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Evaluating Properties of Variable Sampling Interval EWMA Control Charts for Mean Vector[1]

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Abstract

Theoretical and numerical comparison have shown that variable sampling interval (VSI) charts are substantially more efficient than fixed sampling interval(FSI) charts in term of ATS(average time to signal). But the frequency of switching between different sampling intervals is a complicating factor in VSI procedures. VSI EWMA charts for monitoring mean vector of related quality characteristics are investigated. To compare the efficiencies of the proposed charts, the performances are evaluated for matched FSI and VSI charts in terms of average time to signal(ATS) and average number of samples to signal(ANSS). For the switching behavior of the proposed VSI charts, average number of switches(ANSW) are also investigated.

Keywords : accumulate-combine approach, ANSS, ANSW, ATS

1. Introduction

The usual practice in using a control chart to monitor a process is to take sample from the process at FSI and plotting in time order on the chart some statistic computed from the sample. VSI control charts vary the sampling interval as a function of what is observed from the process can detect process changes faster than FSI charts. For VSI chart, Reynolds(1995) showed that if the consecutive observations are independent, then the use of two sampling intervals spaced as apart as possible is optimal.

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One disadvantage of VSI scheme is that frequent switching between different sampling intervals requires more cost and effort to administer the process than corresponding FSI scheme. Amin and Letsinger(1991) described general procedures for VSI scheme and examined switching behavior and runs rules for switching between different sampling intervals on univariate $\overline{X}$-chart. They also presented that the average number of switches to signal(ANSW) of the CUSUM and EWMA procedures exists far fewer than the Shewhart procedure. Reynolds(1995) pays special attention to the frequency of switching between the two sampling intervals which decreases as the smoothing constant λ decreases.

A multivariate FSI EWMA chart for monitoring mean vector of a multivariate normal process using accumulate-combine approach was presented by Lowry et al.(1992). By simulation, they showed that the performances of the multivariate EWMA procedure performs better than the multivariate CUSUM procedures of Pignatiello and Runger(1990), and it performs roughly the same if small shift has occurred. Vargas et al.(2004) presented a comparative study of the performance of CUSUM and EWMA charts in order to detect small changes of process average.

In this paper, we investigate the properties of multivariate VSI control charts for simultaneously controlling the shifts on the means of multivariate normal process in terms of ATS and ANSS. And we also investigate the ANSW and ASWR of the proposed VSI charts. By markov chain method or simulation, the ANSW values for the EWMA procedure are substantially less than those of Shewhart procedure. And we also found that the VSI EWMA chart with accumulate-combine feature will have the lowest ATS when the process changed. Thus, for the same effort, we can note that VSI EWMA charts with accumulate-combine feature will detect most process changes fast.

2. Description of Some Control Procedures

Assume that the quality vector $\underline{X}' = (X_1, X_2, \cdots, X_p)$ are jointly distributed as p-variate normal distribution $N_p(\underline{\mu}, \Sigma)$. We take a sequence of independent random vectors $\underline{X}_1$, $\underline{X}_2$, $\underline{X}_3$, $\cdots$, where $\underline{X}_i = (\ \underline{X}'_{i1}, \cdots, \underline{X}'_{in})'$ is a sample of observations at the sampling time i and $\underline{X}_{ij} = (X_{ij1}, \cdots, X_{ijp})'$. Let $\underline{\theta}_0 = (\underline{\mu}_0, \Sigma_0)$ be the known target process parameters for $\underline{\theta} = (\underline{\mu}, \Sigma)$ of p quality variables.

2.1 Sample Statistic for Combine-Accumulate Approach

The general multivariate statistical quality control chart can be considered as a repetitive tests of significance where each quality characteristic is defined by p quality variables X_1, X_2, $\cdots$, X_p. Therefore, we can obtain a sample statistic for monitoring $\underline{\mu}$ by using the likelihood ratio test(LRT) statistic of the hypothesis

$H_0: \mu = \mu_0$ on the basis of a sample from $N_p(\mu, \Sigma)$ where Σ_0 is known.
Likelihood ratio λ_i at the ith sample can be expressed as

$$\lambda_i = \exp\left[-\frac{n}{2} (\overline{X}_i - \mu_0)' \Sigma_0^{-1} (\overline{X}_i - \mu_0) \right].$$

Let us define $Z_i^2 = -2 \ln \lambda_i$. Then

$$Z_i^2 = n(\overline{X}_i - \mu_0)' \Sigma_0^{-1} (\overline{X}_i - \mu_0). \tag{2.1}$$

If this sample statistic plots above the upper control limit(UCL), the process is deemed out of control and assignable causes of variation are sought. Thus, the sample statistic Z_i^2 can be used as the control statistic for μ of p related quality variables. Alt(1984) described various types of multivariate Shewhart type T^2 control charts based on Hotelling's $T^2 = n(\overline{X}_i - \mu_0)' S^{-1} (\overline{X}_i - \mu_0)$ statistic and provided some recommendations for implementation where S is covariance matrix of the sample.

2.2 ANSS and ATS of VSI Procedure

If the control statistic from a sample falls in the signal region, the chart signals. Let $s(y)$ be a signal function such that

$$s(y) = \begin{cases} 1 & \text{if the chart signals} \\ 0 & \text{otherwise} \end{cases}$$

when the control statistic $Y = y$ is observed. A signal region is designated on the chart such that the control statistic is very unlikely to fall within the signal region when there has been no change in the process. A value of control statistic falling within the signal region is taken as evidence that the process has changed and that action should be taken to correct the problem.

In FSI control chart, $t_{i+1} - t_i$, the length of sampling interval between sampling times, is constant for all $i\,(i = 0, 1, \cdots)$. But for a VSI chart, the sampling times are random variables and $t_{i+1} - t_i$ is a function of chart statistic and depends on the past sample informations $X_1, X_2, \cdots, X_i$. To evaluate the performance of a VSI control chart, it is necessary to obtain time and number of samples separately. Therefore, we use ATS and ANSS for evaluating and comparing the properties of the FSI and VSI charts.

The choice of a sampling interval as a function of control statistics can be represented by a sampling interval function $d(x)$, which specifies the sampling interval to be used when $X_i = x$ is observed. To apply η sampling interval VSI chart, the interval of chart statistic can be divided into in-control region C and out-of-control region C'. Then the in-control region $C = \{x \mid s(x) = 0\}$ can be partitioned into η regions $I_1, I_2, \cdots, I_\eta$ where I_j is the region in which the sampling interval d_j is used. Thus, the sampling interval used between X_i and X_{i+1} is $d(X_1, \cdots, X_i)$. If we use a finite number of interval lengths $d_1, d_2, \cdots, d_\eta$ where $d_1 < d_2 < \cdots < d_\eta$, these possible interval lengths must be chosen to satisfy $l_1 < d_i < l_2$. The minimum possible interval length $l_1 > 0$ might be determined by the physical considerations such as the shortest time required to take a sample. The maximum interval length l_2 might be determined by the maximum amount of time that the process engineers are willing to allow the process to run without sampling.

In multivariate Shewhart chart, the ATS and the variance of the time to signal is given as

$$E(T) = d_0 + \sum_{j=1}^{\eta} \frac{d_j p_j}{q} \tag{2.2}$$

and

$$Var(T) = \sum_{j=1}^{\eta} \frac{d_j^2 p_j}{q} + \frac{(\sum_{j=1}^{\eta} d_j p_j)^2}{q^2}, \tag{2.3}$$

where d_0 is the sampling interval used before the first sample, $p_j = P(d(X_i) = d_j)$ and $q = 1 - \sum_{j=1}^{\eta} p_j > 0$.

In this paper, we assume that the VSI chart is started at time 0 and the interval used before the first sample, is a fixed constant, say d_0. And, control chart signals when control statistic $y \in C'$. When the VSI feature is applied using two sampling intervals, the sampling function is

$$l(y) = \begin{cases} d_1 & \text{if } y \in (g, h] \\ d_2 & \text{if } y \in (0, g] \end{cases}$$

where $g(0 < g < h)$ is the boundary between the regions specifying d_1 and d_2.

2.3 ANSW and ASWR of VSI Procedure

Because a VSI chart switches between different sampling intervals, properties such as the time required to signal will be more difficult to evaluate than for the corresponding FSI chart. In addition, it will be desirable to evaluate properties, such as the amount of switching done by the VSI chart, which are not issue in FSI charts. But, ANSS and ATS do not directly provide information on how frequently switches are made. Therefore, it is necessary to consider a new quantity which measures the frequency of switches in VSI control procedures. Define the average number of switches(ANSW) to be the expected value of the number of switches made from the start of the process until the chart signals. The ANSW can be obtained as follows

$$ANSW = (ANSS - d_0) \cdot P(\text{switch}) \quad (2.4)$$

And, the probability of switch is given by

$$P(\text{switch}) = P(d_1) \cdot P(d_2 | d_1) + P(d_2) \cdot P(d_1 | d_2) \quad (2.5)$$

where $P(d_i)$ is the probability of using sampling interval d_i, and $P(d_i | d_j)$ is the conditional probability of using sampling interval d_i in the current sample given that the sampling interval d_j $(d_i \neq d_j)$ was used in the previous sample. To quantify the amount of switching, average switching rate(ASWR) can be defined as

$$ASWR = ANSW / ANSS. \quad (2.6)$$

A low value of the ASWR will usually be desirable from the administrative point of view, but a value of the ASWR very close to zero may not be achievable in a chart that is responsive to changes in the process.

3. Shewhart Control Chart

The Shewhart chart has a good ability to detect large changes in monitored parameter quickly and is easy to implement the process. However, the Shewhart chart is slow to signal for small or moderate changes in the process. And it uses the information only in the last sample and, unless runs rules are used, ignores all the informations of the previous samples.

A multivariate Shewhart chart for μ based on the sample statistics Z_i^2 in (2.1) signals whenever

$$Z_i^2 \geq h_{Z^2} \quad (3.1)$$

And for VSI Shewhart chart based on Z_i^2, suppose that the sampling interval ;

$$d_1 \text{ is used when } Z_i^2 \in (g_{Z^2}, h_{Z^2}],$$
$$d_2 \text{ is used when } Z_i^2 \in (0, g_{Z^2}],$$

where $g_{Z^2} \leq h_{Z^2}$ and $d_1 < d_2$. If an indication of a production process change is strong enough, then VSI chart signals in the same way as the FSI chart.

For example, the probability of switch when the production process is in-control, $P(d_1 | \mu = \mu_0) = P(d_2 | \mu = \mu_0) = 0.49865$ for an in-control ANSS is 370.4. Then,

$$P(\text{switch}) = P(d_1) \cdot P(d_2 | d_1) + P(d_2) \cdot P(d_1 | d_2) = 0.4973,$$

$$ANSW = 183.703 \text{ switches until a signal}$$

and

$$ASWR = 0.5068.$$

If the process is in-control, the control statistic Z_i^2 has a chi-squared distribution with p degrees of freedom when $\mu = \mu_0$ and $\Sigma = \Sigma_0$. Hence, the design parameters g_{Z^2} and h_{Z^2} can be obtained to satisfy a desired ATS and ANSS. When the process has shifted to μ from the target μ_0, the LRT statistic Z_i^2 has a non-central chi-square distribution with p degrees of freedom with noncentrality parameter $\tau^2 = n(\mu - \mu_0)'\Sigma_0^{-1}(\mu - \mu_0)$. The properties of the chart based on Z_i^2 can be obtained by markov chain or integral equation approach.

Result 3.1 If $X_{ij} = (X_{ij1}, X_{ij2}, \cdots, X_{ijp})'$ is distributed as $N_p(\mu_0, \Sigma_0)$ and X_{ij}'s are independent. Assume that VSI multivariate control chart based on Z_i^2 in (2.1) with two sampling intervals are used as stated above. When the parameters of the distribution shifted as $N(\mu, c\Sigma_0)$ where c is a constant, then

$$ANSS = \frac{1}{1 - F(\frac{h_{Z^2}}{c})}$$

and

$$ATS = d_0 + \frac{d_1[F(\frac{h_{Z^2}}{c}) - F(\frac{g_{Z^2}}{c})] + d_2 F(\frac{g_{Z^2}}{c})}{1 - F(\frac{h_{Z^2}}{c})},$$

where $F(\cdot)$ is a chi-squared distribution function with p degrees of freedom and noncentrality parameter $n(\mu - \mu_0)'\Sigma_0^{-1}(\mu - \mu_0)/c$.

4. EWMA Control Chart

The EWMA control chart is based on an EWMA of the current and past sample information. In EWMA chart, the more recent observations are assigned more weights and the older observations are assigned less weights. Like the Shewhart chart, the EWMA control chart is easy to implement and interpret. Multivariate EWMA(MEWMA) chart takes two ways to use the past sample information, such as combine-accumulate approach and accumulate-combine approach.

4.1 Combine-Accumulate Approach

For FSI EWMA chart for the process mean vector $\underline{\mu}$ based on LRT statistic Z_i^2 $(i=1,2,\cdots)$ in (2.1) is given by

$$Y_{Z^2,i}=(1-\lambda)Y_{Z^2,i-1}+\lambda Z_i^2, \tag{4.1}$$

where $Y_{Z^2,0}=\omega\,(\omega\geq 0)$ and λ $(0<\lambda\leq 1)$ is a smoothing constant. The statistic Z_i^2 is a function of the current sample $\underline{X}_i$, and $Y_{Z^2,i}$ is the value of the EWMA after the observation at the ith sample. This chart signals whenever $Y_{Z^2,i}\geq h_{Z^2(E)}$.

And for VSI EWMA chart based on Z_i^2, suppose the sampling interval ;

d_1 is used when $Z_i^2\in(g_{Z^2(E)},h_{Z^2(E)}]$,
d_2 is used when $Z_i^2\in(0,g_{Z^2(E)}]$,

where $g_{Z^2(E)}\leq h_{Z^2(E)}$ and $d_1<d_2$. The design parameters $g_{Z^2(E)}$, $h_{Z^2(E)}$ can be obtained to satisfy a specified ATS and ANSS by Markov chain approach. And when the parameters are on-target or changed, the performances of this chart can be evaluated by the Markov chain approach.

4.2 Accumulate-Combine Approach

Lowry et al.(1992) proposed a FSI MEWMA chart for $\underline{\mu}$ with accumulate-combine technique. They asserted that MEWMA chart for mean vector is a more straightforward generalization of the corresponding univariate procedure than the multivariate CUSUM statistics of Pignatiello and Runger(1990).

MEWMA chart for mean vector $\underline{\mu}$ is a multivariate extension of univariate EWMA chart. The vectors of EWMA's are defined as

$$\underline{Y}_i=(I-\Lambda)\,\underline{Y}_{i-1}+\Lambda\,\overline{\underline{X}}_i \tag{4.2}$$

$i=1,2,\cdots$ where $\underline{Y}_0=\underline{\mu}_0$ and $\Lambda=diag(\lambda_1,\cdots,\lambda_p)$, $0<\lambda_j\leq 1$ $(j=1,2,\cdots,p)$. Equation (4.2) can be rewritten by repeated substitution as

$$\underline{Y}_i=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\overline{\underline{X}}_k+(I-\Lambda)^i\underline{\mu}_0. \tag{4.3}$$

Then the dispersion matrix of $\underline{Y}_i$ can be obtained as

$$\Sigma_{\underline{Y}_i}=\sum_{k=1}^{i}[\Lambda(I-\Lambda)^{i-k}\Sigma(I-\Lambda)^{i-k}\Lambda]/n, \tag{4.4}$$

where Σ is a known covariance matrix. For FSI MEWMA chart based on accumulate-combine approach for μ signals as soon as

$$T_i^2 = (Y_i - \mu_0)' \Sigma_{Y_i}^{-1} (Y_i - \mu_0) > h_1,$$

where $h_1 (>0)$ is chosen to achieve a specified in-control ANSS. Unless there is any reason to differently weight the elements of smoothing matrix Λ, all diagonal elements of Λ can be set to an equal value. Under the assumption that $\lambda_1 = \lambda_2 = \cdots = \lambda_s = \lambda$, then MEWMA vectors can be written as

$$Y_i = \sum_{k=1}^{i} \lambda(1-\lambda)^{i-k} \overline{X}_k + (1-\lambda)^i Y_0 . \tag{4.5}$$

$i = 1, 2, \cdots$, and the dispersion matrix of Y_i is given by

$$\Sigma_{Y_i} = \frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}] \frac{\Sigma}{n} . \tag{4.6}$$

Prabhu and Runger(1997) stated that good choices for λ depend on the number of variables in the control scheme and the size of the shift in MEWMA chart for μ and they stated that values for λ from 0.1 to 0.5 are good choices. Lowry et al.(1992) also showed that the distribution of T_i^2 depends on μ and Σ only through the noncentrality parameter τ as

$$\tau = [n(\mu - \mu_0)' \Sigma^{-1} (\mu - \mu_0)]^{1/2}$$

and smaller values of λ are more effective in detecting the smaller shifts in the mean vectors. And for VSI EWMA chart based on Y_i, suppose the sampling interval ;

d_1 is used when $T_i^2 \in (g_1, h_1]$,
d_2 is used when $T_i^2 \in (0, g_1]$,

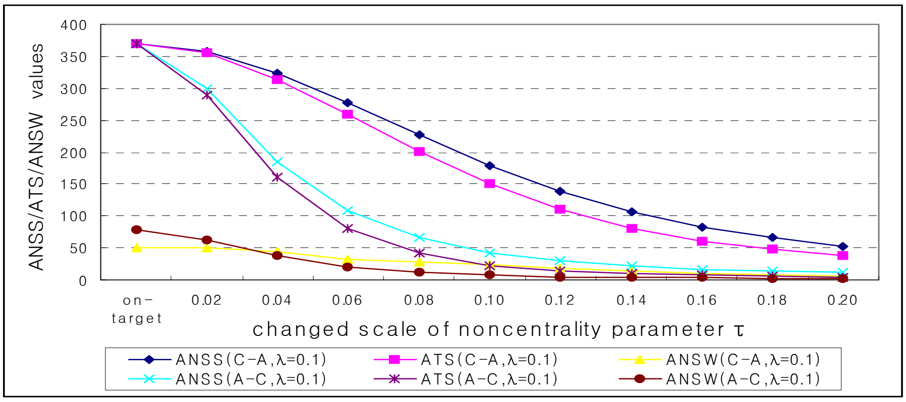
where $g_1 \le h_1$ and $d_1 < d_2$. Since it is difficult to obtain the exact distribution of chart statistic Y_i, process parameters g_1, h_1 can be obtained to satisfy a specified ATS and ANSS. And when the process is in-control or out-of-control states, the vaules ANSS, ATS, ANSW can be evaluated by simulation.

5. Numerical Results and Concluding Remarks

In selecting a control chart for a particular application, the main objectives in terms of statistical properties will be to achieve fast detection of changes in the process while maintaining a low false alarm rate and a resonable average sampling rate. Then

[Table 1] Performances of Shewhart chart

scale of shift	$p=2$			$p=3$			$p=4$		
	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW
in-control	370.4	370.4	183.7	370.4	370.4	183.7	370.4	370.4	183.7
$\tau=0.25$	311.1	305.3	154.0	324.3	319.0	160.6	331.9	327.1	164.4
$\tau=0.5$	202.3	187.5	99.0	228.9	214.5	112.4	246.7	232.8	121.4
$\tau=0.75$	118.0	99.7	55.8	143.5	124.1	68.6	162.7	142.9	78.4
$\tau=1.0$	67.3	50.2	29.5	85.8	66.5	38.8	101.2	80.5	46.5
$\tau=1.5$	23.3	12.7	7.4	30.9	17.9	10.7	37.9	23.0	14.0
$\tau=2.0$	9.4	3.9	1.5	12.3	5.3	2.5	15.1	6.8	3.5
$\tau=2.5$	4.5	1.8	0.3	5.7	2.2	0.5	6.9	2.6	0.7
$\tau=3.0$	2.6	1.2	0.0	3.1	1.4	0.1	3.6	1.5	0.1
$\tau=3.5$	1.7	1.1	0.0	2.0	1.1	0.0	2.2	1.2	0.0
$\tau=4.0$	1.3	1.0	0.0	1.4	1.1	0.0	1.6	1.1	0.0
$\tau=4.5$	1.1	1.0	0.0	1.2	1.0	0.0	1.3	1.0	0.0
$\tau=5.0$	1.0	1.0	0.0	1.1	1.0	0.0	1.1	1.0	0.0

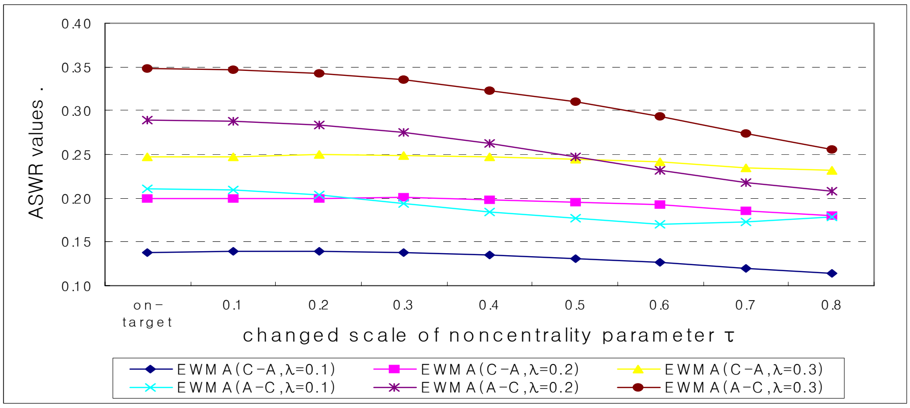


<Figure 1> Performances of EWMA chart ($p=4$)

the expected times required to detect shifts of interest can be compared. For VSI chart, the amount of switching for the different charts can also be compared.

The properties of VSI EWMA chart are determined by the choice of the process parameters λ, h, g, d_1, d_2. For the purposes of comparison and evaluation of different FSI and VSI charts, all charts being considered were set up so that the ANSS and ATS are 370.4 when $\mu=\mu_0$, $d_0=1$ and the sample size for each variable was five for $p=2\sim 4$. For simplicity in our computation, we assume that the target mean vector $\mu_0=\underline{0}'$, all diagonal and off-diagonal elements of Σ_0 are

1 and 0.3, respectively. And we let that the sampling interval of unit time $d=1$ in FSI chart and two sampling intervals used as $d_1=0.1$ and $d_2=1.9$ in VSI chart. After the smoothing constants of the proposed EWMA charts have been determined, the design parameters $g's$ and $h's$ and the ANSS, ATS and ANSW values were calculated by Markov chains with the number of transient states $r=100$ or simulation with 10,000 runs.



<Figure 2> ASWR values of the proposed EWMA chart ($p=4$)

[Table 2] Performances of EWMA chart with combine-accumulate approach($p=3$)

scale of shift	$\lambda=0.1$			$\lambda=0.2$			$\lambda=0.3$		
	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW
in-control	370.4	370.4	50.6	370.4	370.4	72.5	370.4	370.4	90.8
$\tau=0.25$	292.5	276.3	39.9	302.2	288.0	59.0	308.5	296.3	74.9
$\tau=0.5$	162.6	132.9	21.0	178.3	147.8	34.1	191.1	162.5	45.8
$\tau=0.75$	81.5	58.9	9.3	90.5	61.5	16.2	101.3	71.1	23.1
$\tau=1.0$	44.3	32.5	4.6	46.2	26.8	7.4	52.2	29.5	10.8
$\tau=1.5$	18.9	17.2	2.5	16.1	10.0	2.7	16.7	8.2	3.1
$\tau=2.0$	10.9	11.4	2.1	8.2	6.4	2.1	7.7	4.7	1.9
$\tau=2.5$	7.3	8.3	2.0	5.2	4.7	1.8	4.6	3.5	1.5
$\tau=3.0$	5.3	6.3	1.9	3.7	3.7	1.6	3.1	2.8	1.3
$\tau=3.5$	4.0	5.0	1.8	2.8	3.0	1.3	2.4	2.4	1.0
$\tau=4.0$	3.2	4.1	1.7	2.2	2.5	1.1	1.9	2.1	0.7
$\tau=4.5$	2.7	3.5	1.5	1.9	2.2	0.8	1.6	2.0	0.5
$\tau=5.0$	2.3	3.0	1.2	1.6	2.0	0.6	1.3	1.9	0.3

The properties and comparison of the proposed procedures are given in [Table 1] through [Table 3]. From the numerical results, we found the following properties. The properties of EWMA procedure are more efficient than those of

Shewhart procedure. The data in Tables show that accumulate-combine procedure yields small ATS and ANSS than combine-accumulate procedure in EWMA procedure, and VSI schemes are more efficient than FSI schemes. As illustrated in table, smaller smoothing constant λ are more effective in detecting small shifts of mean vector in EWMA charts. It can be seen that the EWMA procedure has substantially fewer switches when compared with

[Table 3] Performances of EWMA chart with accumulate-combine approach($p=3$)

	$\lambda=0.1$			$\lambda=0.2$			$\lambda=0.3$		
scale of shift	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW
in-control	370.5	370.4	78.9	370.4	370.4	108.3	370.4	370.4	129.9
$\tau=0.25$	127.5	101.0	25.4	175.7	154.4	49.0	213.2	195.6	72.3
$\tau=0.5$	37.4	20.0	6.6	57.2	36.7	13.8	78.6	57.1	23.9
$\tau=0.75$	17.2	7.6	3.1	23.1	11.0	4.9	31.5	17.1	8.1
$\tau=1.0$	10.2	4.3	2.1	12.4	5.1	2.6	15.7	6.8	3.6
$\tau=1.5$	5.1	2.2	1.4	5.6	2.4	1.4	6.3	2.6	1.6
$\tau=2.0$	3.2	1.6	1.0	3.5	1.6	1.1	3.7	1.6	1.1
$\tau=2.5$	2.3	1.3	0.8	2.4	1.3	0.9	2.5	1.3	0.9
$\tau=3.0$	1.8	1.1	0.6	1.9	1.1	0.7	1.9	1.1	0.7
$\tau=3.5$	1.4	1.1	0.4	1.5	1.1	0.4	1.5	1.1	0.5
$\tau=4.0$	1.2	1.0	0.2	1.3	1.0	0.3	1.3	1.0	0.3
$\tau=4.5$	1.1	1.0	0.1	1.1	1.0	0.1	1.2	1.0	0.1
$\tau=5.0$	1.0	1.0	0.0	1.1	1.0	0.1	1.1	1.0	0.1

the Shewhart procedure. And we also found that ANSW and ASWR decrease as λ decreases in VSI EWMA procedure.

The optimal selection of λ depends on the size of the shift in the mean vector to be dected quickly. And, it may be possible to improve the performance of the EWMA chart based on accumulate-combine approach at selected off-target conditions with alternate choices of λ. We also found that VSI EWMA chart based on accumulate-combine approach appears to be a good control charting device for detecting variety of shifts in the mean vector of multivariate normal process.

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