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A Note on Convergence of Fuzzy Variables

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Abstract

Liu[Fuzzy Optimization and Decision Making, 2(2003), 87-100] proved that convergence in credibility does not imply convergence a.s. and convergence in mean does not imply convergence a.s. by giving counter-examples. But these examples are not true. In this note, we prove that convergence in credibility implies convergence a.s. and convergence in mean implies convergence a.s.

Keywords : Convergence a.s., Convergence in credibility, Convergence in mean, Fuzzy variable

Recently, Liu(2003) gave some new convergence concepts of fuzzy sequence and discussed the relationship among them. Example 2(Liu(2003)) showed that convergence in credibility does not imply convergence a.s. and Example 3 (Liu(2003)) showed that convergence in mean does not imply convergence a.s. But these are wrong examples. Actually, convergence in credibility imply convergence a.s. and convergence in mean implies convergence a.s. We prove these statement.

Let ξ be a fuzzy variables defined on the possibility space $(\Theta, \mathrm{P}(\Theta), \mathrm{Pos})$. The necessity of a fuzzy event is defined as the impossibility of the opposite event, i.e.,

$$Nes\{\xi \le r\} = 1 - Pos\{\xi > r\}.$$

The credibility of a fuzzy event is defined as the average of its possibility and

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necessity. That is,

$$\mathrm{Cr}\{\xi \le r\} = \frac{1}{2}\left(\mathrm{Pos}\{\xi \le r\} + \mathrm{Nes}\{\xi \le r\}\right).$$

Definition 1.(Liu[2]) Suppose that $\{\xi_i\}$ is a sequence of fuzzy variables. The sequence $\{\xi_i\}$ is said to be convergence a.s. to the fuzzy variable ξ if and if only if there exists a set $A \in \mathcal{A}$ with $C_r\{A\} = 1$ such that

$$\lim_{i \to \infty} |\xi_i(\theta) - \xi(\theta)| = 0$$

for every $\theta \in A$. In that case we write $\xi_i \to \xi$, a.s.

Definition 2.(Liu[2]) Suppose that $\{\xi_i\}$ is a sequence of fuzzy variables.
We say that the sequence $\{\xi_i\}$ convergence in credibility to the fuzzy variable ξ if

$$\lim_{i \to \infty} C_r\{|\xi_i - \xi| \ge \varepsilon\} = 0$$

for every $\varepsilon > 0$.

Liu[2] gave Example 2 and 3 to show that convergence in credibility does not imply convergence a.s. and convergence in mean does not imply convergence a.s., respectively. But these are wrong examples and the statements are wrong. Actually, convergence in credibility does imply convergence a.s.and convergence in mean does imply convergence a.s. We prove these statements in the following theorems.

Theorem 1. Let $\{\xi_i\}$ be a sequence of fuzzy variables. If the sequence $\{\xi_i\}$ converges in credibility to a fuzzy variable ξ, then $\{\xi_i\}$ converges a.s. to ξ.

We first consider the following lemma.

Lemma 1. For any $A \in \mathrm{P}(\Theta)$, $C_r(A) = 0$ if and only if $\mathrm{Pos}(A) = 0$.

Proof. $C_r(A) = 0$ iff $\frac{1}{2}(\mathrm{Pos}(A) + 1 - \mathrm{Pos}(A^c)) = 0$ iff $\mathrm{Pos}(A) = 0$ and $\mathrm{Pos}(A^c) = 1$ iff $\mathrm{Pos}(A) = 0$ since $\max\{\mathrm{Pos}(A), \mathrm{Pos}(A^c)\} = 1$.

Proof of the Theorem 1. Suppose that $\{\xi_i\}$ converges in credibility to ξ. Then we have $\text{Pos}\{|\xi_n - \xi| \geq \varepsilon\} \to 0$ and $\text{Pos}\{|\xi_n - \xi| < \varepsilon\} \to 1$ for any $\varepsilon > 0$. We note that $\{\xi_i\}$ converges a.s. to ξ iff for any $\varepsilon > 0$.

$$\begin{aligned} 0 &= C_r\{|\xi_n - \xi| \geq \varepsilon, \quad i.\ o.\} \\ &= C_r\left\{ \bigcap_{n=1}^{\infty} \bigcup_{j=n}^{\infty} [|\xi_j - \xi| \geq \varepsilon] \right\}. \end{aligned}$$

By Lemma 1, it suffices to show that

$$\text{Pos}\left\{ \bigcap_{n=1}^{\infty} \bigcup_{j=n}^{\infty} [|\xi_j - \xi| \geq \varepsilon] \right\} = 0$$

Now, for every $\varepsilon > 0$, we have that

$$\begin{aligned} \text{Pos}\left\{ \bigcap_{n=1}^{\infty} \bigcup_{j=n}^{\infty} [|\xi_j - \xi| \geq \varepsilon] \right\} &\leq \text{Pos}\left\{ \bigcup_{j=n}^{\infty} \left[|\xi_j - \xi| \geq \frac{1}{k} \right] \right\} \\ &= \sup_{j \geq n} \text{Pos}\{|\xi_j - \xi| \geq \varepsilon\} \to 0 \end{aligned}$$

as $n \to \infty$, since $\text{Pos}\{|\xi_j - \xi| \geq \varepsilon\} \to 0$, which completes the proof.

Theorem 2. Let $\{\xi_i\}$ be a sequence of fuzzy variables. If the sequence $\{\xi_i\}$ converges in mean to a fuzzy variable ξ, then $\{\xi_i\}$ converges a.s. to ξ.

Proof. Convergence in mean implies convergence in credibility by Theorem 6[2], and hence the result follows immediate from Theorem 1.

References

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