lambda) m_i < U_j) \\ &= P(h_L + \frac{(j-1)(h_U - h_L)}{N} \\ &< \lambda x_t + (1-\lambda)(h_L + \frac{1}{2} \times \frac{(2i-1)(h_U - h_L)}{N}) \\ &< h_L + \frac{j(h_U - h_L)}{N}) \\ &= P(h_L + \frac{h_U - h_L}{2N\lambda}(2(j-1) - (1-\lambda)(2i-1)) \\ &< x_t < h_L + \frac{h_U - h_L}{2N\lambda}(2j - (1-\lambda)(2i-1))). \end{aligned} \tag{21}$$

In general, the left and right sides of equation(19) will not be integer. P_{ij} is calculated by basing on distribution function of the negative binomial distribution. For example

$$P_{ij} = P(4.6 < x_t < 6.1) = P(x_t = 5) + P(x_t = 6) = \binom{5-1}{r-1} P^r (1-P)^{5-r} + \binom{6-1}{r-1} P^r (1-P)^{6-r}. \tag{22}$$

Define the vector **R** to be

$$\mathbf{R} = [R_1, R_2, \ldots, R_N]^T. \tag{23}$$

Let **Q** be the matrix obtained from the transition matrix P by deleting row N+1 and column N+1, In other words, **Q** is the transition matrix among the in-control states. ANOS vector R is the solution to the system

$$(\mathbf{I} - \mathbf{Q})\mathbf{R} = \mathbf{1}, \tag{24}$$

where **I** is $N \times N$ identity matrix and **1** is a $1 \times N$ column vector of ones .ANOS given that $Z_0 = \mu_0$ is just the middle entry, that is, the $((N + 1)/2)^{\text{th}}$ entry is the vector **R** (We must choose N to be odd so that there is a unique middle value).

7. COMPARISON OF THE PERFORMANCE

For illustrative purpose, in this section we compare the performance of the proposed $\text{EWMA}_{\text{CCC-}r}$ chart with that of the $\text{CS}_{\text{CCC-}r}$ chart. We may express the null hypothesis H_0 (process is in-control) and the alternative hypothesis H_1 (process is out-of-control) in a formal manner as follows:

$$H_0 : P = P_0, H_1 : P = \kappa P_0. \tag{25}$$

Concretely, we compare ANOS of the proposed $\text{EWMA}_{\text{CCC-}r}$ chart with that of the $\text{CS}_{\text{CCC-}r}$ chart.

ANOS of the $\text{CS}_{\text{CCC-}r}$ chart is defined as the expected number of plotted point required for the $\text{CS}_{\text{CCC-}r}$ chart to detect the first out-of-control observation. Similarly, ANOS of the $\text{EWMA}_{\text{CCC-}r}$ chart is defined as the expected number of plotted point required for the $\text{EWMA}_{\text{CCC-}r}$ chart to detect the first out-of-control observation. The ANOS of the $\text{CS}_{\text{CCC-}r}$ chart under each of the conditions of $NB(r, P_1 = \kappa P_0)$ for several values of κ can be given as a solution of the following equation:

$$\text{ANOS} = \frac{1}{P_{\text{LCSL}_{\text{CS}}} + P_{\text{UCSL}_{\text{CS}}}}, \tag{26}$$

where $P_{\text{UCSL}_{\text{CS}}} = \left(1 - \sum_{i=r}^{UCSL_{CCC-r}} \binom{i-1}{r-1} P_1^r (1-P_1)^{i-r}\right)^2$,

$$P_{\text{LCSL}_{\text{CS}}} = \left(\sum_{i=r}^{LCSL_{CCC-r}} \binom{i-1}{r-1} P_1^r (1-P_1)^{i-r}\right)^2.$$

On the other hand, for the $\text{EWMA}_{\text{CCC-}r}$ chart, the ANOS can be given as solutions of equation (20).

Let the type I error α set at 0.0027, assuming that the probability of the type I error in either direction is equally undesirable. Let the population distribution be the nega-tive distribution $NB(r, P_0)$ given as r = 2.5 and P_0= 0.001, 0.0001.

The upper and lower confirmation control limits of the $\text{CS}_{\text{CCC-}r}$ chart are given respectively as solutions of equations (7) and (8).

The upper and lower control limits of the proposed $\text{EWMA}_{\text{CCC-}r}$ chart are given respectively as solutions of equations (13) and (14).

Figures 2-5 show the in-control ANOSs for various values of the parameters λ and L. For a given value λ , the users must choose the value of L that gives the desired in-control ANOS of $1/\alpha$. In the EWMA chart, a smaller value of λ , say, λ <0.1 is recommended to detect small shifts in quality characteristic. For a given value λ in the range that λ <0.1, a value of L is chosen in the interval $2.5 \le L \le 3.0$ to give the desirable in-control ANOS, in this case, $1/\alpha \cong 370$. In this paper, the six combinations of λ and L are chosen as (λ , L) = (0.06, 2.563), (0.07, 2.626), (0.08, 2.684), (0.06, 2.556), (0.07, 2.609), (0.08, 2.655).

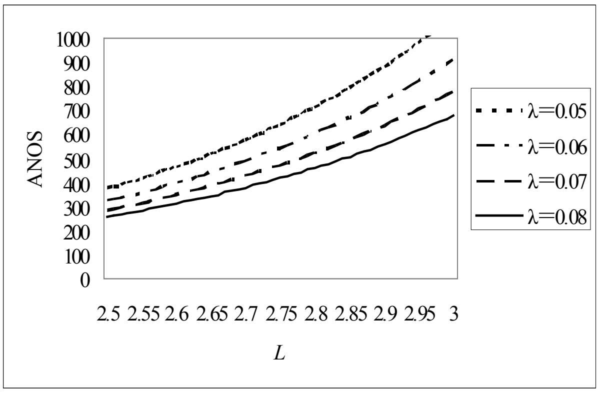


Figure 2. ANOSs for various values of λ and L (P_0= 0.001, r = 2)

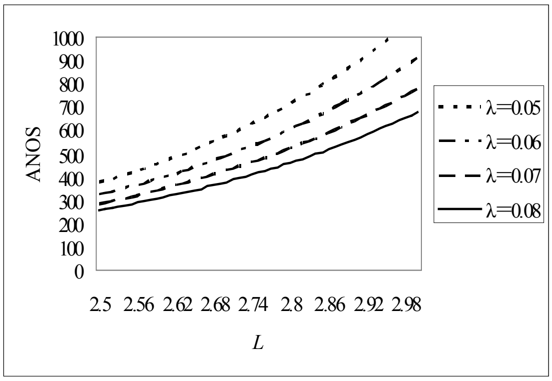


Figure 3. ANOSs for various values of λ and L (P_0= 0.0001, r = 2)

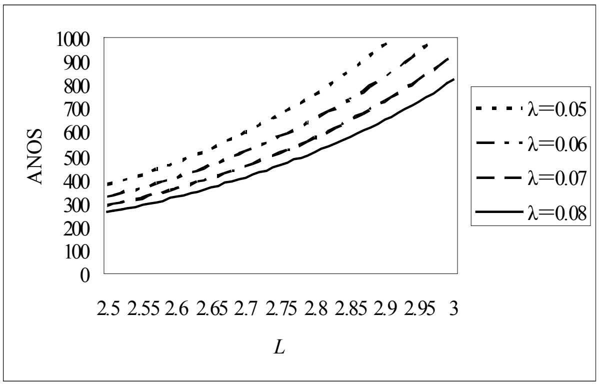


Figure 4. ANOSs for various values of λ and L (P_0= 0.001, r = 5)

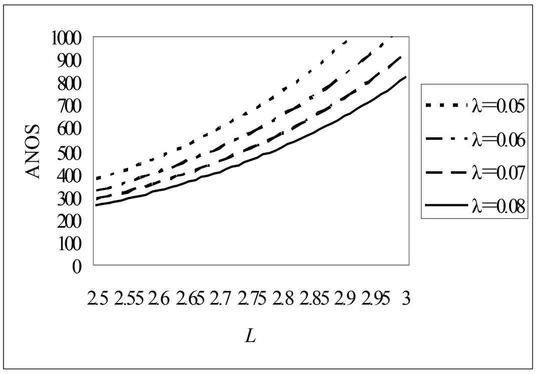


Figure 5. ANOSs for various values of λ and L (P_0= 0.0001, r = 5)

Table 1. Confirmation control limits and actual values of type I error (%) of the $CS_{CCC\text{-}r}$ chart

P_0 = 0.001	$CS_{CCC\text{-}r}$ chart		
	Lower confirmation control limit: $LCSL_{CCC\text{-}r}$	Upper confirmation control limit: $UCSL_{CCC\text{-}r}$	Actual values of type I error (%)
r = 2	299	5111	0.2695
r = 5	1805	9640	0.2699
P_0 = 0.001	Lower confirmation control limit: $LCSL_{CCC\text{-}r}$	Upper confirmation control limit: $UCSL_{CCC\text{-}r}$	Actual values of type I error (%)
r = 2	2991	51141	0.2699
r = 5	18047	96440	0.2700

Table 1 shows the upper and lower confirmation control limits and the actual values of type I error α of the $CS_{CCC\text{-}r}$ chart.

Table 2 shows the upper and lower control limits and the actual values of type I error α of the proposed $EWMA_{CCC\text{-}r}$ chart for each combination of λ and L.

Table 3 shows ANOSs of both the proposed $EWMA_{CCC\text{-}r}$ chart and the $CS_{CCC\text{-}r}$ chart for each combination of λ and L in several cases of κ. It is evident from Table 3 that ANOSs of the proposed $EWMA_{CCC\text{-}r}$ chart are smaller than those of the $CS_{CCC\text{-}r}$ chart for upward (situations where of κ >1) and the downward shifts (situations where κ <1) in P. In situations where κ <1, ANOS indicates the expected number of individual observations required for the chart until judging correctly that P is getting smaller. On the other hand, in situations where κ >1, ANOS indicates the expected number of individual observations required for the chart until judging correctly that P is getting larger. Therefore, the smaller the value of ANOS is, the more superior the performance of a control chart is. Therefore, it can be seen that it is more adequate to apply the proposed $EWMA_{CCC\text{-}r}$ chart to high-yield processes than the $CS_{CCC\text{-}r}$ chart. From comparison of ANOS performance, our purpose has been successfully achieved.

Table 2. Control limits and actual values of type I error(%) of the $EWMA_{CCC\text{-}r}$ chart

$P_0 = 0.001$	$EWMA_{CCC\text{-}r}$ chart				
	λ	L	Lower control limit: h_L	Upper control limit: h_U	Actual values of type I error (%)
$r = 2$	0.06	2.563	1363	2637	0.2695
	0.07	2.626	1293	2707	0.2698
	0.08	2.684	1226	2774	0.2699
$r = 5$	0.06	2.556	3995	6005	0.2694
	0.07	2.609	3890	6110	0.2697
	0.08	2.655	3789	6211	0.2697
$P_0 = 0.0001$	λ	L	Lower control limit: h_L	Upper control limit: h_U	Actual values of type I error (%)
$r = 2$	0.06	2.563	13626	26374	0.2694
	0.07	2.626	12928	27072	0.2696
	0.08	2.684	12252	27748	0.2698
$r = 5$	0.06	2.556	39949	60051	0.2694
	0.07	2.609	38890	61110	0.2697
	0.08	2.655	37882	62118	0.2697

Table 3. Comparison of ANOSs of the proposed $EWMA_{CCC\text{-}r}$ chart with those of the $CS_{CCC\text{-}r}$ chart

(a) In the case that $r = 2$, P_0=0.001

κ	Actual value of ANOS			
	$EWMA_{CCC\text{-}r}$ chart			$CS_{CCC\text{-}r}$ chart
	λ =0.06	0.07	0.08	
	L=2.563	2.626	2.684	
0.5	8	7	7	13
0.6	12	11	11	28
0.7	20	19	19	60
0.8	39	39	39	127
0.9	104	104	104	246
1.0	0.2695*	0.2698*	0.2699*	0.2695*
1.1	366	460	584	406
1.2	138	175	231	355
1.3	70	84	104	285
1.4	45	51	60	224
1.5	34	37	41	178

(b) In the case that $r = 5$, P_0=0.001

κ	Actual value of ANOS			
	$EWMA_{CCC\text{-}r}$ chart			$CS_{CCC\text{-}r}$ chart
	λ =0.06	0.07	0.08	
	L=2.563	2.626	2.684	
0.5	5	4	4	4
0.6	7	7	6	10
0.7	11	11	11	26
0.8	22	22	22	71
0.9	65	66	66	197
1.0	0.2694*	0.2697*	0.2697*	0.2699*
1.1	140	161	185	332
1.2	48	52	57	207
1.3	27	28	30	125
1.4	19	20	20	79
1.5	15	15	16	52

8. CONCLUSIONS

In this paper, we presented an $EWMA_{CCC\text{-}r}$ chart to detect more sensitively small or moderate shifts in the process fraction defectives toward both upward and downward directions in high-yield processes such as the manufacturing process of today. The $EWMA_{CCC\text{-}r}$ chart is constructed by applying the designing method of the EWMA*c* chart to the CCC-*r* chart. According to the designing method of the EWMA*c* chart, the Markov chain approach is used to calculate ANOSs for any shifts in *P*. Comparing ANOS of the proposed $EWMA_{CCC\text{-}r}$ chart with that of the $CS_{CCC\text{-}r}$ chart, it can be seen that the proposed $EWMA_{CCC\text{-}r}$ chart is more sensitive to detect small or moderate shifts in the process fraction defectives toward both upward and downward directions in high-yield processes. Hence, it is adequate to apply the proposed $EWMA_{CCC\text{-}r}$ chart to high-yield processes as an alternative chart for high-yield processes. As *r* increases, the $EWMA_{CCC\text{-}r}$ chart is more and more sensitive to detect small or moderate shifts in the process fraction defectives toward both upward and downward directions, while more and more observations are required to obtain a plotting point on the chart. From the trade-off problem, it is necessary to determine economically values of designing parameters such as number of nonconforming item observed before a point is plotted on the chart, the sampling (inspection) interval and the upper and lower control limits of the $EWMA_{CCC\text{-}r}$ chart. The issue of economical design of the $EWMA_{CCC\text{-}r}$ chart will be left for future research.

ACKNOWLEDGMENT

This research has been supported by the Grant-in-Aid for Scientific Research C(2) No.14580484 from the Japan Society for the Promotion of Science.

REFERENCES

Borror, C. M., Champ, C. W. and Rigdon, S. E. (2002), Poisson EWMA Control Charts, *Journal of Quality Technology*, 30, 352-361.

Gan, F. F. (2002), Monitoring Poisson Observations Using Modified Exponentially Weighted Moving Average Control Charts, *Communications in Statistics-Simulation and Computation*, 19, 103-124.

Montgomery, D. C. (2001), *Introduction to Statistical Quality Control*, 4th ed., John Wily & Sons, New York, NY.

Ohta, H. and Kusukawa, E. (2004), *An Application of Confirmation Sample Control Chart to the High-Yield Processes and Its Economic Design. In H, -J. Lenz and P, -TH. Wilrich (ed), Frontiers in Statistical Quality Control 7, (New York: Physica-Verlag),* chapter 2, 101-118.

Roberts, S. W. (1959), Control Chart Tests Based on Geometric Moving Averages, Technometrics, 1, 239-250.

Roberts, S. W. (1966), A Comparison of Some Control Chart Procedures, *Technometrics*, 8, 411-430.

Steiner, S. H. (1999), Confirming Sample Control Charts, *International Journal of Production Research*, 37, 737-748.

Xie, M., Goh, T. N. and Lu, X. S. (1998), Computer–Aided Statistical Monitoring of Automated Manufacturing Process, *Journal of Computers & Industrial Engineering*, 35, 189-192.