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Estimation for the Double Exponential Distribution Based on Type-II Censored Samples

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Abstract

In this paper, we derive the approximate maximum likelihood estimators of the scale parameter and location parameter of the double exponential distribution based on Type-II censored samples. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

Keywords : Approximate maximum likelihood estimator, Double exponential distribution, Type-II censored sample

1. Introduction

Consider the double exponential or Laplace distribution with the probability density function (pdf)

$$f(x;\theta,\sigma)=\frac{1}{2\sigma}e^{-|x-\theta|/\sigma},\qquad -\infty<x<\infty\,,\qquad \sigma>0\,, \tag{1.1}$$

and the cumulative distribution function (cdf)

$$F(x;\theta,\sigma)=\begin{cases}\frac{1}{2}\exp\left[-\frac{\theta-x}{\sigma}\right], & x\le\theta,\\ 1-\frac{1}{2}\exp\left[-\frac{x-\theta}{\sigma}\right], & x\ge\theta.\end{cases} \tag{1.2}$$

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The double exponential distribution is used to model symmetric data with long tails. This distribution also arises directly when a random variable occurs as the difference of two variables with exponential distributions with the same scale.

Govindarajulu (1966) gave the coefficients of the best linear unbiased estimators for the location and the scale parameters in the double exponential distribution from complete and symmetric censored samples. Raghunandanan and Srinivasan (1971) presented some simplified estimators of the location and the scale parameters of a double exponential distribution. Bain and Engelhardt (1973) discussed the usefulness of the double exponential distribution as a model for statistical studies, and obtained the confidence intervals based on the maximum likelihood estimators for the location and the scale parameters of a double exponential distribution, and Kappenman (1975) obtained the conditional confidence intervals for the parameters of a double exponential distribution. Shyu and Owen (1986a, 1986b) obtained the tolerance intervals for the two-parameter double exponential distribution.

The approximate maximum likelihood estimation method was first developed by Balakrishnan (1989) for the purpose of providing the explicit estimators of the scale parameter in the Rayleigh distribution. Kang (1996) obtained the approximate maximum likelihood estimator (AMLE) for the scale parameter of the double exponential distribution based on Type-II censored samples and he showed that the proposed estimator is generally more efficient than the best linear unbiased estimator and the optimum unbiased absolute estimator. Kang et al. (1997) proposed the AMLE of the location and the scale parameters of the two-parameter exponential distribution with Type-II censoring. Childs and Balakrishnan (1996, 2000) developed the conditional inference procedures for the Laplace distribution based on conventionally Type-II right censored samples and the progressive Type-II censored samples.

In this paper, we derive the AMLEs of the scale parameter σ and the location parameter θ based on Type-II censored sample. We also compare the proposed estimators in the sense of the mean squared error (MSE) for various censored samples.

2. Approximate Maximum Likelihood Estimators

Let us assume that the following Type-II censored sample from a sample of size n is

$$X_{r+1:n} \le X_{r+2:n} \le \cdots \le X_{n-s:n} \tag{2.1}$$

where the first r and the last s observations are censored.

The likelihood function based on the Type-II censored sample in (2.1) is given by

$$L=\frac{n!}{r!s!}\{F(X_{r+1:n})\}^r\{1-F(X_{n-s:n})\}^s\prod_{i=r+1}^{n-s}f(X_{i:n}). \tag{2.2}$$

By putting $Z_{i:n}=(X_{i:n}-\theta)/\sigma$, the likelihood function can be rewritten as

$$L=\frac{n!}{r!s!}\sigma^{-A}\{F(Z_{r+1:n})\}^r\{1-F(Z_{n-s:n})\}^s\prod_{i=r+1}^{n-s}f(Z_{i:n}) \tag{2.3}$$

where $A=n-r-s$ is the size of the censored sample in (2.1), $f(z)$ and $F(z)$ are the pdf and the cdf of the standard double exponential distribution, respectively.

We have the log-likelihood function as follows;

$$\ln L=\ln\left(\frac{n!}{r!s!}\right)-A\ln\sigma+r\ln\{F(Z_{r+1:n})\}+s\ln\{1-F(Z_{n-s:n})\}+\sum_{i=r+1}^{n-s}\ln f(Z_{i:n}). \tag{2.4}$$

By realizing that $\frac{f'(z)}{f(z)}=-\frac{|z|}{z}$, $z\neq 0$, on differentiating with respect to σ in turn and equation to zero, we obtain the estimating equation as

$$\frac{\partial\ln L}{\partial\sigma}=-\frac{1}{\sigma}\left[A+r\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})}Z_{r+1:n}-s\frac{f(Z_{n-s:n})}{1-F(Z_{n-s:n})}Z_{n-s:n}-\sum_{i=r+1}^{n-s}|Z_{i:n}|\right] \tag{2.5}$$
$$=0.$$

Equation (2.5) does not admit an explicit solution for σ. But we can expand the functions

$$\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})} \text{ and } \frac{f(Z_{n-s:n})}{1-F(Z_{n-s:n})}$$

in Taylor series around the points

$$F^{-1}(p_{r+1})=\begin{cases}-\ln\{2(1-p_{r+1})\}, & p_{r+1}\geq 0.5\\ \ln(2p_{r+1}), & p_{r+1}<0.5\end{cases} \tag{2.6}$$

and

$$F^{-1}(p_{n-s})=\begin{cases}-\ln\{2(1-p_{n-s})\}, & p_{n-s}\geq 0.5\\ \ln(2p_{n-s}), & p_{n-s}<0.5\end{cases} \tag{2.7}$$

where $p_i=\frac{i}{n+1}$ and $q_i=1-p_i$, respectively.

We also can expand the functions

$$\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})}Z_{r+1:n} \text{ and } \frac{f(Z_{n-s:n})}{1-F(Z_{n-s:n})}Z_{n-s:n}$$

in Taylor series around the points (2.6) and (2.7), respectively. So we consider three cases as $z_{r+1:n} \geq 0$, $z_{r+1:n} < 0 < z_{n-s:n}$, and $z_{n-s:n} \leq 0$.

Balakrishnan and Cutler (1994) derived explicitly the maximum likelihood estimator of the parameter θ based on symmetrically Type-II censored samples as follows;

If $X_{r+1:n} \leq \theta \leq X_{n-r:n}$

$$\hat{\theta} = \begin{cases} X_{(n+1)/2:n}, & n \text{ is odd} \\ \text{any value in } [X_{n/2:n}, X_{(n/2)+1:n}], & n \text{ is even} \end{cases}$$

and if $\theta < X_{r+1:n}$, the likelihood function in the equation (2.3) is a monotonically increasing function of θ, thus $\hat{\theta} = X_{r+1:n}$ and if $\theta > X_{n-r:n}$, the likelihood function in the equation (2.3) is a monotonically decreasing function of θ, thus $\hat{\theta} = X_{n-r:n}$.

So we consider the maximum likelihood estimator of the parameter θ as follows ;

If $X_{r+1:n} \leq \theta \leq X_{n-s:n}$

$$\hat{\theta} = \begin{cases} X_{(n+1)/2:n}, & n \text{ is odd} \\ [X_{n/2:n} + X_{(n/2)+1:n}]/2, & n \text{ is even} \end{cases}$$

and if $\theta < X_{r+1:n}$, $\hat{\theta} = X_{r+1:n}$ and if $\theta > X_{n-s:n}$, $\hat{\theta} = X_{n-s:n}$.

Case 1 : $z_{r+1:n} \geq 0$

Since $F(Z_{n-s:n}) = 1 - f(Z_{n-s:n})$, we have to expand the functions $\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})}$ or $\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})} Z_{r+1:n}$.

First, the expansion of the function $\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})}$ is required. Therefore, we can approximate this function by

$$\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})} \simeq \alpha_1 + \beta_1 Z_{r+1:n} \tag{2.8}$$

where

$$\alpha_1 = \begin{cases} 1, & p_{r+1} < 0.5 \\ \frac{q_{r+1}}{p_{r+1}} - \frac{q_{r+1}}{(p_{r+1})^2} \ln(2q_{r+1}), & p_{r+1} \geq 0.5 \end{cases}$$

$$\beta_1 = \begin{cases} 0, & p_{r+1} < 0.5 \\ -\dfrac{q_{r+1}}{(p_{r+1})^2}, & p_{r+1} \geq 0.5. \end{cases}$$

By substituting the equation (2.8) into the equation (2.5), we obtain the approximate likelihood equation of equation (2.5) as

$$\frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^*}{\partial \sigma} = -\frac{1}{\sigma}\left[A + r(\alpha_1 + \beta_1 Z_{r+1:n})Z_{r+1:n} - sZ_{n-s:n} - \sum_{i=r+1}^{n-s} |Z_{i:n}|\right] = 0. \quad (2.9)$$

Equation (2.9) is quadratic in σ as follows;

$$A\sigma^2 + B_1\sigma + C_1 = 0, \quad (2.10)$$

where

$$B_1 = r\alpha_1(X_{r+1:n} - \widehat{\theta}) - s(X_{n-s:n} - \widehat{\theta}) - \sum_{i=r+1}^{n-s} |X_{i:n} - \widehat{\theta}|$$

$$C_1 = r\beta_1(X_{r+1:n} - \widehat{\theta})^2.$$

Upon solving equation (2.10) for σ, we derive the AMLE of σ as

$$\widehat{\sigma}_1 = \frac{-B_1 + \sqrt{B_1^2 - 4AC_1}}{2A}. \quad (2.11)$$

Secondly, the expansion of the function $\dfrac{f(Z_{r+1:n})}{F(Z_{r+1:n})} Z_{r+1:n}$ is required. Therefore, we can also approximate this function by

$$\frac{f(Z_{r+1:n})}{F(Z_{r+1:n})} Z_{r+1:n} \simeq \alpha_2 + \beta_2 Z_{r+1:n} \quad (2.12)$$

where

$$\alpha_2 = \begin{cases} 0, & p_{r+1} < 0.5 \\ \left[\dfrac{\ln(2q_{r+1})}{p_{r+1}}\right]^2 q_{r+1}, & p_{r+1} \geq 0.5 \end{cases}$$

$$\beta_2 = \begin{cases} 1, & p_{r+1} < 0.5 \\ \dfrac{q_{r+1}[p_{r+1} + \ln(2q_{r+1})]}{(p_{r+1})^2}, & p_{r+1} \geq 0.5. \end{cases}$$

By substituting the equation (2.12) into the equation (2.5), we obtain the approximate likelihood equation of equation (2.5) as

$$\frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^*}{\partial \sigma} = -\frac{1}{\sigma}\left[A + r(\alpha_2 + \beta_2 Z_{r+1:n}) - sZ_{n-s:n} - \sum_{i=r+1}^{n-s} |Z_{i:n}|\right] = 0. \quad (2.13)$$

From the equation (2.13), we can derive more simple estimator of σ as

$$\widehat{\sigma}_2 = \frac{B_2}{A_2} \quad (2.14)$$

where

$$A_2 = A + r\alpha_2$$

$$B_2 = -r\beta_2(X_{r+1:n} - \widehat{\theta}) + s(X_{n-s:n} - \widehat{\theta}) + \sum_{i=r+1}^{n-s} |X_{i:n} - \widehat{\theta}|.$$

Case 2 : $z_{r+1:n} < 0 < z_{n-s:n}$

Since $F(Z_{r+1:n}) = f(Z_{r+1:n})$ and $F(Z_{n-s:n}) = 1 - f(Z_{n-s:n})$ in this case, so we obtain the likelihood equation as follows;

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{1}{\sigma}\left[A + rZ_{r+1:n} - sZ_{n-s:n} - \sum_{i=r+1}^{n-s} |Z_{i:n}|\right] = 0. \quad (2.15)$$

Hence, in this case, we can obtain the exact maximum likelihood estimator of σ as follows;

$$\widehat{\sigma} = \frac{-r(X_{r+1:n} - \widehat{\theta}) + s(X_{n-s:n} - \widehat{\theta}) + \sum_{i=r+1}^{n-s} |X_{i:n} - \widehat{\theta}|}{A}. \quad (2.16)$$

Case 3 : $z_{n-s:n} \leq 0$

Since $F(Z_{r+1:n}) = f(Z_{r+1:n})$, we have to expand the functions $\dfrac{f(Z_{n-s:n})}{1 - F(Z_{n-s:n})}$ or $\dfrac{f(Z_{n-s:n})}{1 - F(Z_{n-s:n})} Z_{n-s:n}$.

First, the expansion of the function $\dfrac{f(Z_{n-s:n})}{1 - F(Z_{n-s:n})}$ is required. Therefore, we can approximate this function by

$$\frac{f(Z_{n-s:n})}{1 - F(Z_{n-s:n})} \simeq \gamma_1 + \delta_1 Z_{n-s:n} \quad (2.17)$$

where

$$\gamma_1 = \begin{cases} 1, & p_{n-s} > 0.5 \\ \dfrac{p_{n-s}}{q_{n-s}} - \dfrac{p_{n-s}}{(q_{n-s})^2} \ln(2p_{n-s}), & p_{n-s} \leq 0.5 \end{cases}$$

$$\delta_1 = \begin{cases} 0, & p_{n-s} > 0.5 \\ \dfrac{p_{n-s}}{(q_{n-s})^2}, & p_{n-s} \le 0.5. \end{cases}$$

By substituting the equation (2.17) into the equation (2.5), we obtain the approximate likelihood equation of equation (2.5) as

$$\frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^*}{\partial \sigma} = -\frac{1}{\sigma}\left[A + rZ_{r+1:n} - s(\gamma_1 + \delta_1 Z_{n-s:n})Z_{n-s:n} - \sum_{i=r+1}^{n-s}|Z_{i:n}|\right] = 0. \tag{2.18}$$

Equation (2.18) is quadratic in σ as follows;

$$A\sigma^2 + D_1\sigma + E_1 = 0, \tag{2.19}$$

where

$$D_1 = r(X_{r+1:n} - \widehat{\theta}) - s\gamma_1(X_{n-s:n} - \widehat{\theta}) - \sum_{i=r+1}^{n-s}|X_{i:n} - \widehat{\theta}|$$

$$E_1 = -s\,\delta_1(X_{n-s:n} - \widehat{\theta})^2.$$

Upon solving the equation (2.19) for σ, we derive the AMLE of σ as

$$\widehat{\sigma_1} = \frac{-D_1 + \sqrt{D_1^2 - 4AE_1}}{2A}. \tag{2.20}$$

Secondly, the expansion of the function $\dfrac{f(Z_{n-s:n})}{1-F(Z_{n-s:n})}Z_{n-s:n}$ is required. Therefore, we can approximate this function by

$$\frac{f(Z_{n-s:n})}{1-F(Z_{n-s:n})}Z_{n-s:n} \simeq \gamma_2 + \delta_2 Z_{n-s:n} \tag{2.21}$$

where

$$\gamma_2 = \begin{cases} 0, & p_{n-s} > 0.5 \\ -\left[\dfrac{\ln(2p_{n-s})}{q_{n-s}}\right]^2 p_{n-s}, & p_{n-s} \le 0.5 \end{cases}$$

$$\beta_2 = \begin{cases} 1, & p_{n-s} > 0.5 \\ \dfrac{p_{n-s}[q_{n-s} + \ln(2p_{n-s})]}{(q_{n-s})^2}, & p_{n-s} \le 0.5. \end{cases}$$

By substituting the equation (2.21) into the equation (2.5), we obtain the approximate likelihood equation of equation (2.5) as

$$\frac{\partial \ln L}{\partial \sigma} \simeq \frac{\partial \ln L^*}{\partial \sigma} = -\frac{1}{\sigma}\left[A + rZ_{r+1:n} - s(\gamma_2 + \delta_2 Z_{n-s:n}) - \sum_{i=r+1}^{n-s} |Z_{i:n}|\right] = 0. \tag{2.22}$$

From the equation (2.22), we can derive more simple estimator of σ as

$$\widehat{\sigma_2} = \frac{D_2}{E_2} \tag{2.23}$$

where

$$E_2 = A - s\gamma_2$$

$$D_2 = -r(X_{r+1:n} - \widehat{\theta}) + s\delta_2(X_{n-s:n} - \widehat{\theta}) + \sum_{i=r+1}^{n-s} |X_{i:n} - \widehat{\theta}|.$$

From the equations (2.11) and (2.14) in case 1, the equation (2.16) in case 2, the equations (2.20) and (2.23) in case 3, we simulate the mean squared errors of these two estimators of σ in the double exponential distribution for sample size $n = 5, 6, 20, 50$. The simulation procedure is repeated 10,000 times in Type-II censored samples. These values are given in Table 1. From Table 1, the estimator $\widehat{\sigma_2}$ is more efficient than $\widehat{\sigma_1}$ in the sense of the mean squared error when the location parameter θ is known. But the estimator $\widehat{\sigma_1}$ is more efficient than $\widehat{\sigma_2}$ in the sense of the mean squared error when the location parameter θ is unknown. The estimator $\widehat{\sigma_2}$ is more simple than $\widehat{\sigma_1}$. The mean squared errors of all the estimators increase as r or s increases.

Table 1. The relative mean squared errors for the estimators of the scale parameter σ and the location parameter θ.

n	r	s	θ is known.		θ is unknown.		
			$\widehat{\sigma_1}$	$\widehat{\sigma_2}$	$\widehat{\theta}$	$\widehat{\sigma_1}$	$\widehat{\sigma_2}$
5	0	0	0.193714	0.193714	0.295003	0.181052	0.181052
	0	1	0.242616	0.242616	0.262666	0.250216	0.250216
	0	2	0.360291	0.297615	0.340980	0.371605	0.371605
	0	3	0.457265	0.357907	0.850382	0.722410	0.733948
	1	0	0.236289	0.236289	0.255658	0.242926	0.242926
	1	1	0.314070	0.314070	0.190435	0.300607	0.300607
	1	2	0.520739	0.416475	0.227199	0.522884	0.522884
	2	0	0.353766	0.293679	0.340856	0.367665	0.367665
	2	1	0.517632	0.411704	0.224290	0.508512	0.508512
	3	0	0.453048	0.349342	0.865680	0.715933	0.727056
6	0	0	0.163865	0.163865	0.238100	0.158069	0.158069
	0	1	0.195492	0.195492	0.252811	0.188257	0.188257
	0	2	0.235533	0.235533	0.261813	0.287991	0.287991
	0	3	0.337077	0.280101	0.400919	0.422488	0.422888
	1	0	0.194295	0.194295	0.264244	0.187204	0.187204
	1	1	0.240029	0.240029	0.162152	0.230340	0.230340
	1	2	0.303503	0.303503	0.171952	0.317522	0.317522
	2	0	0.235892	0.235892	0.271171	0.288633	0.288633
	2	1	0.306257	0.306257	0.179647	0.316058	0.316058
	3	0	0.352367	0.288195	0.419134	0.445962	0.446464

Table 1. (continued)

n	r	s	θ is known.		θ is unknown.		
			$\widehat{\sigma_1}$	$\widehat{\sigma_2}$	$\widehat{\theta}$	$\widehat{\sigma_1}$	$\widehat{\sigma_2}$
20	0	0	0.048856	0.048856	0.066063	0.048219	0.048219
	0	1	0.051668	0.051668	0.072470	0.050978	0.050978
	0	2	0.054175	0.054175	0.081989	0.053739	0.053739
	0	3	0.057431	0.057431	0.105028	0.057618	0.057618
	1	0	0.051490	0.051490	0.073149	0.050883	0.050883
	1	1	0.054585	0.054585	0.066027	0.053925	0.053925
	1	2	0.057467	0.057467	0.072400	0.056672	0.056672
	1	3	0.061129	0.061129	0.081369	0.060735	0.060735
	2	0	0.054488	0.054488	0.083403	0.054123	0.054123
	2	1	0.057870	0.057870	0.073110	0.057073	0.057073
	2	2	0.061173	0.061173	0.065961	0.060213	0.060213
	2	3	0.065417	0.065417	0.071853	0.064349	0.064349
	3	0	0.057565	0.057565	0.107013	0.057813	0.057813
	3	1	0.061314	0.061314	0.082905	0.060989	0.060989
	3	2	0.065080	0.065080	0.072621	0.064139	0.064139
	3	3	0.069939	0.069939	0.065169	0.068886	0.068886
	10	0	0.099104	0.095265	0.193914	0.189015	0.189028
	0	10	0.098776	0.093990	0.197226	0.192902	0.192934
	10	1	0.109771	0.105013	0.153682	0.178886	0.178901
	10	2	0.122036	0.116601	0.119342	0.174507	0.174529
	1	10	0.109567	0.104609	0.155196	0.181508	0.181539
	2	10	0.123450	0.117411	0.120263	0.177820	0.177839

Table 1. (continued)

n	r	s	θ is known.		θ is unknown.		
			$\widehat{\sigma_1}$	$\widehat{\sigma_2}$	$\widehat{\theta}$	$\widehat{\sigma_1}$	$\widehat{\sigma_2}$
50	0	0	0.020220	0.020220	0.024265	0.020111	0.020111
	0	1	0.020691	0.020691	0.025108	0.020579	0.020579
	0	2	0.021074	0.021074	0.026268	0.020985	0.020985
	0	3	0.021449	0.021449	0.029408	0.021390	0.021390
	1	0	0.020678	0.020678	0.025388	0.020566	0.020566
	1	1	0.021154	0.021154	0.024265	0.021038	0.021038
	1	2	0.021551	0.021551	0.025108	0.021434	0.021434
	1	3	0.021947	0.021947	0.026268	0.021856	0.021856
	2	0	0.021165	0.021165	0.026783	0.021074	0.021074
	2	1	0.021658	0.021658	0.025388	0.021540	0.021540
	2	2	0.022066	0.022066	0.024265	0.021947	0.021947
	2	3	0.022481	0.022481	0.025108	0.022363	0.022363
	3	0	0.021590	0.021590	0.030194	0.021530	0.021530
	3	1	0.022099	0.022099	0.026783	0.022008	0.022008
	3	2	0.022519	0.022519	0.025388	0.022401	0.022401
	3	3	0.022953	0.022953	0.024265	0.022837	0.022837
	25	0	0.040417	0.040018	0.186144	0.123897	0.123901
	25	10	0.066938	0.066079	0.051393	0.091535	0.091546
	10	25	0.067222	0.066748	0.050006	0.091289	0.091291
	30	0	0.046828	0.045022	0.123592	0.086499	0.093181

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