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Seasonal Cointegration Rank Tests for Daily Data[1)]

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Abstract

This paper extends the maximum likelihood seasonal cointegration procedure developed by Johansen and Schaumburg (1999) for daily time series. The finite sample distribution of the associated rank test for daily data is also presented.

Keywords: Cointegration rank test, Daily time series, Seasonal cointegration, Seasonal error correction model

1. Introduction

Johansen and Schaumburg(1999) analyzed the error correction model for seasonal cointegration and showed that asymptotic distribution of the cointegration rank test is asymptotically mixed Gaussian. But Lee(1992) indicated that the finite sample distribution of cointegration rank test may be quite different from the associated asymptotic distribution, especially, in small sample size. Lee(1992), Lee and Siklos(1995) and Darne(2004) discussed this issue for the seasonal case and computed the finite sample critical values of the likelihood ratio(LR) test statistics for cointegration rank test in quarterly and monthly time series.

Johansen and Schaumburg(1999) proved that the asymptotic distributions of cointegration rank test are the same at all seasonal frequencies. (I.e except zero and pi frequency)

Darne(2004) computed the finite sample critical values for cointegration rank test using the method of Johansen and Schaumburg(1999) for monthly time series.

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However, some finite sample critical values are not correct in the sense that they do not match the tabulated numbers in Johansen and Schaumburg(1999) despite large sample size.

Contrary to macroeconomic data, financial data such as stock price and exchange rate have a period with five days of the week. Darne(2003) first computed the finite sample distribution of the rank test for daily data. But he used Lee(1992)'s seasonal cointegration procedure which impose particular parameter restriction on the seasonal error correction model.

In this paper we extend the Johansen-Schaumburg(1999) seasonal cointegration procedure to daily time series. We also provide the finite sample distribution of the LR test for the seasonal cointegration rank and compare with the associated asymptotic distribution.

2. Daily Seasonal Cointegration

Consider an n-dimensional autoregressive process X_t defined by

$$X_t = \sum_{i=1}^{k} \Pi_i X_{t-i} + \Phi D_t + \epsilon_t, \tag{1}$$

where ϵ_t is i.i.d $N_n(0,\Sigma)$, initial values $X_0, X_{-1}, \ldots, X_{-k+1}$ are fixed and D_t means a deterministic term. If we define $(n\times n)$ matrix polynomial $A(z) = I_n - \sum_{i=1}^{k} \Pi_i z^i$, the properties of the process X_t completely depend on roots of the characteristic equation written by $|A(z)|=0$. We consider the process whose characteristic function has five roots on the unit circle $(z_1 = 1, z_2 = e^{2\pi i/5}, z_3 = e^{-2\pi i/5}, z_4 = e^{4\pi i/5}, z_5 = e^{-4\pi i/5})$, i.e., the roots of the characteristic function exist on zero, $2\pi/5$, $-2\pi/5$, $4\pi/5$ and $-4\pi/5$ frequency.

Under the proper regularity condition, model (1) has the following seasonal error correction model proposed by Johansen and Schaumburg(1999),

$$p(L)X_t = \sum_{m=1}^{5} \alpha_m \beta_m' X_t^{(m)} + \sum_{j=1}^{k-5} \Gamma_j p(L) X_{t-j} + \Phi D_t + \epsilon_t \tag{2}$$

where α_m and β_m are $(n\times r_m)$ matrices with full rank r_m, and $\alpha_m = \alpha_{mR} + i\alpha_{mI}$ $\beta_m = \beta_{mR} + i\beta_{mI}$.

The reduced rank matrix $\alpha_m \beta_m'$ with $\beta_m' = \beta_R' - i\beta_I'$ describe the long-run behavior of the series at each of the five frequencies,

$$X_t^{(m)} = \frac{p_m(L)L}{p_m(z_m)z_m} X_t$$

$$p_j(z) = \prod_{m \neq j}^{5} (1 - \bar{z}_m z) = \frac{p(z)}{1 - \bar{z}_j z}$$

$$p(z) = \prod_{m=1}^{5} (1 - \bar{z}_m z) = (1 - z^5) = (1 - z)(1 + z + z^2 + z^3 + z^4)$$

where z_m's are the non seasonal and seasonal unit roots for the daily data and $\bar{z}_m$'s are their complex conjugate.

In our model, $X_t^{(m)}$ are defined as follows:

$$X_t^{(1)} = \frac{1}{5}(L + L^2 + L^3 + L^4 + L^5)X_t$$

$$X_t^{(2)} = \frac{1}{5}[\cos(\frac{2}{5}\pi)L + \cos(\frac{4}{5}\pi)L^2 + \cos(\frac{4}{5}\pi)L^3 + \cos(\frac{2}{5}\pi)L^4 + L^5]X_t$$
$$+ \frac{1}{5}i[-\sin(\frac{2}{5}\pi)L - \sin(\frac{4}{5}\pi)L^2 + \sin(\frac{4}{5}\pi)L^3 + \sin(\frac{2}{5}\pi)L^4]X_t$$

$$X_t^{(3)} = \frac{1}{5}[\cos(\frac{2}{5}\pi)L + \cos(\frac{4}{5}\pi)L^2 + \cos(\frac{4}{5}\pi)L^3 + \cos(\frac{2}{5}\pi)L^4 + L^5]X_t$$
$$+ \frac{1}{5}i[\sin(\frac{2}{5}\pi)L + \sin(\frac{4}{5}\pi)L^2 - \sin(\frac{4}{5}\pi)L^3 - \sin(\frac{2}{5}\pi)L^4]X_t$$

$$X_t^{(4)} = \frac{1}{5}[\cos(\frac{4}{5}\pi)L + \cos(\frac{2}{5}\pi)L^2 + \cos(\frac{2}{5}\pi)L^3 + \cos(\frac{4}{5}\pi)L^4 + L^5]X_t$$
$$+ \frac{1}{5}i[-\sin(\frac{4}{5}\pi)L + \sin(\frac{2}{5}\pi)L^2 - \sin(\frac{2}{5}\pi)L^3 + \sin(\frac{4}{5}\pi)L^4]X_t$$

$$X_t^{(5)} = \frac{1}{5}[\cos(\frac{4}{5}\pi)L + \cos(\frac{2}{5}\pi)L^2 + \cos(\frac{2}{5}\pi)L^3 + cos(\frac{4}{5}\pi)L^4 + L^5]X_t$$
$$+ \frac{1}{5}i[\sin(\frac{4}{5}\pi)L - \sin(\frac{2}{5}\pi)L^2 + \sin(\frac{2}{5}\pi)L^3 - \sin(\frac{4}{5}\pi)L^4]X_t$$

We let

$$X_t^{(2)} = X_t^{(2R)} + iX_t^{(2I)}, \quad X_t^{(3)} = X_t^{(2R)} - iX_t^{(2R)}$$
$$X_t^{(4)} = X_t^{(4R)} + iX_t^{(4I)}, \quad X_t^{(5)} = X_t^{(4R)} - iX_t^{(4I)},$$

where

$$X_t^{(2R)} = \frac{1}{5}[\cos(\frac{2}{5}\pi)L + \cos(\frac{4}{5}\pi)L^2 + \cos(\frac{4}{5}\pi)L^3 + \cos(\frac{2}{5}\pi)L^4 + L^5]X_t$$

$$X_t^{(2I)} = \frac{1}{5}[-\sin(\frac{2}{5}\pi)L - \sin(\frac{4}{5}\pi)L^2 + \sin(\frac{4}{5}\pi)L^3 + \sin(\frac{2}{5}\pi)L^4]X_t$$

$$X_t^{(4R)} = \frac{1}{5}[\cos(\frac{4}{5}\pi)L + \cos(\frac{2}{5}\pi)L^2 + \cos(\frac{2}{5}\pi)L^3 + \cos(\frac{4}{5}\pi)L^4 + L^5]X_t$$

$$X_t^{(4I)} = \frac{1}{5}[-\sin(\frac{4}{5}\pi)L + sin(\frac{2}{5}\pi)L^2 - \sin(\frac{2}{5}\pi)L^3 + \sin(\frac{4}{5}\pi)L^4]X_t.$$

If we express the complex frequency root and their conjugate frequency root simultaneously, the error correction model (2) can be rewritten as follows:

$$\begin{aligned} p(L)X_t = {} & \alpha_1\beta_1' X_t^{(1)} + 2(\alpha_{2R}\beta_{2R}' + \alpha_{2I}\beta_{2I}')X_t^{(2R)} + 2(\alpha_{2R}\beta_{2I}' - \alpha_{2I}\beta_{2R}')X_t^{(2I)} \\ & + 2(\alpha_{4R}\beta_{4R}' + \alpha_{4I}\beta_{4I}')X_t^{(4R)} + 2(\alpha_{4R}\beta_{4I}' - \alpha_{4I}\beta_{4R}')X_t^{(4I)} \\ & + \sum_{j=1}^{k-5}\Gamma_j p(L)X_{t-j} + \Phi D_t + \epsilon_t \end{aligned} \quad (3)$$

Johansen(1988) applied the canonical correlation method to deal with the unknown parameter estimation problem and showed that the asymptotic distribution of LR test statistics for cointegration rank test can be expressed in terms of a Brownian Motion. To apply the same estimation method at seasonal frequency, Lee(1992) imposed restrictions on parameters such as $(\alpha_{2R}\beta_{2I}' - \alpha_{2I}\beta_{2R}') = 0$ and $(\alpha_{4R}\beta_{4I}' - \alpha_{4I}\beta_{4R}') = 0$. Instead of peculiar restrictions on parameters Johansen and Schaumburg(1999) used the switching algorithm in a different way and compared the log-likelihood under the reduced rank r with the log-likelihood obtained from the unrestricted seasonal ECM, which corresponds to $r = n$. We omit the detailed description of the estimation algorithm since Dahl Pedersen(1996), Johansen and Schaumburg(1999), and Löf and Lyhagen(2002) gave full details of the estimation procedures.

Johansen and Schaumburg(1999) introduced the parameters Φ_m and represented ΦD_t by $\sum_{m=1}^{\bar{s}}\Phi_m \bar{z}_m^t$ where $\sum_{m=2}^{\bar{s}}\Phi_m \bar{z}_m^t$ generates the unwanted oscillating trend and Φ_1 generates linear trend. Kunst and Franses(1998) showed that when the cointegration at seasonal frequencies exists, the unrestricted seasonal intercepts indicates a growing amplitude in the seasonal and suggested the restricted seasonal dummy approach which give a better forecast than the unrestricted seasonal dummy approach in most cases. Therefore, in addition to model (3), we consider the following models suggested by Johansen and Schaumburg(1999) with restriction in deterministic term $\sum_{m=1}^{s}\Phi_m \bar{z}_m^t = \sum_{m=1}^{s}\alpha_m \rho_m' \bar{z}_m^t$ for some matrix ρ_m of dimension $(1 \times r_m)$. They showed that with this restriction the unwanted oscillating trend and linear trend can be removed. If we impose this restriction for deterministic terms, seasonal error correction model (3) can be written as follows:

$$p(L)X_t = \sum_{m=1}^{5} \alpha_m \begin{pmatrix} \beta_m' \\ \rho_m' \end{pmatrix} \begin{pmatrix} X_t^{(m)} \\ \bar{z}_m^t \end{pmatrix} + \sum_{j=1}^{k-5} \Gamma_j p(L) X_{t-j} + \sum_{m=6}^{\hat{s}} \Phi_m \bar{z}_m{}^t + \epsilon_t \tag{4}$$

If we do not impose a parameter restriction for deterministic term at zero frequency, the seasonal error correction model (4) can be written as

$$p(L)X_t = \alpha_1 \beta_1' X_t^{(1)} + \sum_{m=2}^{5} \alpha_m \begin{pmatrix} \beta_m' \\ \rho_m' \end{pmatrix} \begin{pmatrix} X_t^{(m)} \\ \bar{z}_m^t \end{pmatrix} + \sum_{j=1}^{k-5} \Gamma_j p(L) X_{t-j} + \Phi_1 + \sum_{m=6}^{\hat{s}} \Phi_m \bar{z}_m{}^t + \epsilon_t. \tag{5}$$

Lee and Silkos(1995) showed that the asymptotic distribution of the LR statistics for seasonal cointegration is unaffected by constant and trend terms, but is affected by seasonal dummies. Because we are interested in cointegration rank test at complex seasonal frequency, the parameter restriction at zero frequency does not affect the rank test at seasonal frequency. Therefore, in the followings we consider model (5) only.

3. Finite Sample Distribution of the Cointegration Rank Test

The simulation results of the finite sample behavior of the cointegration rank test at four seasonal frequencies except zero frequency are presented. Finite sample critical values at zero frequency can be obtained from Lee and Siklos(1995).

Data generating process for model (3) with $D_t = 0$ is the k dimensional seasonal integrated process given by $\Delta_5 X_t = \epsilon_t \quad (t = 1, 2, \ldots, T)$ with $\epsilon_t \sim i.i.d.\ N(0, I_k)$, $k = 1, 2, \cdots, 5$ for $(n - r) = 1, 2, \cdots, 5$. Data generating process for model (5) with restricted deterministic terms is $\Delta_5 X_t = 2 \times 1_k + \epsilon_t$, where 1_k is k dimension vector with elements 1. Four sample sizes are considered throughout, T=50, 100, 200 and 400 and are replicated 100,000 times.

We summarize the simulation results in Table 1 and Table 2. Table 1 displays the empirical critical values of the rank tests obtained from model (3) with no deterministic terms. The empirical critical values of the rank tests obtained from model (5) with restricted seasonal dummies and unrestricted constant are displayed in Table 2.

From table 1 we observe that finite sample empirical quantiles of the cointegration rank tests are similar to the asymptotic ones when the number of independent cointegrating vectors, r, is close to the number of time series, n. In particular, when the dimension $(n - r)$ is less than 3, the difference between the empirical quantiles and asymptotic ones can be ignored. But the difference of two critical values increases as the sample size gets smaller and the dimension $(n-r)$ gets larger, especially when $(n-r)$ is larger than 3. Similar results are observed from Table 2 when the linear trend and restricted seasonal dummy are considered. It is observed that the difference of two critical values gets much larger in this case. In conclusion, when $(n-r)$ is larger than 3 and sample size is small the asymptotic critical values may lead a wrong inference at cointegration test. Johansen and Schaumburg(1999) pointed out that complex seasonal cointegration rank test at other seasonal frequencies except zero and π frequency have the same distribution and we confirm it through the simulation study.

4. Conclusion

In this paper we propose an extension of the seasonal cointegration procedure developed by Johansen and Schaumburg(1999) for daily time series.

When the dimension $(n-r)$ is less than 3, the difference between the asymptotic distribution and the finite sample distribution is negligible. But for small samples the difference gets larger as $(n-r)$, the difference of the number of the times series and the number of the independent cointegration vector, gets larger. Therefore, if we use the asymptotic critical values for the cointegration rank test there is a tendency to reject the null hypothesis more often which recommends the use of finite sample critical values for the cointegration rank test when $(n-r)$ is larger than 3 in small samples.

Table 1. Quantiles of the Seasonal Cointegration rank test at seasonal frequency for model (3) with no deterministic term

$n-r$	T	θ	1%	5%	10%	50%	75%	90%	95%	99%
1	50	$2\pi/5$	0.021	0.115	0.238	1.527	2.984	4.847	6.265	9.463
		$4\pi/5$	0.023	0.116	0.236	1.527	2.986	4.872	6.257	9.500
	100	$2\pi/5$	0.023	0.115	0.231	1.507	2.953	4.846	6.253	9.435
		$4\pi/5$	0.023	0.115	0.239	1.520	2.965	4.825	6.226	9.508
	200	$2\pi/5$	0.022	0.113	0.234	1.508	2.947	4.799	6.205	9.395
		$4\pi/5$	0.023	0.115	0.234	1.511	2.959	4.857	6.239	9.411
	400	$2\pi/5$	0.022	0.115	0.233	1.501	2.938	4.804	6.221	9.514
		$4\pi/5$	0.023	0.114	0.235	1.518	2.966	4.818	6.261	9.535
	∞		0.023	0.114	0.234	1.500	2.950	4.800	6.200	9.450
2	50	$2\pi/5$	4.291	5.886	6.907	11.715	15.034	18.667	21.055	26.314
		$4\pi/5$	4.337	5.926	6.934	11.729	15.108	18.735	21.107	26.416
	100	$2\pi/5$	4.289	5.838	6.831	11.554	14.843	18.349	20.710	25.731
		$4\pi/5$	4.272	5.810	6.827	11.579	14.913	18.425	20.845	25.700
	200	$2\pi/5$	4.232	5.812	6.804	11.515	14.804	18.267	20.626	25.647
		$4\pi/5$	4.236	5.796	6.816	11.459	14.758	18.271	20.631	25.661
	400	$2\pi/5$	4.219	5.768	6.773	11.451	14.750	18.230	20.630	25.624
		$4\pi/5$	4.252	5.773	6.790	11.463	14.748	18.231	20.547	25.616
	∞		4.210	5.740	6.730	11.400	14.600	18.100	20.400	25.300
3	50	$2\pi/5$	17.031	20.339	22.303	30.550	35.741	40.946	44.431	51.441
		$4\pi/5$	17.031	20.418	22.386	30.778	36.022	41.357	44.783	52.009
	100	$2\pi/5$	16.714	19.928	21.841	29.870	34.955	40.135	43.424	50.216
		$4\pi/5$	16.626	19.954	21.899	29.962	35.099	40.201	43.453	50.222
	200	$2\pi/5$	16.523	19.756	21.675	29.626	34.664	39.764	43.088	49.821
		$4\pi/5$	16.628	19.831	21.765	29.656	34.734	39.731	43.036	49.839
	400	$2\pi/5$	16.569	19.723	21.637	29.556	34.531	39.554	42.886	49.582
		$4\pi/5$	16.574	19.730	21.596	29.513	34.510	39.525	42.801	49.348
	∞		16.300	19.400	21.300	29.200	34.100	39.100	42.300	48.900
4	50	$2\pi/5$	38.589	43.686	46.605	58.412	65.566	72.688	77.225	85.940
		$4\pi/5$	39.161	44.303	47.323	59.347	66.666	73.869	78.282	87.578
	100	$2\pi/5$	37.546	42.445	45.306	56.735	63.586	70.250	74.481	82.883
		$4\pi/5$	37.460	42.588	45.459	56.900	63.784	70.539	74.831	83.545
	200	$2\pi/5$	37.113	42.000	44.770	55.950	62.710	69.344	73.473	81.850
		$4\pi/5$	37.166	41.947	44.791	56.092	62.869	69.437	73.636	82.134
	400	$2\pi/5$	36.918	41.726	44.524	55.662	62.420	68.939	73.049	81.216
		$4\pi/5$	36.899	41.749	44.595	55.726	62.403	68.864	73.133	81.461
	∞		36.300	41.100	43.800	54.800	61.500	67.900	72.000	80.300
5	50	$2\pi/5$	69.243	76.331	80.287	95.946	105.180	114.080	119.680	130.980
		$4\pi/5$	70.807	78.010	82.130	98.374	107.950	117.100	122.950	134.500
	100	$2\pi/5$	66.684	73.514	77.331	92.079	100.750	109.130	114.350	124.630
		$4\pi/5$	67.052	73.824	77.754	92.561	101.320	109.780	115.050	125.430
	200	$2\pi/5$	65.825	72.413	76.113	90.568	99.021	107.160	112.380	122.460
		$4\pi/5$	65.932	72.420	76.170	90.646	99.183	107.330	112.550	122.780
	400	$2\pi/5$	65.547	72.000	75.693	90.009	98.438	106.410	111.460	121.550
		$4\pi/5$	65.473	71.979	75.631	90.052	98.488	106.530	111.660	121.840
	∞		64.100	70.500	74.200	88.300	96.600	105.000	110.000	119.000

Table 2. Quantiles of the Seasonal Cointegration rank test at seasonal frequency for model (5) with restricted seasonal dummies and unrestricted constant.

$n-r$	T	θ	1%	5%	10%	50%	75%	90%	95%	99%
1	50	$2\pi/5$	2.234	3.246	3.940	7.502	10.175	13.136	15.183	19.670
		$4\pi/5$	2.230	3.254	3.945	7.495	10.173	13.142	15.236	19.771
	100	$2\pi/5$	2.216	3.249	3.946	7.444	10.072	13.008	15.034	19.449
		$4\pi/5$	2.223	3.258	3.954	7.474	10.097	13.022	15.036	19.374
	200	$2\pi/5$	2.228	3.235	3.928	7.438	10.090	13.006	15.001	19.345
		$4\pi/5$	2.223	3.244	3.933	7.454	10.064	12.954	14.926	19.212
	400	$2\pi/5$	2.218	3.236	3.918	7.422	10.035	12.985	14.943	19.260
		$4\pi/5$	2.246	3.233	3.915	7.432	10.060	12.966	14.994	19.068
	∞		2.230	3.270	3.970	7.510	10.150	13.070	15.120	19.480
2	50	$2\pi/5$	12.704	15.394	17.048	24.092	28.631	33.360	36.429	43.002
		$4\pi/5$	12.656	15.354	16.999	24.207	28.856	33.652	36.680	43.111
	100	$2\pi/5$	12.601	15.217	16.820	23.679	28.155	32.737	35.765	41.871
		$4\pi/5$	12.473	15.143	16.757	23.705	28.157	32.823	35.831	41.985
	200	$2\pi/5$	12.447	15.083	16.718	23.525	27.972	32.514	35.550	41.675
		$4\pi/5$	12.500	15.113	16.692	23.497	27.918	32.431	35.455	41.539
	400	$2\pi/5$	12.445	15.081	16.700	23.430	27.865	32.369	35.308	41.294
		$4\pi/5$	12.452	15.062	16.666	23.421	27.888	32.357	35.258	41.285
	∞		12.540	15.230	16.790	23.610	28.020	32.480	35.470	41.380
3	50	$2\pi/5$	31.851	36.311	38.925	49.596	56.104	62.545	66.772	75.413
		$4\pi/5$	32.031	36.604	39.285	50.153	56.758	63.231	67.465	75.882
	100	$2\pi/5$	31.220	35.611	38.067	48.271	54.535	60.748	64.736	72.733
		$4\pi/5$	31.288	35.587	38.110	48.474	54.729	60.958	64.940	72.889
	200	$2\pi/5$	30.909	35.161	37.713	47.763	53.865	59.896	63.876	71.863
		$4\pi/5$	30.876	35.218	37.700	47.819	53.960	60.092	64.013	71.733
	400	$2\pi/5$	30.732	35.042	37.551	47.575	53.708	59.748	63.552	71.501
		$4\pi/5$	30.829	35.095	37.570	47.569	53.683	59.721	63.577	71.261
	∞		30.990	35.290	37.840	47.880	53.950	60.000	63.900	71.640
4	50	$2\pi/5$	60.217	66.610	70.241	84.753	93.372	101.780	107.000	117.410
		$4\pi/5$	61.147	67.771	71.486	86.302	95.166	103.700	109.210	119.900
	100	$2\pi/5$	58.237	64.365	67.864	81.451	89.505	97.204	102.190	111.920
		$4\pi/5$	58.373	64.556	68.098	81.827	89.944	97.820	102.900	112.880
	200	$2\pi/5$	57.473	63.475	66.881	80.297	88.151	95.741	100.680	110.120
		$4\pi/5$	57.658	63.586	66.996	80.451	88.360	96.018	100.730	110.350
	400	$2\pi/5$	57.265	63.157	66.551	79.812	87.544	95.084	99.813	109.480
		$4\pi/5$	57.221	63.223	66.590	79.891	87.636	95.106	99.923	109.380
	∞		57.490	63.440	66.890	80.200	88.070	95.600	100.450	109.900
5	50	$2\pi/5$	98.164	106.650	111.510	130.120	140.960	151.410	157.950	170.830
		$4\pi/5$	100.700	109.650	114.580	133.940	145.310	156.060	162.830	176.390
	100	$2\pi/5$	94.090	101.980	106.360	123.440	133.320	142.770	148.670	160.300
		$4\pi/5$	94.501	102.470	106.930	124.170	134.200	143.780	149.890	161.730
	200	$2\pi/5$	92.497	100.160	104.470	121.040	130.690	140.000	145.800	157.240
		$4\pi/5$	92.567	100.290	104.620	121.170	130.900	140.170	145.740	157.130
	400	$2\pi/5$	91.767	99.479	103.760	120.240	129.820	138.780	144.550	155.390
		$4\pi/5$	91.892	99.487	103.780	120.330	129.820	138.940	144.570	155.750
	∞		92.100	99.930	104.150	111.730	130.170	139.380	145.060	156.420

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