Journal of Korean
Data & Information Science Society
2005, Vol. 16, No. 4, pp. 1147~1157

A Study on the Time-Dependent Bonus-Malus System in Automobile Insurance[1)]

Jung-Chul Kang[2)]

Abstract

Bonus-Malus system is generally constructed based on claim frequency and Bayesian credibility model is used to represent claim frequency distribution. However, there is a problem with traditionally used credibility model for the purpose of constructing bonus-malus system. In traditional Bonus-Malus system adopted credibility model, individual estimates of premium rates for insureds are determined based solely on the total number of claim frequency without considering when those claims occurred.

In this paper, a new model which is a modification of structural time series model applicable to counting time series data are suggested. Based on the suggested model relatively higher premium rates are charged to insured with more claim records.

Keywords : Automobile insurance, Bayesian credibility model, Bonus-Malus system, Structural time series model, Time-Dependent model

1. Introduction

Nowadays, we have been enjoying the nation-wide advanced automobile culture with the increasingly growing needs of automobile as a result of rapid economic growth. Further, our deriving habits have become more stable than before. These kinds of social movements lead to the increase of the automobile insurance market size, and automobile insurance is now the largest line of business in the sector of non-life insurance with nearly a half of the total premium incomes. However, it is believed that our automobile insurance has not been much developed from the

---

1) This research was supported by a grant from NURI.

2) Professor, Department of Banking and Insurance, Dong-Eui University, Busan 614-714, Korea.
E-mail : jckang@deu.ac.kr

viewpoint of rate-making. One of the most intriguing research subjects is strongly related to searching for a more advanced rate-making method designed properly in relation to our own automobile market, especially by way of the so-called bonus-malus systems(simply called BMS).

From a practical point of view, it is commonly recognised that the current BMS, using the traditional theory of experience rating employed currently in most of developed countries including Korea, possesses several crucial problems to be solved in a near future. That is, the systems create the possibility of malus evasion in the case that the future loss of bonus is estimated to exceed the claim amount occurred. In real world, most of policyholders would not be likely to report their small claim amounts to the insurance company in order to avoid the potential possibility of premium increase; hence, maintaining the systems lead to "bonus-hunger-type" insurance arrangements - this is contrary to the natural objective of BMS.

The above mentioned problems would be mainly caused from the current system not taking into full account the following two factors. Firstly, in the current system the claim frequency is assumed to be a time-invariant variable but when investigating the actual experience data we could easily find out that the claim frequency is varying with time(i.e. a time-varying variable). The other is that the current system does not allow for the claim amounts, instead just concentrating on the claim frequency.

The primary purpose of this thesis is to search for an alternative method supplementing appropriately the weakness of the traditional BMS(i.e. risk differentiation).

Now, the outline of this thesis is briefly described. In Chapter 2, we review the model used for the construction of Bonus-Malus system. In Chapter 3, a new model which is a modification of structural time series model applicable to counting time series data is suggested. We also perform a simulation comparison between the traditional BMS and the newly proposed BMS. In Chapter 4, we make some valuable conclusions and provide some suggestions for the future research as an extension of the thesis.

2. Classical Model for Construction of Bonus-Malus System

In most countries, bonus-malus system is constructed based solely on claim frequency. The net premium ideally required from the insured can then be identified with its own frequency λ. Consider a policyholder observed for t years and denote by k_j the number of accidents in which he was at fault incurred during year j. So the information concerning the policyholder is a vector $(k_1, \cdots, k_t)$. The observation k_j are the realization of random variable K_j, assumed independendly and identically distributed.

An optimal bonus-malus rule will give the best estimator of an individual's expected number of accident at time $(t+1)$ given the available information for the first t periods $(k_1, \cdots, k_t)$. Let us denote this estimator as $\hat{\lambda}_{t+1}|(k_1, \cdots, k_t)$. With each group of observations $(k_1, \cdots, k_t)$, we must associate a number $\hat{\lambda}_{t+1}|(k_1, \cdots, k_t)$, which is the best estimator of λ at time $t+1$.

By Bayes theorem, the posterior structure function, given the claims history $(k_1, \cdots, k_t)$, is equal to

$$u(\lambda|k_1, \cdots, k_t) = \frac{p(k_1, \cdots, k_t|\lambda)u(\lambda)}{\overline{p}(k_1, \cdots, k_t)}, \tag{2.1}$$

where

$$\overline{p}(k_1, \cdots, k_t) = \int_0^\infty p(k_1, \cdots, k_t|\lambda)u(\lambda)d\lambda \tag{2.2}$$

is the distribution of claims during the t observation years in the portfolio.

The most classical choice for a loss function is a squared loss, $L(\hat{\lambda}, \lambda) = (\hat{\lambda} - \lambda)^2$. In that case, we find the minimum of

$$\int_0^\infty (\lambda_{t+1} - \lambda)^2 u(\lambda|k_1, \cdots, k_t)d\lambda, \tag{2.3}$$

which leads to

$$\hat{\lambda}_{t+1}|(k_1, \cdots, k_t) = \int_0^\infty \lambda u(\lambda|k_1, \cdots, k_t)d\lambda. \tag{2.4}$$

This is the posterior expectation of Λ, where Λ is itself a random variable with distribution representing the expected risks inherent in the given portfolio.

The company must impose on the group of the insureds who underwent the claims history $(k_1, \cdots, k_t)$ a net premium equal to their a posterior claim frequency. What they use for the modeling of claim frequency is Bayesian model as follows. The Poisson distribution is often used for the description of random and independent events such as automobile accidents. Indeed, under well known assumptions, the distribution of the number of accidents during a given period can be written as

$$\Pr(K_t^{(i)} = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \tag{2.5}$$

where k is the realization of the random variable $K_t^{(i)}$ for policyholder i in a given period t and λ is the Poisson parameter. Equation (2.5) assumes that all the policyholders have the same claim frequency. A more general model allows parameter λ to vary among individuals. If we assume that this parameter is a random variable and follows a gamma distribution with parameter α and β, the distribution of the number of accidents during a given period becomes

$$\Pr(K_t^{(i)}=k)=\frac{\Gamma(k+\alpha)}{k!\,\Gamma(\alpha)}\times\frac{\beta^{\alpha}}{(1+\beta)^{k+\alpha}}, \tag{2.6}$$

which corresponds to a negative binomial distribution.

Applying the negative binomial distribution, the posterior distribution of λ is a gamma distribution with parameters $(\alpha+k)$ and $(\beta+t)$. Therefore, the Bayes estimator of an individual's expected number of accidents at time $(t+1)$ is the mean of the a posterior gamma distribution with $(\alpha+k)$ and $(\beta+t)$. Based on this Poisson distribution model, gamma prior with parameters α and β, and squared loss function, optimal estimator of $\lambda_{t+1}|(k_1,\cdots,k_t)$ is given as follows and it can be expressed into the form which is called credibility formula.

$$\begin{aligned}\hat{\lambda}_{t+1}|(k_1,\cdots,k_t) &= \frac{\alpha+\sum_{t=1}^{T}K_t}{\beta+T} \\ &= Z\cdot\frac{\sum_{t=1}^{T}K_t}{T}+(1-Z)\cdot\frac{\alpha}{\beta},\end{aligned} \tag{2.7}$$

where $Z=\dfrac{T}{\beta+T}$.

The bonus-malus system defined by the above formula has several important properties.

(i) The insured has to pay a premium proportional to the estimate of his claim frequency.

(ii) At each stage of this sequential process, the mean of the individual claim frequencies is equal to the overall mean α/β.

(iii) The premium depends only on k, the total number of reported accidents. It does not depend on the way these accidents are distributed over the years.

(iv) The bonus-malus system suggested here is a particular case of the well-known credibility formula, which postulates that the net premium modified by experience should be put in the form of a linear combination of the a priori premium α/β and the observations $(\sum_{t=1}^{T}K_t)/T$. The posterior mean is a weighted average of the a priori premium and the observations.

The simplest premium calculation principle for an insurance company consists of requiring the policyholder to pay a net premium plus a security loading proportional to that net premium. It is the expected value principle. This principle means that the policyholder who underwent claims history $(k_1, \cdots, k_t)$ will have to pay a premium.

$$\begin{aligned} P_{t+1}(k_1, \cdots, k_t) &= (1+\delta)\, \hat{\lambda}_{t+1}(k_1, \cdots, k_t) \\ &= (1+\delta) \cdot \frac{\alpha + k}{\beta + t} \end{aligned} \tag{2.8}$$

where $k = \sum_{t=1}^{T} K_t$ and δ is a safety loading.

3. GENERALIZATION OF BONUS-MALUS SYSTEM

3.1 An Application of Time-Dependent Model

In most countries, BMS is constructed based solely on claim frequency. Here, we are going to build up a BMS based only on the number of accidents reported to the company, not on their amounts.

Even though the Bayesian model, explained previously in Chapter 2, could be a useful tool for constructing BMS in automobile insurance, this method still contains some problems in the application of this method. For example, it would be worth to compare two insureds, the insured A and the insured B, with following claim records. For 5 years, the insured A has one claim for each of the first two years and thereafter no claim was reported. On the contrary, the insured B has one claim throughout recent two years. How are insurance companies going to assign premiums for these two insureds? It would be reasonable to charge more severely for recent claims, but the Bayesian model is designed to consider only the frequency of claims regardless of the claim-occurrence-time. Hence, we need to build up a new BMS tougher than the traditional BMS, as described further below.

Most claim records show that the frequency of claims is changing over time(i.e. time-varying parameter) as well as autocorrelated with each other. Bailey and Simon(1959) indicates that there exists a time-varying parameter in automobile losses. In the traditional BMS, it is normally assumed that the model components are time-invariant and past data collected are treated equally without taking into account the age of the data. In equation (2.8) we can see that optimal estimator of λ_{T+1} is determined based solely on the total number of claim frequency without considering when those claims occurred. So, there is a necessity to explore a new model allowing for the claim-occurrence-time(here, we shall call it the time-dependent BMS). What kind of model can then be used for the construction of a tougher bonus-malus system? We suggest here to employ a time-series model.

One of the important models for economic time series is the basic structural model. Structural time series models are discussed by Harvey and Todd(1983). A characteristic of these models is that they express the time series observations Y_t in terms of trend and seasonal components. These components have a direct interpretation in terms of quantities of interest, in contrast to the implicitly defined components in ARIMA models(Box and Jenkins, 1976). The basic structural time series model is given as

$$Y_t = m_t + g_t + \varepsilon_t, \quad t = 1, \cdots, T, \tag{3.1}$$

where m_t, g_t, and ε_t are trend, seasonal, and irregular components, respectively. The irregular components ε_t are serially uncorrelated normally distributed random variable with zero mean and variance σ^2_ε. The process of generating the trend is of the form

$$m_t = m_{t-1} + b_{t-1} + \eta_t, \quad t = 1, \cdots, T \tag{3.2}$$

and

$$b_t = b_{t-1} + \xi_t, \quad t = 1, \cdots, T, \tag{3.3}$$

where η_t and ξ_t are each a normally distributed independent white noise process with zero mean and variance σ^2_η and σ^2_ξ, respectively. It is further assumed that error terms η_t, ξ_t and ε_t are independent of each other. If $g_t = 0$ in (3.1), the model provides a local approximation to a linear trend model. The level m_t and the slope b_t each change slowly over time according to random walk process(Harvey and Todd, 1983). The process of generating the seasonal component is

$$g_t = -\sum_{i=1}^{s-1} g_{t-i} + \omega_t, \quad t = 1, \cdots, T, \tag{3.4}$$

where ω_t is a sequence of serially uncorrelated normal random variable with zero mean and variance σ^2_ω, and s is the number of period in a seasonal cycle.

Under the assumption of this basic structural time series model, more weights are given to more recent observation in the estimation of level, trend, and seasonal components. The model in (3.1) without slope and seasonal components is called the local level model. Suppose that we wish to estimate the local level of a series of observations. Under the assumption of local level model which contains only time-varying level component m_t, optimal estimator of m_{t+1} conditional on the information up to time T is equivalent to the weighted moving average of past observations.

However, since the claim frequency variable is a counting variable, we cannot use this structural time series model directly for setting up a BMS. Instead of the above model, we can use the following model in order to obtain the same kind of estimates which is a weighted average with more weights assigned to more recent observations.

$$\begin{aligned} K_t|\mu_t &\sim Poisson(\mu_t) \\ \mu_{t-1}|K_1, \cdots, K_{t-1} &\sim Gamma(\alpha_{t-1}, \beta_{t-1}) \\ \mu_t|K_1, \cdots, K_{t-1} &\sim Gamma(\alpha_{t|t-1}, \beta_{t|t-1}), \end{aligned} \tag{3.5}$$

where $\alpha_{t|t-1} = \omega \cdot \alpha_{t-1}$, $\beta_{t|t-1} = \omega \cdot \beta_{t-1}$, $0 < \omega \le 1$. Once the observation k_t becomes available, the posterior distribution, $p(\mu_t|k_t)$, is Gamma distributed with parameters $(\alpha_t = \alpha_{t|t-1} + k_t, \beta_t = \beta_{t|t-1} + 1)$.

Assuming this model for K_t and μ_t, the resulting predictive distribution of K_t given K_1 through K_{t-1} is a negative binomial distribution with parameters $(\alpha_{t|t-1}, \beta_{t|t-1}/(1+\beta_{t|t-1}))$. And the optimal forecast of K_{t+1} at time t is given as follows which is a weighted average of past claim frequencies and it charges more severely to recent claims.

$$\begin{aligned} \widehat{K}_{t+1|t} &= \frac{\alpha_t}{\beta_t} \\ &= \frac{\sum_{j=0}^{t-1} w^j \cdot K_{t-j}}{\sum_{j=0}^{t-1} w^j}. \end{aligned} \tag{3.6}$$

In the above weighted average weights decline exponentially(Harvey and Fernandes, 1989).

In our case, the pure premium to be charged is related to the frequency of accident. Hence, we can now build a table of premiums to be charged as a function of accident k and number of years t. Then we normalize the posterior premium in a way that the premium for new insured is 100($k=0$ and $t=0$). We then obtain

$$P'_{t+1}(k_1, \cdots, k_t) = \frac{\beta_0}{\alpha_0} \cdot \frac{(\alpha_{t|t-1} + k_t)}{(\beta_{t|t-1} + 1)} \times 100. \tag{3.7}$$

3.2 Simulation Study

An apparent consequence of the implementation of a BMS is a progressive decrease of the observed average premium level, due to a concentration of policyholders in the high discount classes. Since it is practically impossible to enforce severe penalty, most policyholders trend to cluster in the lowest BMS classes. Consequently many insurers suffered great losses, it is most difficult to obtain an increase in the basic premium.

A crucial characteristic of a BMS is thus the distribution of policyholders among classes, once the system has reached the stationary state. Among other things this allows forecasting the average premium level. The simulation was used to compute the stationary distribution of the insureds among the classes, as well as the stationary average level. Given a variety of systems, stationary levels are difficult to compare. Therefore, a "Relative Stationary Average Level"(RSAL) is used, which is defined as(Lemaire, 1988)

$$\text{RSAL} = \frac{\text{ststionary average level} - \min imum\ \text{level}}{\max imum\ \text{level} - \min imum\ \text{level}}. \tag{3.8}$$

It measures the position of the average driver, once the BMS has reached a steady-state condition. It evaluates the degree of clustering of policies in the lowest classes of the BMS. Expressed as a percentage, this is an index that determines the relative position of the average policyholder, when the lowest premium is set equal to zero and the highest to 100. A low value of RSAL indicates a high clustering of policies in the lowest BMS classes. A high RSAL suggests a better spread of policies among classes.

Insurances consist in a transfer of risk from the policyholder to the carrier. Without experience rating, the transfer is total(i.e., the variability of the insureds' payments is zero). With experience rating, personalized premiums from the policyholder will vary from year to year according to claims history; cooperation between drivers is weakened. Solidarity between policyholders can be evaluated by a measure of the variability of annual premiums. The coefficient of variation(standard deviation divided by mean) is selected, as it is a dimension-less parameter.

A simulation study was performed under the assumptions of a closed portfolio, i.e. no policies entered or left portfolio. Claims were generated for the ith policy by a Poisson distribution with mean Λ. The risk parameter Λ were simulated using a fitted Gamma distribution. Using the value of the parameters, the program was run for a portfolio of 50,000 new policyholders during a period of 30 years.

Figure 3.3 presents the evolution of mean premium level for each model. In the case of $\omega = 1.0$, the premium income after 30 years is somewhat equal to the starting one. That is, when the weighting parameter ω is 1.0, we can conclude that the time-dependent model is the same as the Bayesian model. For $\omega < 1.0$, it is worth nothing that the company income has a steep decrease during the first five years, the time taken by the best drivers to reach the minimal class. After that, the income continues to decrease more slowly, but irregularly. As a conclusion, the mean premium level shows atendency to decrease more steeply as the weighting parameter ω decrease, in particular during the first 10 years.

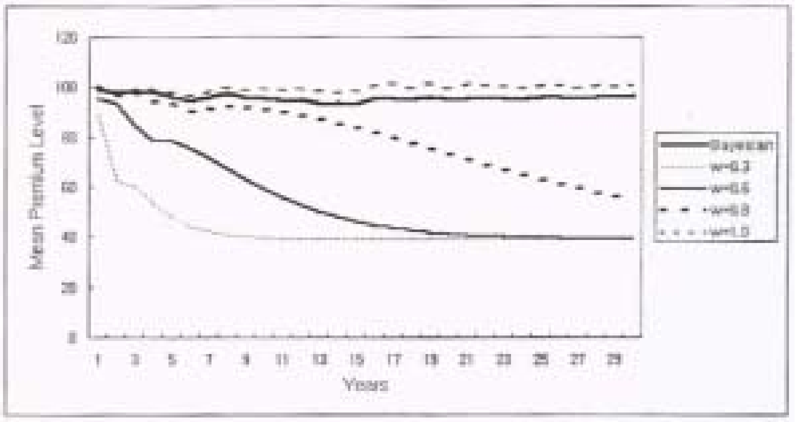

Figure 3.3 The Evolution of Mean Premium Level

Figure 3.4 shows the evolution of the coefficient of variation with time. Typically, the coefficient of variation starts at zero for the first policy year, increase until the best policyholders reach the maximum discount, then decreases until stationarity is reached.

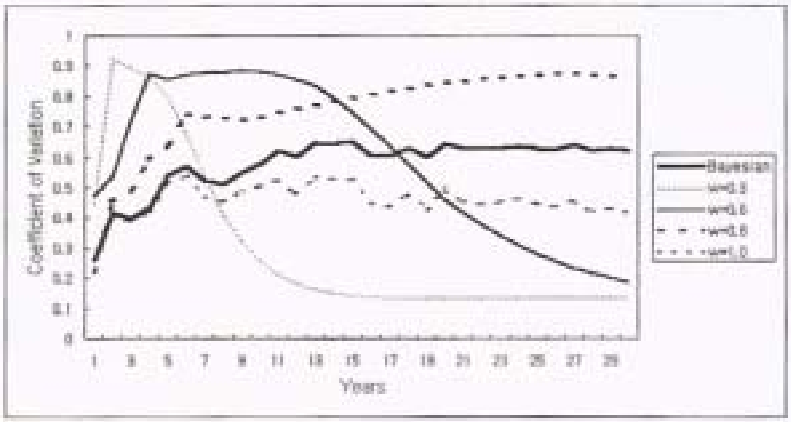

Figure 3.4 The Evolution of Coefficient of Variation

4. CONCLUSIONS AND FUTURE WORK

Bonus-Malus system in automobile insurance rewards claim-free policyholders by premium discounts and penalizes policyholders with claims by premium surcharges. The purpose of adopting bonus-malus system is to alleviate differences in risk propensity. Bonus-Malus system is generally constructed based on claim frequency and Bayesian credibility model is used to represent claim frequency distribution.

However, there is a problem with traditionally used credibility model for the purpose of constructing bonus-malus system. In traditional Bonus-Malus system adopted credibility model, individual estimates of premium rates for the insureds are determined based solely on the total number of claim frequency without considering when those claims occurred. In this paper, a new model which is a modification of structural time series model applicable to counting time series data was suggested. Based on the suggested model relatively higher premium rates are charged to the insureds with more recent claim records.

Many efforts have recently been focused on the study of discrete time series. In developing such models the integer-valued first-order autoregressive(INAR(1)) process, introduced independently by McKenzie(1986) and Al-Osh and Alzaid(1987), has received considerable attention. We plan to study integer-valued first-order autoregressive process for the construction of bonus-malus system. Finally, an effort will be made to rationalize the forms of our models in which the coefficients vary stochastically.

References

1. Al-Osh, M. A. and Alzaid, A. A. (1987). First-Order Integer-Valued Autoregressive(INAR(1)) Process, *Journal of Time Series Analysis,* 8, 261-275.
2. Bailey, R. A. and Simon, L. J. (1959). An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car, *Proceeding of the Casualty Actuarial Society,* 46, 159-164.
3. Box, G. and Jenkins, G. (1976). *Time Series Analysis, Forecasting and Control*, San Francisco: Holden-Day.
4. Harvey, A. and Fernandes, C. (1989). Time Series Models for Count or Qualitative Observations*, Journal of Business and Economic Statistics*, 7, 407-417.
5. Harvey, A. and Todd, P. (1983). Forecasting Economic Time Series with Structural and Box-Jenkins Models: A Case Study, *Journal of Business*

and Economic Statistics, 1, 299-315.
6. Lemaire, J. (1988). A Comparative Analysis of Most European and Japanese Bonus-Malus Systems, *Journal of Risk and Insurance*, 55, 660-681.
7. Mckenzie, E. (1986). Autoregressive Moving-Average Processes with Negative Binomial and Geometric Marginal Distribution, *Advances in Applied Probability,* 18, 679-705.

[received date : Sep 2005, accepted date : Nov. 2005]