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Estimation for the Extreme Value Distribution Based on Multiply Type-II Censored Samples[1]

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Abstract

We derive the approximate maximum likelihood estimators of the scale parameter and location parameter of the extreme value distribution based on multiply Type-II censored samples. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

Keywords : Approximate maximum likelihood estimator, extreme value distribution, location and scale parameters, mean squared error, multiply Type-II censored sample

1. Introduction

A random variable X is said to have a extreme value distribution with the probability density function (pdf)

$$f(x;\theta,\sigma)=(1/\sigma)e^{(x-\theta)/\sigma}\exp\{-e^{(x-\theta)/\sigma}\},\ -\infty<x<\infty,\ -\infty<\theta<\infty,\ \sigma>0 \quad (1.1)$$

and the cumulative distribution function (cdf)

$$F(x;\theta,\sigma)=1-\exp\{-e^{(x-\theta)/\sigma}\},\quad -\infty<x<\infty,\ -\infty<\theta<\infty,\ \sigma>0 \quad (1.2)$$

where θ and σ are the location and scale parameters, respectively.

The extreme value distribution has been extensively used to model natural phenomena such as rainfall and floods, and also in modeling lifetimes and material strengths. The maximum likelihood estimation for the parameters of the extreme value distribution leads to likelihood equations that have to be solved numerically,

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even when the complete sample is available.

Multiply Type-II censored sampling arises in a life-testing experiment whenever the experimenter does not observe the failure times of some units placed on a life-test. Another situation where multiply censored samples arise naturally is when some units failed between two points of observation with exact times of failure of these units unobserved.

Balakrishnan and Varadan (1991) obtained the approximate maximum likelihood estimators (AMLEs) of the location and scale parameters in the extreme value distribution with censoring. Balakrishnan et al. (1995) derived the estimators for the location and scale parameters of the extreme value distribution under multiply Type-II censoring. Fei et al. (1995) studied the Weibull distribution and the extreme-value distribution under multiply Type-II censoring. They compared the mean squared errors of the maximum likelihood estimators, AMLEs, and best linear unbiased estimators of the parameters in the extreme value distribution. Kang (1996) obtained the AMLE for the scale parameter of the double exponential distribution based on Type-II censored samples and he showed that the proposed estimator is generally more efficient than the best linear unbiased estimator and the optimum unbiased absolute estimator. Kang (2003) proposed the AMLEs of the location and the scale parameters of the two-parameter exponential distribution with multiply Type-II censoring.

Recently, Balakrishnan et al. (2004) discussed point and interval estimation for the extreme value distribution under progressively Type-II censoring. Kang et al. (2004) considered the problem of estimating the scale parameter of the Weibull distribution based on multiply Type-II censored samples. They also proposed two estimators by using the approximate maximum likelihood estimation method for Weibull and extreme value distributions. Kang and Park (2005) derived the AMLEs of the scale parameter of the half-logistic distribution based on multiply Type-II censored samples.

In this paper, we derive the AMLEs of the scale parameter σ and the location parameter θ based on multiply Type-II censored sample. We also compare the proposed estimators in the sense of the mean squared error (MSE) for various censored samples.

2. Approximate Maximum Likelihood Estimators

Let us assume that the following multiply Type-II censored sample from a sample of size n is

$$X_{a_1:n} < X_{a_2:n} < \cdots < X_{a_s:n} \tag{2.1}$$

where $1 \le a_1 < a_2 < \cdots < a_s \le n$.

$$a_0 = 0,\ a_{s+1} = n+1,\ F(x_{a_0:n}) = 0,\ F(x_{a_{s+1}:n}) = 1. \tag{2.2}$$

The likelihood function based on the multiply Type-II censored sample (2.1) can be written as

$$L = n! \prod_{j=1}^{s} f(x_{a_j:n}) \prod_{j=1}^{s+1} \frac{[F(x_{a_j:n}) - F(x_{a_{j-1}:n})]^{a_j - a_{j-1} - 1}}{(a_j - a_{j-1} - 1)!}. \tag{2.3}$$

The random variable $Z_{i:n} = (X_{i:n} - \theta)/\sigma$ then has a standard extreme value distribution with pdf and cdf;

$$f(z) = e^z \exp\{-e^z\},\ F(z) = 1 - \exp\{-e^z\}, \quad -\infty < z < \infty.$$

From the equation (2.3), the likelihood equations for θ and σ are obtained as

$$\begin{aligned}\frac{\partial \ln L}{\partial \theta} &= -\frac{1}{\sigma}\Bigg[(a_1 - 1)\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})} - (n - a_s)\frac{f(Z_{a_s:n})}{1 - F(Z_{a_s:n})} \\ &\quad + \sum_{j=1}^{s} \frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})} + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\frac{f(Z_{a_j:n}) - f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\Bigg] \\ &= 0\end{aligned} \tag{2.4}$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma} &= -\frac{1}{\sigma}\Bigg[s + (a_1 - 1)\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})} Z_{a_1:n} \\ &\quad - (n - a_s)\frac{f(Z_{a_s:n})}{1 - F(Z_{a_s:n})} Z_{a_s:n} + \sum_{j=1}^{s} \frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})} Z_{a_j:n} \\ &\quad + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\frac{f(Z_{a_j:n})Z_{a_j:n} - f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\Bigg] \\ &= 0.\end{aligned} \tag{2.5}$$

The equations (2.4) and (2.5) do not admit explicit solutions for θ an σ. Let

$$\xi_i = F^{-1}(p_i) = \ln[-\ln q_i],\ \text{where}\ p_i = \frac{i}{n+1},\ q_i = 1 - p_i.$$

First, we may expand the following functions in Taylor series around the points ξ_{a_1}, ξ_{a_s}, ξ_{a_j} and $(\xi_{a_j}, \xi_{a_{j-1}})$, respectively.

$$\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})} Z_{a_1:n} \simeq \alpha_1 + \beta_1 Z_{a_1:n} \tag{2.6}$$

$$\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})} Z_{a_s:n} \simeq \varkappa_1 + \delta_1 Z_{a_s:n} \tag{2.7}$$

$$\frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})} Z_{a_j:n} \simeq \varkappa_j + \delta_j Z_{a_j:n} \tag{2.8}$$

$$\frac{f(Z_{a_j:n})Z_{a_j:n} - f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \simeq \alpha_j + \beta_j Z_{a_j:n} + \gamma_j Z_{a_{j-1}:n} \tag{2.9}$$

where

$$\alpha_1 = \frac{f(\xi_{a_1})}{p_{a_1}} \xi_{a_1}{}^2 \left[\frac{f(\xi_{a_1})}{p_{a_1}} - \{1 - e^{\xi_{a_1}}\} \right]$$

$$\beta_1 = \frac{f(\xi_{a_1})}{p_{a_1}} \left[1 + \{1 - e^{\xi_{a_1}}\} \xi_{a_1} - \frac{f(\xi_{a_1})}{p_{a_1}} \xi_{a_1} \right]$$

$$\varkappa_1 = -\frac{f(\xi_{a_s})}{q_{a_s}} \xi_{a_s}{}^2 \left[\frac{f(\xi_{a_s})}{q_{a_s}} + \{1 - e^{\xi_{a_s}}\} \right]$$

$$\delta_1 = \frac{f(\xi_{a_s})}{q_{a_s}} \left[1 + \{1 - e^{\xi_{a_s}}\} \xi_{a_s} + \frac{f(\xi_{a_s})}{q_{a_s}} \xi_{a_s} \right]$$

$$\varkappa_j = e^{\xi_{a_j}} \xi_{a_j}{}^2$$

$$\delta_j = 1 - e^{\xi_{a_j}} \xi_{a_j} - e^{\xi_{a_j}}$$

$$\alpha_j = -\frac{\xi_{a_j}{}^2 f'(\xi_{a_j}) - \xi_{a_{j-1}}{}^2 f'(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} + \left[\frac{\xi_{a_j} f(\xi_{a_j}) - \xi_{a_{j-1}} f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right]^2$$

$$\beta_j = \frac{f(\xi_{a_j})}{p_{a_j} - p_{a_{j-1}}} \left[1 + \{1 - e^{\xi_{a_j}}\} \xi_{a_j} - \frac{\xi_{a_j} f(\xi_{a_j}) - \xi_{a_{j-1}} f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right]$$

$$\gamma_j = -\frac{f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \left[1 + \{1 - e^{\xi_{a_{j-1}}}\} \xi_{a_{j-1}} - \frac{\xi_{a_j} f(\xi_{a_j}) - \xi_{a_{j-1}} f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right].$$

By substituting the equations (2.6), (2.7), (2.8), and (2.9) into the equation (2.5), we can derive an estimator of σ as follows;

$$\widehat{\sigma_1} = \frac{-B_1 + C_1 \widehat{\theta}}{A_1} \tag{2.10}$$

where

$$A_1 = s + (a_1 - 1)\,\alpha_1 - (n - a_s)\varkappa_1 + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)\alpha_j + \sum_{j=1}^{s} \varkappa_j$$

$$B_1 = (a_1-1)\beta_1 X_{a_1:n} - (n-a_s)\delta_1 X_{a_s:n} + \sum_{j=1}^{s}\delta_j X_{a_j:n} - \sum_{j=2}^{s}(a_j - a_{j-1}-1)(\beta_j X_{a_j:n} + \gamma_j X_{a_{j-1}:n})$$

$$C_1 = (a_1-1)\beta_1 - (n-a_s)\delta_1 + \sum_{j=2}^{s}(a_j - a_{j-1}-1)(\beta_j+\gamma_j) + \sum_{j=1}^{s}\delta_j .$$

Second, we can also expand the following functions in Taylor series;

$$\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})} \simeq \alpha_2 + \beta_2 Z_{a_1:n} \tag{2.11}$$

$$\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})} \simeq \kappa_2 + \delta_2 Z_{a_s:n} \tag{2.12}$$

$$\frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})} \simeq \kappa_{2j} + \delta_{2j} Z_{a_j:n} \tag{2.13}$$

where

$$\alpha_2 = \frac{f(\xi_{a_1})}{p_{a_1}}\left[1 - \{1 - e^{\xi_{a_1}}\}\xi_{a_1} + \frac{f(\xi_{a_1})}{p_{a_1}}\xi_{a_1}\right]$$

$$\beta_2 = \frac{f(\xi_{a_1})}{p_{a_1}}\left[\{1 - e^{\xi_{a_1}}\} - \frac{f(\xi_{a_1})}{p_{a_1}}\right]$$

$$\kappa_2 = \frac{f(\xi_{a_s})}{q_{a_s}}\left[1 - \{1 - e^{\xi_{a_s}}\}\xi_{a_s} - \frac{f(\xi_{a_s})}{q_{a_s}}\xi_{a_s}\right]$$

$$\delta_2 = \frac{f(\xi_{a_s})}{q_{a_s}}\left[\{1 - e^{\xi_{a_s}}\} + \frac{f(\xi_{a_s})}{q_{a_s}}\right]$$

$$\kappa_{2j} = 1 + e^{\xi_{a_j}}\xi_{a_j} - e^{\xi_{a_j}}$$

$$\delta_{2j} = -e^{\xi_{a_j}}$$

By substituting the equations (2.9), (2.11), (2.12), and (2.13) into the equation (2.5), we can derive an estimator of σ as follows;

$$\widehat{\sigma_2} = \frac{-B_2 + \sqrt{B_2{}^2 - 4A_2C_2}}{2A_2} \tag{2.14}$$

where

$$A_2 = s + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\alpha_j$$

$$B_2 = (a_1 - 1)\alpha_2 X_{a_1:n} - (n - a_s)\varkappa_2 X_{a_s:n} + \sum_{j=1}^{s}\varkappa_{2j} X_{a_j:n}$$
$$+ \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\beta_j X_{a_j:n} + \gamma_j X_{a_{j-1}:n})$$
$$- \left[(a_1 - 1)\alpha_2 - (n - a_s)\varkappa_2 + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\beta_j + \gamma_j) + \sum_{j=1}^{s}\varkappa_{2j} \right]\widehat{\theta}$$

$$C_2 = (a_1 - 1)\beta_2(X_{a_1:n} - \widehat{\theta})^2 - (n - a_s)\delta_2(X_{a_s:n} - \widehat{\theta})^2 + \sum_{j=1}^{s}\delta_{2j}(X_{a_j:n} - \widehat{\theta})^2.$$

Third, we can expand the following functions in Taylor series;

$$\frac{f(Z_{a_j:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \simeq \alpha_{1j} + \beta_{1j} Z_{a_j:n} + \gamma_{1j} Z_{a_{j-1}:n} \tag{2.15}$$

and

$$\frac{f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \simeq \alpha_{2j} + \beta_{2j} Z_{a_j:n} + \gamma_{2j} Z_{a_{j-1}:n} \tag{2.16}$$

where

$$\alpha_{1j} = \frac{f(\xi_{a_j})}{p_{a_j} - p_{a_{j-1}}}\left[1 - \{1 - e^{\xi_{a_j}}\}\xi_{a_j} + \frac{\xi_{a_j} f(\xi_{a_j}) - \xi_{a_{j-1}} f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right]$$

$$\beta_{1j} = \frac{f(\xi_{a_j})}{p_{a_j} - p_{a_{j-1}}}\left[\{1 - e^{\xi_{a_j}}\} - \frac{f(\xi_{a_j})}{p_{a_j} - p_{a_{j-1}}} \right]$$

$$\gamma_{1j} = \frac{f(\xi_{a_j}) f(\xi_{a_{j-1}})}{[p_{a_j} - p_{a_{j-1}}]^2}$$

$$\alpha_{2j} = \frac{f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}}\left[1 - \{1 - e^{\xi_{a_{j-1}}}\}\xi_{a_{j-1}} + \frac{\xi_{a_j} f(\xi_{a_j}) - \xi_{a_{j-1}} f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right]$$

$$\beta_{2j} = -\frac{f(\xi_{a_j}) f(\xi_{a_{j-1}})}{[p_{a_j} - p_{a_{j-1}}]^2} = -\gamma_{1j}$$

$$\gamma_{2j} = \frac{f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}}\left[\{1 - e^{\xi_{a_{j-1}}}\} + \frac{f(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}} \right].$$

By substituting the equations (2.6), (2.7), (2.8), (2.15), and (2.16) into the equation (2.5), we can derive an estimator of σ as follows;

$$\widehat{\sigma}_3 = \frac{-B_3 + \sqrt{B_3^{\,2} - 4A_3C_3}}{2A_3} \tag{2.17}$$

where

$$
\begin{aligned}
A_3 &= s+(a_1-1)\,\alpha_1 - (n-a_s)\kappa_1 + \sum_{j=2}^{s}\kappa_j \\
B_3 &= (a_1-1)\,\beta_1 X_{a_1:n} - (n-a_s)\,\delta_1 X_{a_s:n} + \sum_{j=1}^{s}\delta_j X_{a_j:n} \\
&\quad + \sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{1j}X_{a_j:n}-\alpha_{2j}X_{a_{j-1}:n}) \\
&\quad - \left[(a_1-1)\,\beta_1-(n-a_s)\,\delta_1+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{1j}-\alpha_{2j})+\sum_{j=1}^{s}\delta_j\right]\widehat{\theta} \\
C_3 &= \sum_{j=1}^{s}(a_j-a_{j-1}-1)\left\{\beta_{1j}(X_{a_j:n}-\widehat{\theta})^2+2\,\gamma_{1j}(X_{a_j:n}-\widehat{\theta})(X_{a_{j-1}:n}-\widehat{\theta}) \right. \\
&\quad \left. -\gamma_{2j}(X_{a_{j-1}:n}-\widehat{\theta})^2\right\}.
\end{aligned}
$$

Last, by substituting the equations (2.11), (2.12), (2.13), (2.15), and (2.16) into the equation (2.5), we can derive an estimator of σ as follows;

$$
\widehat{\sigma_4} = \frac{-B_4+\sqrt{B_4^{\,2}-4sC_4}}{2s} \tag{2.18}
$$

where

$$
\begin{aligned}
B_4 &= (a_1-1)\,\alpha_2 X_{a_1:n} - (n-a_s)\kappa_2 X_{a_s:n} + \sum_{j=1}^{s}\kappa_{2j}X_{a_j:n} \\
&\quad + \sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{1j}X_{a_j:n}-\alpha_{2j}X_{a_{j-1}:n}) \\
&\quad - \left[(a_1-1)\,\alpha_2-(n-a_s)\kappa_2+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{1j}-\alpha_{2j})+\sum_{j=1}^{s}\kappa_{2j}\right]\widehat{\theta} \\
C_4 &= C_2 + C_3.
\end{aligned}
$$

From the equations (2.10), (2.14), (2.17), and ((2.18), we can show that $\widehat{\sigma_1}$ and $\widehat{\sigma_3}$ are same and $\widehat{\sigma_2}$ and $\widehat{\sigma_4}$ are same if $a_j-a_{j-1}=0$.

Now, we consider the estimator of the location parameter θ. From the equations (2.15) and (2.16), we expand the function as follows;

$$
\frac{f(Z_{a_j:n})-f(Z_{a_{j-1}:n})}{F(Z_{a_j:n})-F(Z_{a_{j-1}:n})} \simeq \alpha_{3j}+\beta_{3j}Z_{a_j:n}+\gamma_{3j}Z_{a_j:n} \tag{2.19}
$$

where

$$
\alpha_{3j}=\alpha_{1j}-\alpha_{2j},\quad \beta_{3j}=\beta_{1j}-\beta_{2j},\quad \gamma_{3j}=\gamma_{1j}-\gamma_{2j}.
$$

By substituting the equations (2.11), (2.12), (2.13), and (2.19) into the equation

(2.4), we can derive an estimator of θ as follows;

$$\hat{\theta} = \frac{E}{D} \tag{2.20}$$

where

$$\begin{aligned}
D &= A_0 C_1 - A_1 C_0 \\
E &= A_1 B_0 - A_0 B_1 \\
A_0 &= (a_1 - 1)\alpha_2 - (n - a_s)\kappa_2 + \sum_{j=1}^{s} \kappa_{2j} + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)\alpha_{3j} \\
B_0 &= (a_1 - 1)\beta_2 X_{a_1:n} - (n - a_s)\delta_2 X_{a_s:n} + \sum_{j=1}^{s} \delta_{2j} X_{a_j:n} \\
&\quad + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)(\beta_{3j} X_{a_j:n} + \gamma_{3j} X_{a_{j-1}:n}) \\
C_0 &= (a_1 - 1)\beta_2 - (n - a_s)\delta_2 + \sum_{j=1}^{s} \delta_{2j} + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)(\beta_{3j} + \gamma_{3j}).
\end{aligned}$$

From the proposed estimators, the mean squared errors of these estimators are simulated by Monte Carlo method for sample sizes $n = 20$ and 50, and various choices of censoring. The simulation procedure is repeated 10,000 times in multiply Type-II censored samples. These values are given in Table 1.

From Table 1, we have the following results;

(i) the MSEs of the estimators $\hat{\sigma}_1$ and $\hat{\sigma}_3$ are same in the most cases and the estimators $\hat{\sigma}_1$ and $\hat{\sigma}_3$ are more efficient than the other estimators in the sense of MSE.

(ii) The MSE of all the estimators generally increases as k increases and the sample size n decreases.

(iii) The MSEs of the estimators for the right censored cases are larger than the left censored cases when the number of censoring k is fixed.

Table 1. The relative mean squared errors for the estimators of the location parameter θ and scale parameter σ.

n	k	a_j	MSE				
			$\widehat{\theta}$	$\widehat{\sigma_1}$	$\widehat{\sigma_2}$	$\widehat{\sigma_3}$	$\widehat{\sigma_4}$
20	0	1～20	0.057115	0.031391	0.046821	0.031391	0.046821
	1	1～19	0.058348	0.034809	0.055924	0.034809	0.055924
		2～20	0.057169	0.032829	0.048613	0.032829	0.048613
	2	1～18	0.059900	0.038384	0.064683	0.038384	0.064683
		3～20	0.057367	0.034308	0.051145	0.034308	0.051145
		2～19	0.058367	0.036491	0.058319	0.036491	0.058319
	3	1～17	0.062853	0.042562	0.074587	0.042562	0.074587
		4～20	0.057508	0.036096	0.053480	0.036096	0.053480
		2～18	0.059862	0.040475	0.067818	0.040475	0.067818
		3～19	0.058517	0.038258	0.061740	0.038258	0.061740
	4	2～17	0.062763	0.045116	0.078522	0.045116	0.078522
		4～19	0.058602	0.040417	0.065070	0.040417	0.065070
		3～18	0.059934	0.042712	0.072277	0.042712	0.072277
		2～4 7～14 16～20	0.057271	0.032835	0.044199	0.030777	0.056398
	5	3～17	0.062746	0.047960	0.084211	0.047960	0.084211
		4～18	0.059954	0.045342	0.076727	0.045342	0.076727
		2～6 10～19	0.058611	0.036623	0.055118	0.034847	0.065181
	6	4～17	0.062679	0.051258	0.090078	0.051258	0.090078
		1 2 6～9 12～15 17～20	0.057520	0.031733	0.040455	0.032622	0.062574
50	0	1～50	0.022142	0.012412	0.014231	0.012412	0.014231
	1	1～49	0.022251	0.013055	0.015514	0.013055	0.015514
		2～50	0.022177	0.012615	0.014437	0.012615	0.014437
	2	1～48	0.022413	0.013642	0.016610	0.013642	0.016610
		3～50	0.022202	0.012845	0.014670	0.012845	0.014670
		2～49	0.022286	0.013266	0.015744	0.013266	0.015744
	3	1～47	0.022600	0.014241	0.017719	0.014241	0.017719
		4～50	0.022214	0.013036	0.014905	0.013036	0.014905
		2～48	0.022444	0.013870	0.016866	0.013870	0.016866
		3～49	0.022309	0.013515	0.016012	0.013515	0.016012
	4	2～47	0.022627	0.014490	0.018010	0.014490	0.018010
		4～49	0.022319	0.013721	0.016277	0.013721	0.016277
		3～48	0.022464	0.014138	0.017164	0.014138	0.017164
		2～4 7～14 16～50	0.022193	0.012624	0.014344	0.012521	0.015988
	5	3～47	0.022645	0.014778	0.018349	0.014778	0.018349
		4～48	0.022473	0.014358	0.017464	0.014358	0.017464
		2～6 10～19 21～50	0.022193	0.012637	0.014260	0.012470	0.015980
	6	4～47	0.022651	0.015020	0.018691	0.015020	0.018691
		1 2 6～9 12～15 17～50	0.022192	0.012455	0.014160	0.012686	0.017233

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