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Influence of Time to Walk Back and Comparing for the Self-balancing Production Line

Daisuke Hirotani
Department of Artificial Complex Systems Engineering
Hiroshima University, 1-4-1, Kagamiyama, Higashi-Hiroshima, 739-8521, Japan
Tel: +81-82-424-7705, Fax: +81-82-422-7024, E-mail: dhiro@hiroshima-u.ac.jp

Myreshka
Department of Artificial Complex Systems Engineering
Hiroshima University, 1-4-1, Kagamiyama, Higashi-Hiroshima, 739-8521, Japan
Tel: +81-82-424-7705, Fax: +81-82-422-7024, E-mail: myreshka@hiroshima-u.ac.jp

Katsumi Morikawa
Department of Artificial Complex Systems Engineering
Hiroshima University, 1-4-1, Kagamiyama, Higashi-Hiroshima, 739-8521, Japan
Tel: +81-82-424-7704, Fax: +81-82-422-7024, E-mail: mkatsumi@hiroshima-u.ac.jp

Katsuhiko Takahashi†
Department of Artificial Complex Systems Engineering
Hiroshima University, 1-4-1, Kagamiyama, Higashi-Hiroshima, 739-8521, Japan
Tel: +81-82-424-7705, Fax: +81-82-422-7024, E-mail: takahasi@pel.sys.hiroshima-u.ac.jp

Abstract. In traditional production lines, such as assembly lines, each worker is usually assigned to a fixed task, which is beneficial since it reduces the amount of training needed for workers to master their assigned tas ks. However, when workers complete their tasks at different speeds, the slowest worker will determine the overall pace of the production line and limit production. To avoid this problem, the self-balancing production line was introduced. In this type of production line, each worker works dynamically, thus they can maintain balanced production. Previous research analyzing the performance of these lines has ignored the walk-back time associated with dynamic workers. U-shaped production lines have also been analyzed and policies for such lines have been proposed. However, the walk-back time cannot be ignored in practice, and research taking this factor into account is needed to enable balanced production and thus the maximum production rate. In this paper, we propose production policies for a production line with the walk-back time taken into account, and define and analyze the conditions for self-balancing. Furthermore, we have compar ed the performance of such a line with that of other production lines under the same conditions, and the results show the superiority of this line in certain cases.

Keywords: Self-Balancing, Blocking, Starving, Production, Line, Convergence

1. INTRODUCTION

In a traditional assembly line, each worker is typically assigned to a particular fixed task and continuously repeats the assigned task with the goal of achieving assembly line balance. For this type of production line, the assignment of workers to balance work has been studied (Scholl, 1995). In this kind of line, when there is an imbalance among worker speeds, the slowest worker will set the pace of the overall work. As a result, the production rate will be lower than it could be. To solve this problem, the self-balancing production line was introduced. The application of such lines has been studied in at least two commercial environments: apparel manufacturing and distribution warehousing (Bartholdi *et al.* 1999). In this type of production line, each worker works dynamically: when the last worker completes an item, he/she walks back and takes over the next item from his/her predecessor. The predecessor then walks back, takes over the next item from his/her predecessor, and so

† : Corresponding Author

on until the first worker walks back and starts a new item. Since faster workers spend more time processing items without being held up by slower workers, balanced production can be maintained. For this sort of line, the maximum production rate is achieved when workers are sequenced from slowest to fastest (Bartholdi *et al.* 1996). Also, other conditions for a line with three workers have been numerically derived through simulation (Bartholdi *et al.* 1999), and the performance of a production line with n workers has been analyzed mathematically (Hirotani *et al.* 2003). In other literature, policies for a U-shaped production line have been proposed (Zavadlav *et al.* 1996). In all of this research, however, the time needed for a worker to walk back to take over an item from an upstream worker has been ignored, even though this factor will significantly affect performance. Therefore, we consider the walk-back time in this paper. In detail, we enumerate particular policies, and compare these. After that, to determine the best of these policies, we derive the conditions under which a line is balanced, calculate the production rate under these conditions, and further analyze the performance. In addition, we compare the performance of this line with that of a U-shaped production line under the same conditions. Our comparison shows that the balanced line is superior in some cases.

In Section 2, we explain our assumptions, describe the characteristics of this production line, and derive the behavior and formulation of this model. In Section 3, we discuss production-line policies that take the walk-back time into account. In Section 4, we derive the best policy by comparing the cycle time, the production rate, and the convergence area. Then, in Section 5, we analyze the performance under the best policy and compare this with the performance of a U-shaped production line. We conclude in Section 6.

2. THE PRODUCTION LINE

In this section, assumptions, and workers' behavior are explained, and the characteristics of this line are described.

2.1 Assumptions

In this research, we assume the production line operates as follows.

1. Workers sequentially process one identical item.
2. Workers are sequenced from one to n, as in a flow-shop production line, and no worker never passes over a downstream worker.
3. Each worker processes his/her work while moving along the line, and worker i processes at a constant velocity v_i in the production line. Thus, a continuous line is considered. This was not assumed in previous papers (Bartholdi *et al.* 1996, Bartholdi *et al.* 1999), which considered practical applications in, for example the fast-food industry.
4. When the last worker finishes processing an item, worker n walks back to worker n-1 and takes over the next item from worker n-1. Worker n-1 then walks back to worker n-2 and takes over the next item from worker n-2. Similarly, all workers walk back to their upstream worker and take over the next item from the upstream worker, and worker 1 introduces a new item into the system. However, the item is not necessarily taken over and processed immediately. The velocity at which the worker walks back is defined as v_b and is constant for all workers, and the time needed to take over an item is ignored.
5. The position of worker i when he/she starts to process an item is given by x_i (Figure 1). The position at iteration t is defined as $x_i^{(t)}$, and the position of worker i when he/she finishes processing an item is given by $x_{i,e}^{(t)}$. Note that $x_{n+1}^{(t)} = x_{n,e}^{(t-1)} = 1$, and $x_1^{(t)} = 0$ for any iteration t. This is because the last worker always finishes an item, and the first worker always starts the processing of an item.

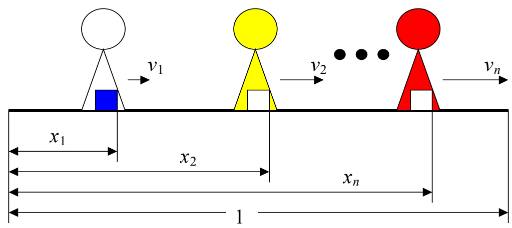


Figure 1. Production line and position of n workers

2.2 Self-Balancing and Convergence

When workers are sequenced from slowest to fastest and the walk-back time is ignored, the production line can maintain balance (Bartholdi *et al.* 1996). Subsequently, the position of workers will converge to a unique fixed point. Let x_i* be the position of worker i at the fixed point; this can be given as follows (Bokkobas 1990, Bartholdi *et al.* 1996).

$$x_i^* = \frac{\sum_{j=1}^{i-1} v_j}{\sum_{j=1}^{n} v_j} \tag{1}$$

Under this condition, the production rate can be calculated as the sum of each worker's velocity $v_i(i=1, 2, \ldots, n)$ as follows (Bartholdi *et al.* 1996).

$$\sum_{i=1}^{n} v_i \tag{2}$$

When the walk-back time is considered, the fixed point shown by Eq. (1) and the production rate shown by Eq. (2) change. In this case, the actual processing time per unit time is increased for each worker, and so the production rate is decreased.

2.3 Imbalance

When workers are not sequenced from slowest to fastest, a slower worker may prevent the work of his/her faster predecessor being further processed. This condition is called blocking. When blocking occurs, the faster upstream worker will work at the same speed as the slower downstream worker until the last worker finishes an item because, from assumption 2, no worker can pass a downstream worker. Two kinds of blocking exist: one is the blocking caused by the initial position of workers, and the other is blocking caused by a worker's velocity.

On the other hand, when the distance that a worker walks back along a line is long, the walk-back time becomes considerable, and an item will not be processed until the last worker finishes an item. This condition is called starving. After starving occurs, the worker must continue to walk back at the next iteration. Note that starving never occurs when the time to walk back is ignored, because the item can be taken over immediately.

When blocking or starving occurs, the position of workers will not converge to a fixed point; thus, the production rate is reduced.

2.4 Behavior and Formulation of the Model

Figure 2 shows a time chart for three workers when the walk-back time is ignored. In this figure, the horizontal axis represents the position of workers and the vertical axis represents time. Zero on the horizontal axis indicates the head of the line and one indicates the end of the line. Since each worker works while moving downstream, the diagonal lines represent the workers' positions while they work. Also, since the walk-back time and takeover time are ignored in this figure, horizontal lines with slope zero represent the workers walking back and taking over an item. When worker 3 finishes an item (i.e., the position becomes equal to one), he/she walks back to worker 2 and takes over the next item from worker 2. At the same instant, worker 2 walks back to worker 1 and takes over the next item from worker 1, and worker 1 walks back to position zero and starts to process a new item.

The cycle time is derived by calculating the time spent in each iteration; i.e., the distance over which worker i processes an item and walks back in that iteration divided by the velocity. Therefore, the cycle time of worker i at iteration t, defined as $a_i^{(t)}$, is

$$a_i^{(t)} = \frac{x_{i,e}^{(t)} - x_i^{(t)}}{v_i} + \frac{x_{i,e}^{(t)} - x_i^{(t)}}{v_b} \quad (i = 1, 2, \cdots, n) \tag{3}$$

Using this, after the steady-state cycle time of worker i, defined as a_i, is derived, we can calculate the production rate using a_i as the reciprocal of the cycle time:

$$\min_i\{1/a_i\} \quad (i=1, 2, \ldots, n) \tag{4}$$

In Figure 2, the small square denotes worker 1 being blocked. After the blocking occurs, worker 1 processes an item at the same velocity as worker 2, until worker 3 finishes an item. Therefore, the cycle time increases, and the production rate decreases. If neither blocking nor starving occurs, convergence of the positions of each worker and the maximum production rate can be obtained.

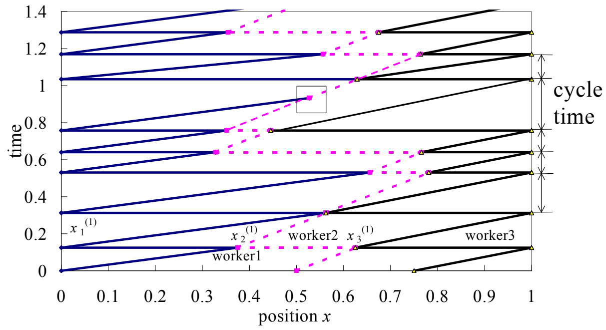


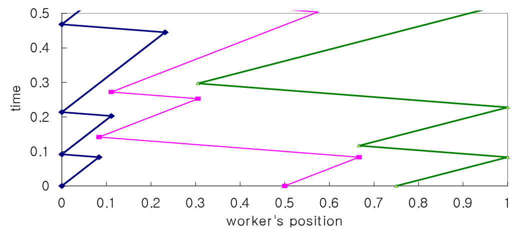
Figure 2. A time chart for three workers when walk-back time is ignored (□: blocking)

3. PRODUCTION POLICIES

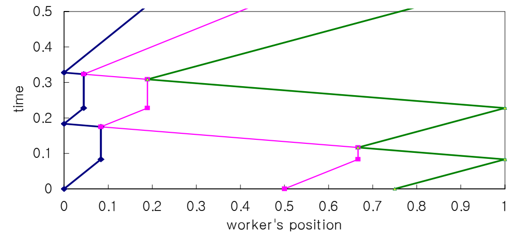
In this section, we consider production policies, taking the walk-back time into account according to two measures: (a) when each worker starts to walk back, and (b) where an item is taken over from an upstream worker. Based on measures (a) and (b), we enumerate four policies. For each policy, the time chart for three workers is shown (Table 1) in Figure 3.

Table 1. Production policies when the walk-back time is considered

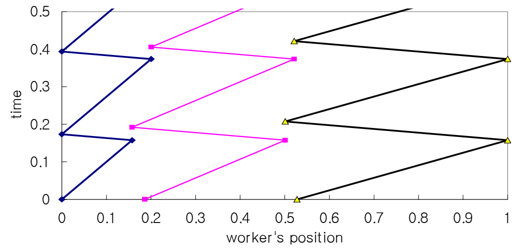
Policy / Measure		(a)	(b)
1	Same processing time for all workers	The position when a product is completed	After the work end of the iteration
2	Waiting for the downstream worker	The position when a product is completed	Immediately after take-over
3	Walking back simultaneously	The position when a product is completed	After the work end of the iteration
4	Processing until work is taken over by a downstream worker	The position to meet a worker who walks back	Immediately after take-over



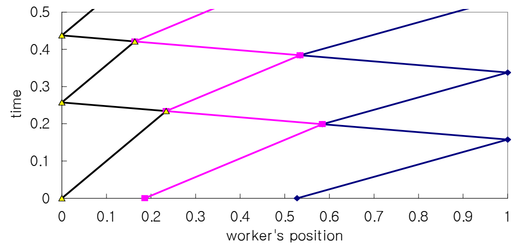
(1) Policy 1



(2) Policy 2



(3) Policy 3



(4) Policy 4

Figure 3. An example of each policy for three workers

3.1 Same Processing Time for all Workers

Under this policy, an item is processed by each worker for as long as the last worker processes an item during that iteration. Therefore, when a worker finishes an item in that iteration, the worker must start to walk back, therefore, the item is not immediately taken over from the upstream worker. As shown in Figure 3(1) (Policy 1), workers 1 and 2 process items during the same time as worker 3 in all iterations. Under this policy, the starting position for worker i when neither blocking nor starving occurs can be formulated as

$$\begin{cases} x_1^{(t+1)} = 0 \\ x_i^{(t+1)} = x_{i-1}^{(t)} + v_{i-1}\left(\dfrac{1-x_n^{(t)}}{v_n}\right) \qquad (i = 2,\cdots,n) \end{cases} \tag{5}$$

In Eq. (5), the first term is the position of worker i-1 at iteration t-1, and the second term is the distance over which worker i-1 processes an item in iteration t. This formula is the same as for the case when walk-back time is ignored (Bartholdi *et al*. 1996).

3.2 Waiting for the Downstream Worker

Under this policy, when the last worker finishes an item, the others have to stop processing an item and wait until their item is taken over by the downstream worker. In Figure 3(2) (Policy 2), when worker 3 finishes processing an item, workers 1 and 2 have to wait until their item is taken over. Under this policy, the starting position for the processing done by worker i when neither blocking nor starving occurs can be formulated as

$$\begin{cases} x_1^{(t+1)} = 0 \\ x_i^{(t+1)} = x_{i-1}^{(t)} + v_{i-1}\left(\dfrac{1-x_n^{(t)}}{v_n} - \dfrac{x_n^{(t)} - x_{i-1}^{(t)}}{v_b}\right) \qquad (i = 2,\cdots,n) \end{cases} \tag{6}$$

In Eq. (6), the first term is the same as in Eq. (5), and the second term is the distance over which worker i-1 processes an item in iteration t. The second term differs from that in Eq. (5), and can be derived from the processing time of the last worker in iteration t minus the time between when the last worker starts to process and when worker i-1 starts to process.

3.3 Walking Back Simultaneously

Under this policy, when the last worker finishes an item, all workers start to walk back immediately. Therefore, all workers except for the last worker have to leave the items they are working on at each item's current location. In Figure 3(3) (Policy 3), when worker 3 completes an item, at the same instant, workers 1 and 2 stop processing the items they are working on, leave the item where it is, and start to walk back like worker 3. Under this policy, the starting position for the processing done by worker i when neither blocking nor starving occurs can be formulated as

$$\begin{cases} x_1^{(t+1)} = 0 \\ x_i^{(t+1)} = x_{i-1}^{(t)} + v_{i-1}\left(\dfrac{1-x_n^{(t)}}{v_n} + \dfrac{1-x_n^{(t)}}{v_b} - \dfrac{x_i^{(t)} - x_{i-1}^{(t)}}{v_b}\right) \\ \qquad (i = 2,\cdots,n) \end{cases} \tag{7}$$

In Eq. (7), the time that each worker processes an item in iteration t (the terms in parentheses) differs from that in Eq. (5). This value is derived from the processing time of the last worker plus the walk-back time of the last worker minus the walk-back time of each worker.

3.4 Processing until Work Is Taken Over by a Downstream Worker

Under this policy, each worker processes an item until he/she meets a downstream worker who has walked back along the line. In Figure 3(4) (Policy 4), worker 3 finishes an item and then walks back upstream along the line. When worker 3 meets worker 2 processing the next item, worker 3 takes over that item from worker 2. After this, worker 2 starts to walk upstream while worker 3 starts to process the item. Under this policy, the starting position for the processing done by worker i when neither blocking nor starving occurs can be formulated as

$$\begin{cases} x_1^{(t+1)} = 0 \\ x_i^{(t+1)} = x_{i-1}^{(t)} + v_{i-1}\left\{ \dfrac{1-x_n^{(t)}}{v_n} - \dfrac{x_n^{(t)} - x_{i-1}^{(t)}}{v_b} \right. \\ \qquad \left. + \dfrac{1}{v_{i-1}+v_b}\left(x_{i-1}^{(t)} + v_{i-1}\left(\dfrac{1-x_n^{(t)}}{v_n} - \dfrac{x_n^{(t)} - x_{i-1}^{(t)}}{v_b} \right)\right)\right\} \\ \qquad (i = 2, \cdots, n) \end{cases} \tag{8}$$

In Eq. (8), the time that each worker processes an item in iteration t (the terms in parenthesis) also differs from that in Eq. (5). This value is derived from the processing time of the last worker minus the time between when the last worker starts to process an item and when worker i-1 starts to process an item in iteration t. Furthermore, the time during which each worker processes an item between when the last worker walks back and worker i-1 starts to process an item is subtracted.

4. COMPARING THE PRODUCTION POLICIES

In this section, we compare the four policies defined above to determine which is best.

4.1 Cycle Time

In Policy 1, an item may remain on the line without being processed. The item state can be called "idle" in this case. To calculate the idle time, we consider the walk-back time. First, since the time that each worker processes an item is the same for all workers, the walk-back time is different for each worker depending on the distance. The walk-back time for worker i during iteration t is given by

$$\frac{x_{i+1}^{(t)} - x_i^{(t)}}{v_b}$$

The idle time is then derived as the cumulative error of the walking-back time. Therefore, we consider the item rather than the iteration. The idle time for item j is

$$\frac{1}{v_b}\left\{ \sum_{l=1}^{j+n-2} (1 - x_n^{(l)}) - \sum_{l=1}^{j-1} x_2^{(l)} \right\}$$

This is because both the upstream and downstream side for each worker is considered. Therefore, only both the first and the last worker need to be considered. If the production line is balanced by appropriate sequencing for item j, the line is also balanced for item j+1. Thus, the idle time in item j+1 can be calculated using the convergence point shown by Eq. (1):

$$\frac{1}{v_b}\left\{ \sum_{l=1}^{j+n-2} (1 - x_n^{(l)}) - \sum_{l=1}^{j-1} x_2^{(l)} \right\} + \frac{1}{v_b}\left(\frac{v_n - v_1}{\sum_{i=1}^{n} v_i} \right)$$

Note that if the formula of the starting position is the same, the convergence point is also the same. That is why Eq. (1) is available. The first term represents the idle time for item j, and the second term represents the additional idle time between items j and j+1. When the production line is balanced, $v_1 < v_n$ is necessary because of the convergence conditions (Bartholdi *et al.* 1999). Therefore, even if the production line is balanced, the idle time increases as the number of iterations increases. This means that balanced production cannot be maintained. Therefore, we exclude this policy from the following analysis.

4.2 Production Rate

We next calculate the production rate for the three remaining policies. First, we define the convergence point for the position of workers by setting $x_i^{(t)} = x_i^{(t+1)}$ as x_i^*. Second, we calculate the cycle time using Eq. (3). Finally, we derive the production rate from the cycle time using Eq. (4). As an example, we examine a production line with three workers under Policy 3. We initially assume that the production line is balanced, and that the position of each worker for both iterations t and t+1 converge to a fixed point. Thus, we can formulate the following two equations using Eq. (7):

$$x_2^* = x_1^* + v_1\left(\frac{1-x_3^*}{v_3} + \frac{1-x_3^*}{v_b} - \frac{x_3^* - x_2^*}{v_b} \right)$$

$$x_3^* = x_2^* + v_2\left(\frac{1-x_3^*}{v_3} + \frac{1-x_3^*}{v_b} - \frac{x_2^* - x_1^*}{v_b} \right)$$

By solving these equations, we can derive the convergence point as

$$x_2^* = \frac{v_1v_2v_3 + (v_1v_2 + v_1v_3)v_b + v_1v_b^2}{3v_1v_2v_3 + 2(v_1v_2 + v_1v_3 + v_2v_3)v_b + (v_1 + v_2 + v_3)v_b^2}$$

$$x_3^* = \frac{2v_1v_2v_3 + (2v_1v_2 + v_1v_3 + v_2v_3)v_b + (v_1 + v_2)v_b^2}{3v_1v_2v_3 + 2(v_1v_2 + v_1v_3 + v_2v_3)v_b + (v_1 + v_2 + v_3)v_b^2}$$

Similarly, for a production line with n workers, the convergence point can be calculated as

$$x_i^* = \frac{\sum_{k=1}^{n} C_{i-1} V_{n,k} v_b^{\,n-k}}{\sum_{k=1}^{n} k V_{n,k} v_b^{\,n-k}} \quad (i = 2, 3, \cdots, n) \tag{9}$$

where $V_{n,i}$ is a product-sum of all combinations of the velocities of i workers chosen from v_1 to v_n, and C_i is the coefficient in $V_{n,i}$, which is the number which contains v_1 to v_i in $V_{n,k}$. For instance, $C_2V_{3,1}=1*v_1+1*v_2+0*v_3=v_1+v_2$. Note that $V_{n,0}=1$ and $C_0V_{n,k}=0$ for all n and k.

Finally, the production rate can be derived from Eqs. (3) and (4):

$$\frac{3v_1v_2v_3v_b + 2(v_1v_2 + v_1v_3 + v_2v_3)v_b^{\,2} + (v_1 + v_2 + v_3)v_b^{\,3}}{v_1v_2v_3 + (v_1v_2 + v_1v_3 + v_2v_3)v_b + (v_1 + v_2 + v_3)v_b^{\,2} + v_b^{\,3}}$$

Similarly, the production rate of a balanced line with n workers can be calculated:

$$\text{Policy 2:} \quad \frac{\sum_{i=1}^{n} V_{n,i} v_b^{\,n-i+1}}{\sum_{i=0}^{n} V_{n,i} v_b^{\,n-i}}$$

$$\text{Policies 3 and 4:} \quad \frac{\sum_{i=1}^{n} i V_{n,i} v_b^{\,n-i+1}}{\sum_{i=0}^{n} V_{n,i} v_b^{\,n-i}} \tag{10}$$

As shown, the production rate is the same under Policies 3 and 4, while that of Policy 2 is lower. Therefore, we exclude Policy 2 from the following analysis.

4.3 Convergence Area

We next compare the convergence areas of Policies 3 and 4, because these policies have the same production rate. A policy with a larger convergence area is regarded as a better policy. We analyzed the convergence area for three workers through simulation (Figure 4). The velocities of worker 3 and for walking back were fixed as 5 and 10, respectively, and the velocities of workers 1 and 2 were varied from 0 to 10. The convergence area of Policy 4 was larger than that of Policy 3. Furthermore, this area covered that of Policy 3. This can be confirmed analytically.

First, the error of the convergence point for worker i at iteration t is defined as $\varepsilon_i^{(t)}$. Using this notation, $x_i^{(t)} = x_i^* + \varepsilon_i^{(t)}$ is substituted into Eqs. (7) and (8). This is because the workers' positions converge to a fixed point and the line is balanced if $\varepsilon_i^{(t)}$ becomes zero. Under Policy 3, $x_i^{(t)} = x_i^* + \varepsilon_i^{(t)}$ is substituted into Eq. (7):

$$x_i^{(t+1)} = x_i^* + \left(1 - \frac{v_{n-1}}{v_b}\right)\varepsilon_{i-1}^{(t)} - v_{i-1}\left\{\left(\frac{1}{v_n} - \frac{1}{v_b}\right)\varepsilon_n^{(t)} + \frac{1}{v_b}\varepsilon_i^{(t)}\right\}$$

Since $x_i^{(t+1)} = x_i^* + \varepsilon_i^{(t+1)}$ from the above assumption, $\varepsilon_i^{(t+1)}$ is derived as

$$\varepsilon_i^{(t+1)} = \left(1 - \frac{v_{n-1}}{v_b}\right)\varepsilon_{i-1}^{(t)} - v_{i-1}\left\{\left(\frac{1}{v_n} - \frac{1}{v_b}\right)\varepsilon_n^{(t)} + \frac{1}{v_b}\varepsilon_i^{(t)}\right\}$$

Next, we consider Policy 4, for which $x_i^{(t+1)} = x_i^* + \varepsilon_i^{(t+1)}$ is substituted into Eq. (7):

$$x_i^{(t+1)} = x_i^* + \varepsilon_{i-1}^{(t)} - \frac{v_{i-1}v_b}{v_{i-1} + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

For similar reasons as above, $\varepsilon_i^{(t+1)}$ is derived as

$$\varepsilon_i^{(t+1)} = \varepsilon_{i-1}^{(t)} - \frac{v_{i-1}v_b}{v_{i-1} + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

For both policies, $\varepsilon_i^{(t+1)} < \varepsilon_i^{(t)}$ is one of the convergence conditions. Therefore, it is shown below to be the same for both policies.

$$\varepsilon_{i-1}^{(t)} - \frac{v_{i-1}v_b}{v_{i-1} + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)} < \varepsilon_i^{(t)}$$

Then, i=2 is substituted on the left side to remove the first term:

$$-\frac{v_{i-1}v_b}{v_{i-1} + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

From the above formula, if

$$\frac{v_{i-1}v_b}{v_{i-1} + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)} < 1,$$

the error becomes zero as the number of iterations increases. When this formula is expanded, $v_1 < v_3$ is derived.

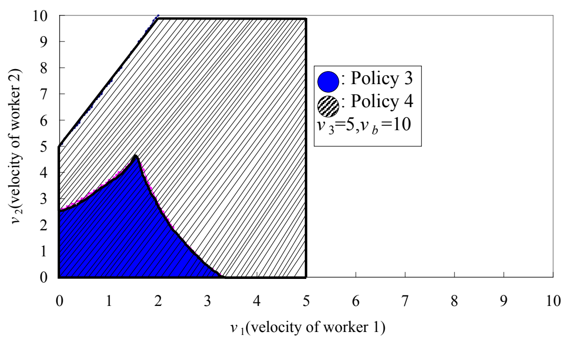


Figure 4. Convergence area for three workers

Similarly, if the condition $\varepsilon_i^{(t+2)} < \varepsilon_i^{(t)}$ is derived, the convergence area of Policy 4 is larger than that of Policy 3.

Therefore, Policy 4 is the best of the four policies we have considered.

5. ANALYZING THE BEST PRODUCTION POLICY

In this section, we analyze the performance and properties of the best policy, Policy 4.

5.1 Imbalance

First, we analyze the blocking and starving conditions (from Section 2.3) for Policy 4, starting with derivation of the blocking conditions. Blocking is a problem only when a worker is faster than the successive worker because blocking occurs only when the upstream worker catches up to the downstream worker. Furthermore, the additional condition can be derived based on time. Considering this, the behavior of a worker in processing an item can be expressed as a straight line: for worker i, $y_1=v_it_1+b_1$, and for worker i-1, $y_2=v_{i-1}t_2+b_2$. Blocking occurs if two straight lines intersect. If we assume that the time at which the last worker starts to walk back is set as zero, b_1 and b_2 are easily derived:

$$b_1 = x_i^{(t)} - \frac{v_1}{v_b}(1-x_i^{(t)})$$

$$b_2 = x_{i-1}^{(t)} - \frac{v_{i-1}}{v_b}(1-x_{i-1}^{(t)})$$

This is because these line passes the following point: $(y_1, t_1)=(x_i^{(t)}, (1-x_i^{(t)})/v_b)$, $(y_2, t_2)=(x_{i-1}^{(t)}, (1-x_{i-1}^{(t)})/v_b)$. The time at which both workers meet is defined as t_b. If workers i and i-1 meet, $t_1=t_2=t_b$, and $y_1=y_2$ are satisfied. Therefore, simultaneous equations can be set, and t_b is derived as

$$t_b = \frac{1}{v_i - v_{i-1}}(x_{i-1}^{(t)} - x_i^{(t)} - \frac{v_{i-1}}{v_b}(1-x_{i-1}^{(t)}) + \frac{v_i}{v_b}(1-x_i^{(t)}))$$

If t_b is a time after an item is taken over, blocking does not occur. The taking-over time consists of the processing time of the last worker, and the time between when the last worker begins to walk back and when the worker i starts to process an item at iteration t. The time is

$$\Delta_i^{(t)} = \frac{1}{v_i+v_b}\left[1-\left\{x_i^{(t)} + v_i\left(\frac{1-x_n^{(t)}}{v_n} - \frac{x_n^{(t)}-x_i^{(t)}}{v_b}\right)\right\}\right]$$

This is derived as follows. Two equations representing the processing and the walking back are solved as simultaneous equations, since the two workers meet. Therefore, if the positions of workers at iteration t are given, their positions at the next iteration can be derived. Based on these positions, the blocking caused by the velocities of workers i-1 and i at iteration t+1 can be evaluated, and the conditions enabling blocking are

$$\begin{cases} v_{i-1} > v_i \\ x_{i-1}^{(t)} - x_i^{(t)} > (v_i - v_{i-1})\left(\frac{1-x_n^{(t)}}{v_n} + \Delta_i^{(t)}\right) \\ \qquad + \left(\frac{v_{i-1}}{v_b}(1-x_{i-1}^{(t)}) - \frac{v_i}{v_b}(1-x_i^{(t)})\right) \end{cases} \tag{11}$$

Both the velocity and initial position of each worker determine whether Eq. (11) is satisfied. To analyse the influence of the initial position, we investigate the blocking caused by the initial position in Section 5.3.

Starving never occurs under Policy 4. This can be proven through a time chart such as those in Figure 3. In Figure 3(4), the behavior of all workers when walking back at any iteration is expressed as a straight line. This means that processing of an item is necessary to transfer to the next iteration. Thus, it is evident that starving will never occur under Policy 4.

5.2 Balance

For a production line with n workers, the convergence conditions are derived from the two independent aspects described below. First, we let $\varepsilon_i^{(t)}$ be the error for worker i from a convergence point defined as x_i^* at iteration t, and consider how the error changes after one iteration. To enable convergence to a fixed point, this error should decrease for all workers as the number of iterations increases. Since worker 1 always starts processsing a new item, he/she starts to process each item at position 0. We consider the error of worker 2 first. Eq. (8) can be transformed by assuming the position at iteration t as $x_i^{(t)} = x_i^* + \varepsilon_i^{(t)}$, and $\varepsilon_i^{(t+1)}$ is derived as

$$\varepsilon_i^{(t+1)} = \varepsilon_{i-1}^{(t)} - \frac{v_{i-1}v_b}{v_{i-1}+v_b}\left(\frac{1}{v_n}+\frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

Because worker 1 always starts at position 0 for all iterations, $\varepsilon_1^{(t+1)} = 0$. The terms of $\varepsilon_2^{(t+1)}$ can then be derived as

$$\varepsilon_2^{(t+1)} = -\frac{v_1 v_b}{v_1+v_b}\left(\frac{1}{v_n}+\frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

Next, we derive the error of worker 3 by substituting the error of worker 2 into Eq. (8):

$$\varepsilon_3^{(t+1)} = \varepsilon_2^{(t)} - \frac{v_2 v_b}{v_2 + v_b}\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

$$= -\left(\frac{v_2 v_b}{v_2 + v_b} - \frac{v_1 v_b}{v_1 + v_b}\right)\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

Similarly, the error can be derived until worker n. Consequently, the error $\varepsilon_i^{(t+1)}$ at one iteration of worker i (i=2, 3, …, n) can be expressed with $\varepsilon_n^{(t)}$ as

$$\varepsilon_i^{(t+1)} = -\left(\sum_{k=1}^{i-1}(-1)^{i+k-1}\frac{v_k v_b}{v_k + v_b}\right)\left(\frac{1}{v_n} + \frac{1}{v_b}\right)\varepsilon_n^{(t)}$$

If the multiplier of $\varepsilon_i^{(t+1)}$ in the formula is less than one, the error decreases and converges to zero as the number of iterations increases. Therefore, the convergence condition is

$$\left(\sum_{k=1}^{i-1}(-1)^{i+k-1}\frac{v_k v_b}{v_k + v_b}\right)\left(\frac{1}{v_n} + \frac{1}{v_b}\right) < 1 \quad (i = 2,\ 3,\ ...,\ n) \tag{12}$$

Second, when every worker starts to process items at a convergence point shown by Eq. (9), the position will not move out of the fixed point during any iteration regardless of the velocity of workers. However, when we consider the walk-back time, all workers do not simultaneously start processing items in any iteration except iteration one. The delay of the start of processing for worker i compared to the start for worker n can be shown as

$$\frac{x_n^* - x_i^*}{v_b} \tag{13}$$

As an example, we consider a production line with three workers to illustrate the above conditions, and to show how this work differs from previous research. The convergence conditions for three workers can be rewritten from Eq. (12):

$$\frac{v_1 v_b}{v_1 + v_b}\left(\frac{1}{v_3} + \frac{1}{v_b}\right) < 1,$$

$$\left(\frac{v_2 v_b}{v_2 + v_b} - \frac{v_1 v_b}{v_1 + v_b}\right)\left(\frac{1}{v_3} + \frac{1}{v_b}\right) < 1$$

For an assumed velocity of workers walking back, v_b = 10, Figure 5 shows the convergence areas derived above for a production line with three workers, and the effect of velocity v_b is shown in Figure 6. In Figure 6, the convergence area is inversely proportional to v_b. Moreover, when v_b becomes infinite, the convergence area becomes identical to that when the walk-back time is ignored (Hirotani *et al*. 2003).

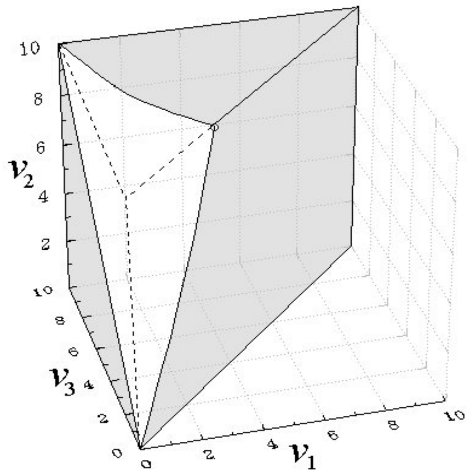


Figure 5. Convergence area for three workers

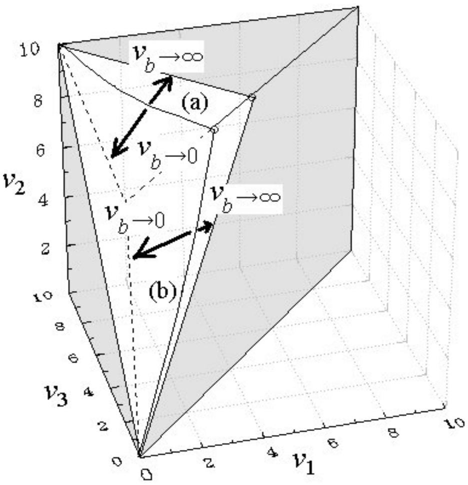
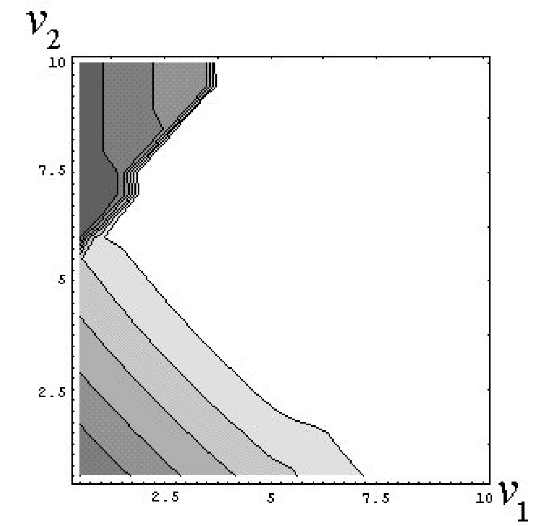
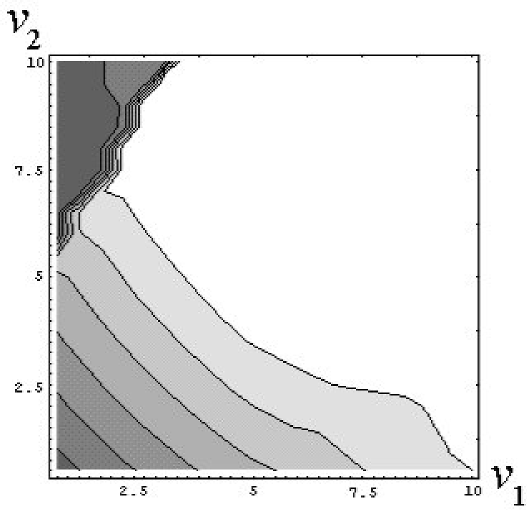


Figure 6. Effect of v_b on the convergence area

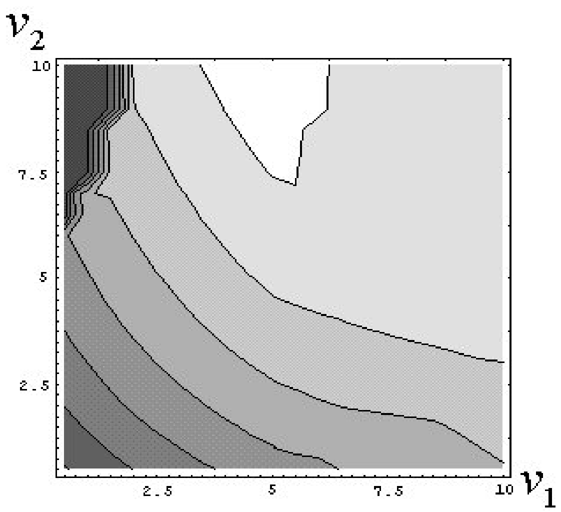
We have also analyzed the production rate. The results for the production rate and the ratio to the maximum production rate are shown in Figures 7 and 8, respectively. These figures are obtained for v_3 = 5 and v_b = 10, 20, and 40. Figure 8 also shows the convergence area for three workers. In Figures 7 and 8, darker areas represent areas of lower production rates and ratios. Figure 7 shows that the production rate increases as the velocity of walking back increases because more time is spent processing items rather than walking. Figure 8 shows that the area where the maximum production rate is achieved is almost the same as the convergence area. Therefore, we should be able to achieve the maximum production rate by satisfying the convergence conditions. Otherwise, the production rate decreases as it moves further from the convergence area. The convergence conditions assuming the initial position $x_2^{(0)}$ = 0.5 and $x_3^{(0)}$ = 0.75 are (v_1, v_2) = (20, 5) under condition (a), (v_1,v_2) = (13.3, 5) under condition (b), and (v_1, v_2) = (11.4, 5) under condition (c). These conditions cannot be shown in Figure 8.



(1) $v_b = 10$

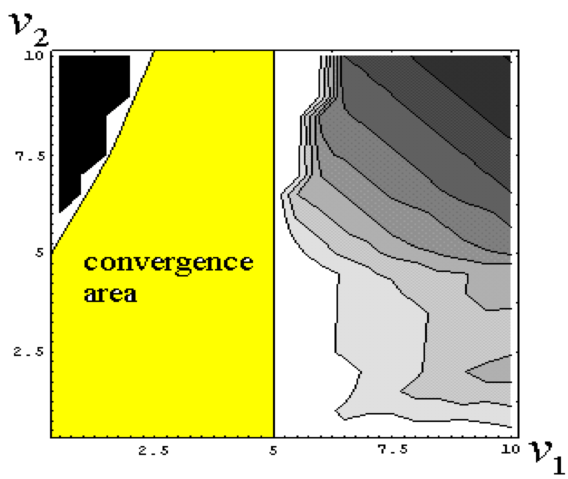


(2) $v_b = 20$

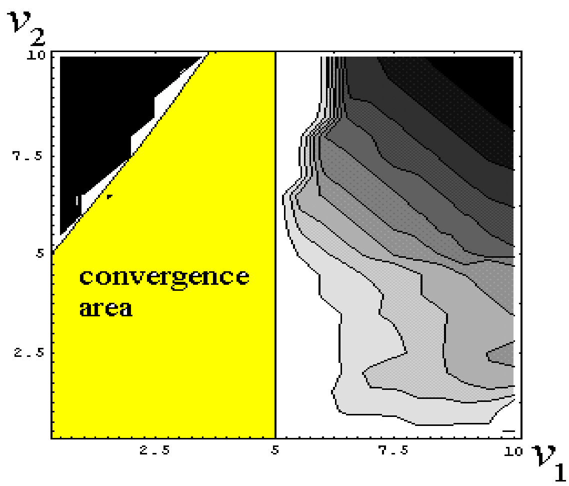


(3) $v_b = 40$

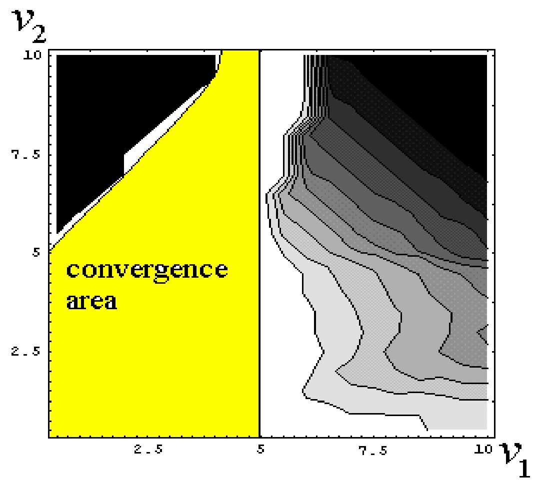
Figure 7. Production rate under various combinations of processing and walking-back velocities



(1) $v_b = 10$



(2) $v_b = 20$



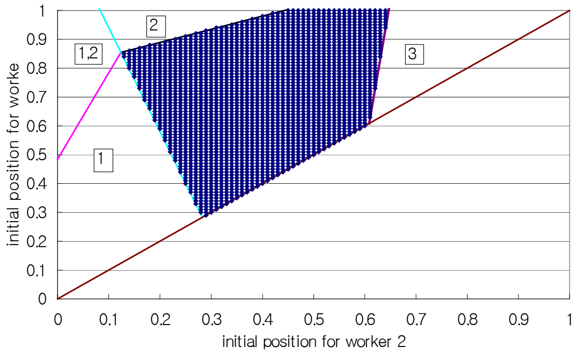
(3) $v_b = 40$

Figure 8. Ratio of the production rate to the maximum rate

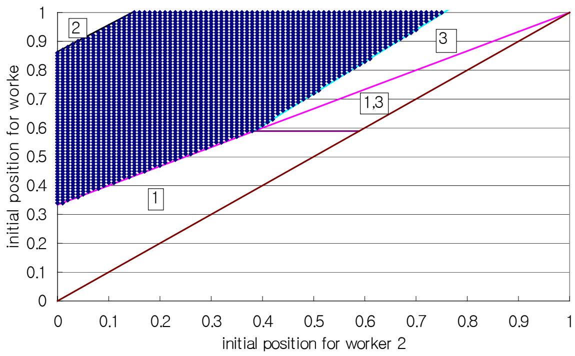
5.3 Initial position for a balanced production line

Satisfying Eq. (11) depends on the velocities and initial positions of workers. To determine the effect of the initial positions, we now consider the first blocking caused by the initial position of workers.

For production lines with three workers, Figure 9 shows the number of iterations during which blocking occurs. In this figure, the hatched areas show the areas where no blocking occurs, and the other areas are where blocking occurs during the indicated iteration. Because three workers are considered, blocking can occur between workers 1 and 2 or between workers 2 and 3. For three workers, three iterations are needed to evaluate whether blocking occurs. Similarly, n iterations are required for n workers using Eq. (8) analytically.



(1) Worker 1 and 2



(2) Worker 2 and 3

Figure 9. Number of iterations during which blocking occurs

Eq. (8) is rewritten for iteration n+1 by summing the workload for each worker:

$$\begin{cases} x_1^{(t)} = 0 \\ x_i^{(t)} = \sum_{j=1}^{i-1} v_j t_p(i,j,t) & (i \le t+1, i = 2,3,\ldots) \\ x_i^{(t)} = x_{i-t}^{(0)} + \sum_{j=i-t}^{i-1} v_j t_p(i,j,t+1) & (i > t+1, i = 2,3,\ldots) \end{cases}$$

where t_p (i, j, t) is the processing time for each iteration:

$$t_p(i,j,t) = \{(\frac{1-x_n^{(t-i+j-1)}}{v_n} - \frac{x_n^{(t-i+j-1)} - x_j^{(t-i+j-1)}}{v_b}) + \frac{1}{v_b+v_j}(1-(x_j^{(t-i+j-1)} + v_j(\frac{1-x_n^{(t-i+j-1)}}{v_n}) - \frac{x_n^{(t-i+j-1)} - x_j^{(t-i+j-1)}}{v_b})))\}$$

Using the above equations and the blocking condition described by Hirotani *et al*. (2003), we calculate the differences from the blocking condition as

$$\begin{aligned} x_{i-1,e}^{(n+1)} - x_{i,e}^{(n+1)} &= \sum_{j=1}^{i-1} v_j t_p(i,j,t) - \sum_{j=1}^{i} v_j t_p(i,j,t) \\ &= \sum_{j=1}^{i-1} (v_j - v_{j+1}) t_p(i,j,t) - v_1 t_p(i,j,t) \end{aligned}$$

If the value of the above formula is negative, the blocking condition is not satisfied. The necessary condition for blocking is $v_{i-1}<v_i$ between worker i and i-1 (Hirotani *et al*. 2003). However, this is not shown in the above formula. Furthermore, t_p (i, j, t) is only changed if blocking occurs on the way. Therefore, the value of the above formula is negative, and blocking never occurs.

5.4 Comparison with other production lines

We next compare the performance of self-balancing production lines with that of U-shaped production lines. The beginning and end of a typical U-shaped production line are close together. One worker can operate both the first and last machines, so a new item enters the line only after one product is completed, and work-in-process (WIP) is held constant. Several policies have been proposed for a U-shaped line (Zavadlav *et al*. 2003). Here, we consider a U-shaped production line with the carousel-type allocation policy as an alternative to the self-balancing line. This is because we want to consider a continuous line where all workers can process items in all areas. Under this policy, all workers take charge of all machines in the same order (Nakade *et al*. 2003). The production rate is considered as a performance measure. We have already analyzed the production rate for the self-balancing line as given by Eq. (10). The production rate for carousel-type allocation is given by

$$\text{production rate} = \min_{i=1,2,\ldots,n} \{nv_i\} \qquad (14)$$

This is because all workers are eventually blocked by the slowest worker. We compared the performance assuming that

(1) The walking-back time is ignored for the U-shaped production line because the last machine is close to the first machine making the walking time negligible.
(2) If a worker is blocked by a downstream worker, that worker's processing velocity becomes equal to that of the slower worker.

Figure 10 shows the area where each production line with three workers is better assuming $v_3 = 5$ in both lines, and $v_b = 10$ in the self-balancing line. The shaded areas show where either type of line might be superior depending on the worker sequence. Apart from these areas, if the velocity of either worker 1 or worker 2 is low, the self-balancing line is superior. Otherwise, the U-shaped line is superior. As the velocity imbalance increases, all workers on the U-shaped line have to work at the speed of the slowest worker. Therefore, there is a trade-off between the two types of line.

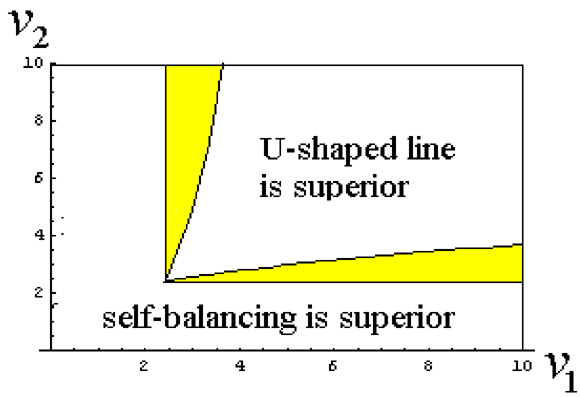


Figure 10. Self-balancing line vs. U-shaped line

6. CONCLUDING REMARKS

In this research, we have focused on a self-balancing line where the walk-back time is taken into account. First, four policies are enumerated, and the cycle time, production rate, and convergence area for each were compared to find the best policy. When the number of workers is n, the conditions for imbalance and convergence to a fixed point were formulated and the influence of the initial positions of workers was analysed. In addition, the line performance was compared with that of a U-shaped production line with a carousel-type allocation rule. As a result, we found that a self-balancing production line was superior in some cases when the walk-back time is considered.

In this research, we assumed the velocity of workers was constant regardless of the work contents. Further research that takes stochastic velocity into account would be useful.

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