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Bayesian Hypothesis Testing for Two Lognormal Variances with the Bayes Factors

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Abstract

The Bayes factors with improper noninformative priors are defined only up to arbitrary constants. So it is known that Bayes factors are not well defined due to this arbitrariness in Bayesian hypothesis testing and model selections. The intrinsic Bayes factor and the fractional Bayes factor have been used to overcome this problem. In this paper, we suggest a Bayesian hypothesis testing based on the intrinsic Bayes factor and the fractional Bayes factor for the comparison of two lognormal variances. Using the proposed two Bayes factors, we demonstrate our results with some examples.

Keywords : Fractional Bayes Factor, Intrinsic Bayes Factor, Lognormal Variance, Reference Prior

1. Introduction

It has been well known that Bayes factors with proper priors have been very successful in testing or model selection problem. However, in Bayesian analysis, limited information and time restrictions often force to the use of noninformative priors such as Jeffreys' priors or reference priors. These noninformative priors are usually improper density and the Bayes factors under improper priors are not well defined because these priors are defined only up arbitrary constants.

Suppose that hypotheses (or models) $H_1,\ H_2, \cdots,\ H_q$ are under consideration, with the data $\boldsymbol{x}=(x_1, x_2, \cdots, x_n)$ having probability density function $f_i(\boldsymbol{x} \mid \boldsymbol{\theta}_i)$ under model $H_i, i=1,2,\cdots,q$. The parameter vectors $\boldsymbol{\theta}_i$ are unknown. Let

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$\pi_i(\theta_i)$ be the prior distribution of model H_i, and let p_i be the prior probabilities of model H_i, $i=1,2,\cdots,q$. Then the posterior probability that the model H_i is true is

$$P(H_i \mid x) = \left(\sum_{j=1}^{q} \frac{p_j}{p_i} \cdot B_{ji} \right)^{-1}, \tag{1.1}$$

where B_{ji} is the Bayes factor of model H_j to model H_i defined by

$$B_{ji} = \frac{m_j(x)}{m_i(x)} = \frac{\int f_j(x \mid \theta_j)\pi_j(\theta_j)d\theta_j}{\int f_i(x \mid \theta_i)\pi_i(\theta_i)d\theta_i}. \tag{1.2}$$

The B_{ji} is interpreted as the comparative support of the data for the model j to i. The computation of B_{ji} needs specification of the prior distribution $\pi_i(\theta_i)$ and $\pi_j(\theta_j)$. Usually, one can use noninformative priors, often improper, such as uniform prior, Jeffreys prior and reference prior. Denote the noninformative prior as π_i^N. The use of improper priors $\pi_i^N(\cdot)$ in (1.2) causes the B_{ji} to contain unspecified constants.

Spiegalhalter and Smith (1982), O'Hagan (1995) and Berger and Pericchi (1996) have made efforts to compensate for that arbitrariness. Berger and Pericchi (1996) introduced the intrinsic Bayes factor (IBF) using a data-splitting idea, which would eliminate the arbitrariness of improper priors. O'Hagan (1995) proposed the fractional Bayes factor (FBF). To remove the arbitrariness in Bayes factor, he used a portion of the likelihood with a so-called the fraction b. These two approaches mentioned above have shown to be quite useful in many statistical areas.

Kim (2000) analyzed comparisons of two exponential means. Kim and Ibrahim (2000) derived an explicit form of the IBF for generalized linear models. Kim, Kang and Kim (2000) calculated the IBF for exponential model comparison with censored data. Kim and Kim (2000) proposed a Bayesian testing for the comparison of two exponential means using FBF with intrinsic prior. Bae, Kim and Kim (2000) derived the FBF with intrinsic prior for the equality of two independent normal means with unknown variance.

The log-normal distribution is widely used to analysis positively skewed data in biomedical research. For the statistical inference of the lognormal distribution, Zhou and Gao (1997) studied the confidence interval of the mean. Zhou, Gao and Hui (1997) suggested the methods for comparing two means. Moon, Shin and Kim (2000) proposed the Bayesian tesing procedures for the equality of two lognormal means based on the intrinsic Bayes factor, and Moon and Kim (2001a, 2001b) proposed the Bayesian testing procedures factor based on the intrinsic Bayes factor for comparing several lognormal means. Moon (2003) derived the intrinsic priors with fractional Bayes factor, and calculated the Bayes factor based on this intrinsic priors for comparing two lognormal means.

In this paper, we consider the testing problem for comparing two lognormal variances. The outline of the remaining sections is as follows. In Section 2, using the reference priors, we provide the Bayesian testing procedure based on the fractional Bayes factor and intrinsic Bayes factor for the testing equality of two lognormal variances. In Section 3, some examples and conclusions of our Bayesian test procedure are given.

2. Bayesian Test Procedures

2.1 Preliminaries

It has known that the use of improper priors $\pi_i^N(\cdot)$ in (1.2) causes the B_{ji} to contain unspecified constants. To solve this problem, O'Hagan (1995) proposed the fractional Bayes factor for Bayesian testing and model selection problem as follow.

When the $\pi_i^N(\theta_i)$ is noninformative prior under H_i, equation (1.2) becomes

$$B_{ji}^N = \frac{\int f_j(x \mid \theta_j)\pi_j^N(\theta_j)d\theta_j}{\int f_i(x \mid \theta_i)\pi_i^N(\theta_i)d\theta_i}.$$

Then the fractional Bayes factor (FBF) of model H_j versus model H_i is

$$B_{ji}^F = B_{ji}^N \cdot \frac{\int f_i^b(x \mid \theta_i)\pi_i^N(\theta_i)d\theta_i}{\int f_j^b(x \mid \theta_j)\pi_j^N(\theta_j)d\theta_j} = B_{ji}^N \cdot \frac{m_i^b(x)}{m_j^b(x)},$$

and $f_i(x \mid \theta_i)$ is the likelihood function and b specifies a fraction of the likelihood, $0 \le b \le 1$, which is to be used as a prior density. He proposed three ways for the choice of the fraction b. A commonly suggested choice is $b = m/n$, where m is the size of the minimal training sample, assuming this is well defined. (see O'Hagan, 1995, 1997).

Berger and Pericchi (1996) proposed the intrinsic Bayes factor (IBF) for Bayesian testing and model selection. The arithmetic intrinsic Bayes factor (AIBF) is given by

$$B_{ji}^{AI} = B_{ji}^{N} \cdot \frac{1}{L} \sum_{l=1}^{L} B_{ij}^{N}(x(l)),$$

where

$$B_{ij}^{N}(x(l)) = \frac{m_i(x(l))}{m_j(x(l))} = \frac{\int f_i(x(l) \mid \theta_i)\pi_i^N(\theta_i)d\theta_i}{\int f_j(x(l) \mid \theta_j)\pi_j^N(\theta_j)d\theta_j}.$$

Here $x(l)$ is minimal training sample and L is the number of all possible minimal training samples.

2.2 Bayesian Hypothesis Testing

Let X be a two parameter lognormal distribution with density function

$$f(x \mid \eta) = \frac{1}{\sqrt{2\pi\eta x}} \exp\{-\frac{1}{2\eta}(\log x - \mu)^2\}, \quad 0 < x < \infty, \tag{2.1}$$

where $-\infty < \mu < \infty$, $\eta > 0$, $\mu = E(\log X)$ and $\eta = Var(\log X)$. The lognormal distribution will be denoted as the $LN(\mu, \eta)$.

Suppose that we have independent random samples $X_{ij} \sim LN(\mu, \eta_i)$, $i = 1, 2$, $j = 1, \cdots, n_i$ where μ is common and unknown. We interested in testing the equality of two lognormal variances, that is,

$$H_1: [\exp(\eta_1) - 1]\exp(2\mu + \eta_1) = [\exp(\eta_2) - 1]\exp(2\mu + \eta_2),$$
$$H_2: [\exp(\eta_1) - 1]\exp(2\mu + \eta_1) \neq [\exp(\eta_2) - 1]\exp(2\mu + \eta_2).$$

But, because the location parameter μ of two populations are common, it equals to test the equality of two parameters η_1 and η_2, that is,

$$H_1: \eta_1 = \eta_2 \quad \text{vs.} \quad H_2: \eta_1 \neq \eta_2. \tag{2.2}$$

Our interest is to develop a Bayesian test based on FBF and IBF for $H_1: \eta_1 = \eta_2$ versus $H_2: \eta_1 \neq \eta_2$ under the noninformative priors.

2.2.1 Bayesian Hypothesis Testing using the Fractional Bayes Factor

Under the hypothesis H_1 in (2,2), the reference prior for $\eta (\equiv \eta_1 = \eta_2)$ and μ is given by

$$\pi_1(\mu, \eta) = \eta^{-1}.$$

Then the likelihood function under H_1 is

$$\begin{aligned} L(\mu, \eta \mid x) &= \left(\frac{1}{\sqrt{2\pi}}\right)^n \left[\prod_{i=1}^{2}\prod_{j=1}^{n_i} x_{ij}^{-1}\right] \exp\{-\frac{1}{2\eta}\left[\sum_{j=1}^{n_1}(\log x_{1j} - \bar{x}_1)^2\right. \\ &\quad \left.+ \sum_{j=1}^{n_2}(\log x_{2j} - \bar{x}_2)^2 + n_1(\bar{x}_1 - \mu)^2 + n_2(\bar{x}_2 - \mu)^2\right]\}, \end{aligned}$$

where $n = n_1 + n_2$, $\bar{x}_1 = \sum_{j=1}^{n_1} \log x_{1j}/n_1$ and $\bar{x}_2 = \sum_{j=1}^{n_2} \log x_{2j}/n_2$. Then the element of fractional Bayes factor under H_1 is given by

$$\begin{aligned} m_1^b(x) &= \int_0^\infty \int_{-\infty}^\infty L^b(\mu, \eta \mid x)\pi_1(\mu, \eta) d\mu d\eta \\ &= \left(\frac{1}{\sqrt{2\pi}}\right)^{(nb-1)} (bn)^{-\frac{1}{2}} \left(\frac{b}{2}\right)^{-\frac{nb-1}{2}} \Gamma\left[\frac{nb-1}{2}\right]\left[\prod_{i=1}^{2}\prod_{j=1}^{n_i} x_{ij}^{-b}\right] \\ &= \times \left[\sum_{j=1}^{n_1}(\log x_{1j} - \bar{x}_1)^2 + \sum_{j=1}^{n_2}(\log x_{2j} - \bar{x}_2)^2 + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{n}\right]^{-\frac{nb-1}{2}}. \end{aligned}$$

For the H_2 in (2.2), the reference prior for μ, η_1 and η_2 is

$$\pi_2(\mu, \eta_1, \eta_2) = \eta_1^{-1}\eta_2^{-1}.$$

Then the likelihood function under H_2 is

$$\begin{aligned} L(\mu, \eta_1, \eta_2 \mid x) &= \left(\frac{1}{\sqrt{2\pi}}\right)^n \left[\prod_{i=1}^{2}\prod_{j=1}^{n_i} x_{ij}^{-1}\right] \eta_1^{-\frac{n_1 b}{2}} \eta_2^{-\frac{n_2 b}{2}} \\ &\quad \times \exp\{-\frac{1}{2\eta_1}\left[\sum_{j=1}^{n_1}(\log x_{1j} - \bar{x}_1)^2 + n_1(\bar{x}_1 - \mu)^2\right]\} \\ &\quad \times \exp\{-\frac{1}{2\eta_2}\left[\sum_{j=1}^{n_2}(\log x_{2j} - \bar{x}_2)^2 + n_2(\bar{x}_2 - \mu)^2\right]\}. \end{aligned}$$

Thus the element of FBF under H_2 gives as follows.

$$\begin{aligned} m_2^b(x) &= \int_{-\infty}^\infty \int_0^\infty \int_0^\infty L^b(\mu, \eta_1, \eta_2 \mid x)\pi_2(\mu, \eta_1, \eta_2) d\eta_1 d\eta_2 d\mu \\ &= \left(\frac{1}{\sqrt{2\pi}}\right)^{nb} \left(\frac{b}{2}\right)^{-\frac{nb}{2}} \left[\prod_{i=1}^{2}\prod_{j=1}^{n_i} x_{ij}^{-b}\right] \Gamma\left[\frac{n_1 b}{2}\right] \Gamma\left[\frac{n_2 b}{2}\right] T_2(x; b), \end{aligned}$$

where

$$T_2(x;b) = \int_{-\infty}^{\infty}[\sum_{j=1}^{n_1}(\log x_{1j}-\bar{x}_1)^2+n_1(\bar{x}_1-\mu)^2]^{-\frac{n_1 b}{2}}$$
$$\times[\sum_{j=1}^{n_2}(\log x_{2j}-\bar{x}_2)^2+n_2(\bar{x}_2-\mu)^2]^{-\frac{n_2 b}{2}}d\mu.$$

Therefore the B_{21}^N is given by

$$B_{21}^N = \frac{\sqrt{n}\Gamma[\frac{n_1}{2}]\Gamma[\frac{n_2}{2}]T_2(x;1)}{\sqrt{\pi}\Gamma[\frac{n-1}{2}][T_1(x)]^{-\frac{n-1}{2}}}, \tag{2.3}$$

where

$$T_1(x)=[\sum_{j=1}^{n_1}(\log x_{1j}-\bar{x}_1)^2+\sum_{j=1}^{n_2}(\log x_{2j}-\bar{x}_2)^2+\frac{n_1 n_2}{n}(\bar{x}_1-\bar{x}_2)^2],$$

$$\frac{m_1^b(x)}{m_2^b(x)} = \frac{\sqrt{\pi}\Gamma[\frac{nb-1}{2}][T_1(x)]^{-\frac{nb-1}{2}}}{\sqrt{n}\Gamma[\frac{n_1 b}{2}]\Gamma[\frac{n_2 b}{2}]T_2(x;b)}.$$

Thus FBF of H_2 versus H_1 is given by

$$B_{21}^F = \frac{\Gamma[\frac{n_1}{2}]\Gamma[\frac{n_2}{2}]\Gamma[\frac{nb-1}{2}]}{\Gamma[\frac{n_1 b}{2}]\Gamma[\frac{n_2 b}{2}]\Gamma[\frac{n-1}{2}]}\frac{T_2(x;1)[T_1(x)]^{-\frac{nb-1}{2}}}{T_2(x;b)[T_1(x)]^{-\frac{n-1}{2}}}. \tag{2.4}$$

Note that the calculation of FBF of H_2 versus H_1 is requires only an one dimensional integration.

2.2.2 Bayesian Hypothesis Testing using the Intrinsic Bayes Factor

The element B_{21}^N in (2.3) of the intrinsic Bayes factor is computed in the fractional Bayes factor. So using minimal training sample, we only calculate the marginal densities $m^N(x(l))$ under H_1 and H_2, respectively. The minimal training samples in our situation are size of 3. So we have $x_1(l)=(x_{11}, x_{12,} x_{21})$ and $x_2(l)=(x_{11}, x_{21}, x_{22})$.

The marginal densities $m_1^N(\mathbf{x}_1(l))$ based on $\mathbf{x}_1(l)=(x_{11},x_{12},x_{21})$ under H_1 is given by

$$m_1^N(\mathbf{x}_1(l)) = \int_0^\infty\int_0^\infty f(x_{11},x_{12},x_{21}\mid\mu,\eta)\pi_1(\mu,\eta)d\mu d\eta$$
$$= \sqrt{\frac{1}{3}}\frac{1}{\pi}\frac{1}{x_{11}x_{12}x_{21}}\left[(x_{11}^2+x_{12}^2+x_{21}^2)-\frac{1}{3}(x_{11}+x_{12}+x_{21})^2\right]^{-1}.$$

Also for the minimal training sample $\mathbf{x}_2(l)$, the marginal densities $m_1^N(\mathbf{x}_2(l))$ under H_1 is given by

$$m_1^N(\mathbf{x}_2(l)) = \sqrt{\frac{1}{3}}\frac{1}{\pi}\frac{1}{x_{11}x_{21}x_{22}}\left[(x_{11}^2+x_{21}^2+x_{22}^2)-\frac{1}{3}(x_{11}+x_{21}+x_{22})^2\right]^{-1}.$$

And the marginal density $m_2^N(\mathbf{x}_1(l))$ under H_2 is given by

$$m_2^N(\mathbf{x}_1(l)) = \int_{-\infty}^\infty\int_0^\infty\int_0^\infty f(x_{11},x_{12},x_{21}\mid\mu,\eta_1,\eta_2)\pi_2(\mu,\eta_1,\eta_2)d\eta_1 d\eta_2 d\mu$$
$$= \frac{1}{\pi}\frac{1}{x_{11}x_{12}x_{21}}S(x_{11},x_{12},x_{21}),$$

where

$$S(x_{11},x_{12},x_{21})=\int_{-\infty}^\infty[(\log x_{11}-\mu)^2+(x_{12}-\mu)^2]^{-1}[(x_{21}-\mu)^2]^{-\frac{1}{2}}d\mu.$$

Similarly, for the minimal training sample $\mathbf{x}_2(l)$, the marginal density $m_2^N(\mathbf{x}_2(l))$ under H_2 is given by

$$m_2^N(\mathbf{x}_2(l)) = \frac{1}{\pi}\frac{1}{x_{11}x_{21}x_{22}}S(x_{11},x_{21},x_{22}),$$

where

$$S(x_{11},x_{21},x_{22})=\int_{-\infty}^\infty[(\log x_{11}-\mu)^2+(x_{21}-\mu)^2]^{-1}[(x_{22}-\mu)^2]^{-\frac{1}{2}}d\mu.$$

Thus the $B_{12}^N(\mathbf{x}_1(l))$ and $B_{12}^N(\mathbf{x}_2(l))$ are given by

$$B_{12}^N(\mathbf{x}_1(l))=\frac{\sqrt{3}[3(x_{11}^2+x_{12}^2+x_{21}^2)-(x_{11}+x_{12}+x_{21})^2]^{-1}}{S(x_{11},x_{12},x_{21})}$$

and

$$B_{12}^{N}(\ \boldsymbol{x}_2(l)) = \frac{\sqrt{3}[3(x_{11}^2 + x_{21}^2 + x_{22}^2) - (x_{11} + x_{21} + x_{22})^2]^{-1}}{S(x_{11}, x_{21}, x_{22})},$$

respectively. Therefore the AIBF of H_2 versus H_1 is given by

$$B_{21}^{AI} = B_{21}^{N} \cdot \frac{1}{2}\left[\frac{1}{L_1}\sum_{l=1}^{L_1} B_{12}^{N}(\ \boldsymbol{x}_1(l)) + \frac{1}{L_2}\sum_{l=1}^{L_2} B_{12}^{N}(\ \boldsymbol{x}_2(l))\right]. \quad (2.5)$$

Note that the calculation of the AIBF of H_2 versus H_1 is requires an one dimensional integration.

3. Numerical Studies

In this section, we will give some examples to show the usefulness of our test procedures by real data set and artificial data.

【Example 1】 The data given here arose in test on the endurance of deep groove ball bearing (Lawless(1982)). These data were assumed to come from Weibull distribution. But a probability plot of the data showed them to also be consonant with a lognormal model. The data are the number of million revolutions before failure for each the 23 ball bearings in life test. Suppose that the data are divided into the following two groups to test the hypothesis model $H_1: \eta_1 = \eta_2$ and $H_1: \eta_1 \neq \eta_2$.

Group 1	17.88	33.00	45.60	51.84	51.96	55.56
	67.80	68.64	93.12	105.84	128.04	
Group 2	28.92	41.52	42.12	48.40	54.12	68.88
	84.12	98.64	105.12	127.92	173.40	

By the logarithmic transformations of the given data, the data follow the normal distribution and the F -test is used for comparing the equality of two variances. From the data, the following values can obtained: $(\ \overline{x}_1, s_1^2) = (4.07, 0.53)$ and $(\ \overline{x}_2, s_2^2) = (4.24, 0.55)$. Then the F-statistic and the corresponding p-value are obtained as 1.0773 and 0.4492 , and we can see that the variances of two groups are equal as we expected. The values of FBF and AIBF of H_2 vs. H_1 are $B_{21}^{F} = 0.3212$ and $B_{21}^{AI} = 0.1170$, respectively. We assume that the prior probabilities, p_1 and p_2 are equal. Then the posterior probabilities for H_1

obtained by $B^{F}_{21}=0.3212$ and $B^{AI}_{21}=0.1170$ in (1.1) are 0.7569 and 0.8952, respectively. Thus we may conclude that the difference between two groups in terms of the posterior probabilities is fairly small.

【Example 2】 This example shows a simulation of 10 and 15 values from the $LN(0,1)$ and $LN(0,3^2)$, respectively. The data sets are given as follows:

Group 1	6.138	0.744	1.070	0.220	0.750	0.452	0.639	1.152
	3.511	1.419						
Group 2	0.938	28.515	15.867	9.228	0.042	0.589	243.639	0.107
	0.089	1.258	0.002	8.607	0.042	1.275	1.348	

The given data follow the normal distribution by the logarithmic transformations, and the sample means and the sample variances are obtained as follows : $(\bar{x}_1, s_1^2)=(0.03, 0.92)$ and $(\bar{x}_2, s_2^2)=(-0.07, 9.15)$. By using these values, the F-statistic and the corresponding p-value can be obtained by 9.9592 and 0.0078 . From this result, we can see that the variances of two groups are not equal as we expected. The values of FBF and AIBF of H_2 vs. H_1 are given by $B^{F}_{21}=53.6465$ and $B^{AI}_{21}=19.9909$, respectively, assuming that the prior probabilities, p_1 and p_2 are equal. Then the posterior probabilities for H_1 by $B^{F}_{21}=53.6465$ and $B^{AI}_{21}=19.9909$ in (1.1) are 0.0183 and 0.0476, respectively. Thus there are strong evidence for H_2 in terms of the posterior probabilities.

We developed a Bayesian test procedures for testing the equality of two lognormal variances. Under the reference priors, the intrinsic Bayes factor of Berger and Pericchi (1996) and the fractional Bayes factor of O'Hagan (1995) are computed. Through some examples, we can conclude that two Bayes factors and F-test perform reasonably.

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