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Acceptance Sampling Plans in the Rayleigh Model

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Abstract

Assume that the life times of the units under test follow the Rayleigh distribution and the test is terminated at a pre assigned time. Acceptance sampling plans are developed for this situation. The minimum sample size necessary to ensure the specified average life are obtained and the operating characteristic values of the sampling plans and producer's risk are given. An example is given to illustrate the methodology.

Keywords : Acceptance sampling, Censored data, operating characteristic curve, Producer's risk, Rayleigh distribution

1. Introduction

The Rayleigh distribution is frequently employed by engineers and other scientists as a model for data resulting from investigations involving wave propagation, radiation and related inquiries as well as in the analysis of target error data. Some types of electrovacuum devices have the feature that they age rapidly with time even though they may have no manufacturing defects, the Rayliegh distribution is quite appropriate for modeling the lifetimes of such units as it possesses a linearly increasing hazard rate. Other uses and motivations for this distribution can be found in Johnson et al. (1994).

Inferential procedures in this distribution have been discussed by many authors, see Cohen and Whitten (1988) and Johnson et al. (1994). However, little attention has been paid to acceptance sampling based on life tests for this distribution. Acceptance sampling is one of the major fields in statistical quality control. It is used by the consumer to decide whether to accept or reject lots of products shipped from the producer. Acceptance sampling is often used when the inspection of the product is costly or destructive, more details are given by Duncan

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(1974) and Montegomery (2000).

The problem we are considering is that of finding the minimum sample size necessary to ensure a certain average life when the life test is terminated at a pre assigned time (t) and when the number of failures does not exceed a given acceptance number (c). The lot is accepted if the specified average life can be established with a pre assigned probability (P^*) specified by the consumer. This kind of life tests are dicussed by Sobel Tischendrof (1959) for the exponential model. Weibull model is considered by Goode and Kao (1961), while Gupta and Groll (1961) considered the Gamma model. Kantam and Rosaiah (1998) and Kantam et al. (2001) considered the half logistic and the log logistic distributions. Baklizi (2003) has considered the Pareto model. In section 2 we present the acceptance sampling plans and the operating characteristic. The results and a descriptive example are given in section 3. Discussion of the findings and suggestions for further research are given in the final section.

2. The Sampling Plans

The cumulative distribution function and the probability density function of the Rayleigh distribution are given respectively by

$$F(t,\sigma) = 1 - e^{-\frac{t^2}{2\sigma^2}}, \qquad t > 0, \sigma > 0. \tag{1}$$

$$f(t,\sigma) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}, \qquad t > 0, \sigma > 0. \tag{2}$$

A sampling plan consists of

n: The number of units on test.

c: An acceptance number such that if c or fewer failures occur during the test time (t), the lot is accepted.

t/σ_0 : where σ_0 is the specified average life.

The consumer risk is fixed such that it does not exceed $1 - P^*$. Assume that the lot is large enough to be considered infinite so that the theory of the binomial distribution is applied. What we want is the minimum sample size (n) such that

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \le 1 - P^* \tag{3}$$

where $p = F(t, \sigma_0)$. Notice that p is a function of t/σ_0. Thus the experiment needs only to specify this ratio. The minimum values of n satisfying inequality (3) were obtained for P^*=0.75, 0.9, 0.95, 0.99, and t/σ_0 =0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712. This choice is consistent with that of Gupta and Groll (1961) and Kantam and Rosaiah (1998, 2001).

The operating characteristic of the sampling plan ($n, c, t/\sigma_0$) gives the probability of accepting the lot. This probability is given by

$$L(p)=\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \tag{4}$$

where $p = F(t,\sigma)$ considered as a function of σ. Values of the operating characteristic as a function of σ/σ_0 for some selected sampling plans are given in table 2. The producer's risk is the probability of rejecting the lot when $\sigma \geq \sigma_0$. Under the sampling plan under consideration, and given a value for the producer's risk, say 0.05, one may be interested in knowing what value of $\sigma \geq \sigma_0$ will ensure the producer's risk less than or equal to 0.05. This value of $\sigma \geq \sigma_0$ is the smallest number $\sigma \geq \sigma_0$ for which $F\left(\frac{t}{\sigma_0}\frac{\sigma_0}{\sigma}\right)$ satisfies the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \geq 0.95 \tag{5}$$

For a given sampling plan $(n, c, t/\sigma_0)$ at specified confidence level P^* the minimum values of $\sigma \geq \sigma_0$ satisfying inequality (5) are computed and presented in table 3.

3. The Results and a Descriptive Example

The results are given in tables 1 - 3. Assume that an experimenter wants to establish that the true unknown average life is at least 1000 hours with confidence $P^* = 0.95$. The experiment will be stopped at $t = 942$ hours. Let the acceptance number be $c = 2$, then the required n from table 1 is 16. If during 942 hours no more than 2 failures out of 16 are observed then the experimenter can assert with confidence 0.95 that the average life is at least 1000 hours.

For the sampling plan $(n = 16, c = 2, t/\sigma_0 = 0.942)$ and $P^* = 0.95$ the operating characteristic values from table 2 are

σ/σ_0	2	4	6	8	10	12
O.C	0.89951	0.99714	0.99972	0.99995	0.99999	1

This means that if the true mean life is twice the specified mean life then the producer's risk is about 0.11 while it is about zero when the true mean life is about 6 or more times the specified mean life.

Table 3 can be used to get the value of $\sigma \geq \sigma_0$ for various choices of c, t/σ_0 in order that the producer's risk may not exceed 0.05. For example, the value of $\sigma \geq \sigma_0$ is 2.86 for $c = 2$, $t/\sigma_0 = 0.942$, $p^* = 0.95$. This means that the product should have an average life of 2.86 times the specified average life of 1000 hours in order that the product be accepted with probability 0.95.

4. Concluding Remarks

This paper deals with single sample plan for truncated life test assuming that the life time of the units follow the Rayleigh distribution. The producer's risk and the operating characteristics are investigated for large lot. However, the suggested sampling plan discussed in this paper can be extended by considering truncation in double sampling plan (see C. C. Craig 1968). Other extension can be made by considering repetitive sampling plan (Sherman 1965) , that is to say, for large sample size, count the defectives, d then

$$\begin{cases} \textit{accept the lot} & \textit{if} \quad d \leq c_1 \\ \textit{reject the lot} & \textit{if} \quad d > c_2 \\ \textit{repeat} & \textit{if} \quad c_1 < d \leq c_2 \end{cases}$$

if $c_1 = c_2$ then we have the single sampling plan discussed that in this paper. More investigation can be made by studying the efficiency comparison with other sampling plan such as sequential probability ratio plan.

Table 1. Minumum values of n necessary to ensure an average life exceeding σ_0 with probability P^* and an acceptance number c

		t/σ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	8	4	2	2	1	1	1	1
	1	15	7	4	3	2	2	2	2
	2	21	10	7	5	3	3	3	3
	3	28	14	9	6	4	4	4	4
	4	34	17	11	8	6	5	5	5
	5	41	20	13	10	7	6	6	6
	6	47	23	15	11	8	7	7	7
	7	53	26	17	13	9	8	8	8
	8	59	29	19	14	10	9	9	9
	9	66	32	21	16	11	10	10	10
	10	72	35	23	17	12	11	11	11
0.90	0	12	6	3	2	1	1	1	1
	1	21	10	6	4	3	2	2	2
	2	29	14	8	6	4	3	3	3
	3	36	17	11	8	5	4	4	4
	4	43	21	13	9	6	5	5	5
	5	50	24	15	11	7	6	6	6
	6	57	27	17	13	8	7	7	7
	7	64	31	19	14	10	8	8	8
	8	71	34	21	16	11	9	9	9
	9	77	37	24	17	12	10	10	10
	10	84	41	26	19	13	11	11	11
0.95	0	16	7	4	3	2	1	1	1
	1	25	12	7	5	3	2	2	2
	2	33	16	10	7	4	3	3	3
	3	41	20	12	9	5	4	4	4
	4	49	23	14	10	6	5	5	5
	5	56	27	17	12	8	6	6	6
	6	64	30	19	14	9	7	7	7
	7	71	34	21	15	10	9	8	8
	8	78	37	23	17	11	10	9	9
	9	85	41	25	18	12	11	10	10
	10	92	44	28	20	13	12	11	11
0.99	0	24	11	6	4	2	1	1	1
	1	35	16	9	6	4	3	2	2
	2	44	21	12	8	5	4	3	3
	3	53	25	15	10	6	5	4	4
	4	61	29	17	12	7	6	5	5
	5	70	33	20	14	9	7	6	6
	6	77	37	22	16	10	8	7	7
	7	85	40	25	18	11	9	8	8
	8	93	44	27	19	12	10	9	9
	9	100	48	29	21	13	11	10	10
	10	108	51	32	23	14	12	11	11

Table 2. Operating characteristic values for the sampling plan $(n, c, t/\sigma_0)$ for a given P^* when $c = 2$

			σ/σ_0					
P^*	n	t/σ_0	2	4	6	8	10	12
0.75	21	0.628	0.97786	0.99952	0.99996	0.99999	1	1
	10	0.942	0.95678	0.99897	0.9999	0.99998	1	1
	7	1.257	0.92497	0.998	0.99981	0.99996	0.99999	1
	5	1.571	0.87952	0.99635	0.99964	0.99993	0.99998	0.99999
	3	2.356	0.68698	0.98577	0.99846	0.9997	0.99992	0.99997
	3	3.141	0.33307	0.94016	0.99241	0.99846	0.99957	0.99985
	3	3.972	0.37604	0.94408	0.99283	0.99854	0.99959	0.99986
	3	4.712	0.17557	0.87474	0.98131	0.99596	0.99884	0.99959
0.90	29	0.628	0.94713	0.99869	0.99987	0.99998	0.99999	1
	14	0.942	0.92096	0.99788	0.99979	0.99996	0.99999	1
	8	1.257	0.88577	0.99662	0.99966	0.99994	0.99998	0.99999
	6	1.571	0.80538	0.99311	0.99929	0.99987	0.99996	0.99999
	4	2.356	0.49935	0.96865	0.99636	0.99928	0.9998	0.99993
	3	3.141	0.33307	0.94016	0.99241	0.99846	0.99957	0.99985
	3	3.972	0.10371	0.84047	0.97548	0.99465	0.99846	0.99946
	3	4.712	0.02142	0.68698	0.94013	0.98577	0.99573	0.99846
0.95	33	0.628	0.93116	0.99821	0.99983	0.99997	0.99999	1
	16	0.942	0.89951	0.99714	0.99972	0.99995	0.99999	1
	10	1.257	0.84074	0.99479	0.99947	0.9999	0.99997	0.99999
	7	1.571	0.7237	0.9886	0.99879	0.99977	0.99994	0.99998
	4	2.356	0.49935	0.96865	0.99636	0.99928	0.9998	0.99993
	3	3.141	0.33307	0.94016	0.99241	0.99846	0.99957	0.99985
	3	3.972	0.10371	0.84047	0.97548	0.99465	0.99846	0.99946
	3	4.712	0.02142	0.68698	0.94013	0.98577	0.99573	0.99846
0.99	44	0.628	0.86149	0.99569	0.99957	0.99992	0.99998	0.99999
	21	0.942	0.79639	0.99273	0.99925	0.99986	0.99996	0.99999
	12	1.257	0.73945	0.98961	0.99891	0.99979	0.99994	0.99998
	8	1.571	0.63969	0.98276	0.99811	0.99964	0.9999	0.99997
	5	2.356	0.3431	0.94464	0.99312	0.99861	0.99961	0.99987
	4	3.141	0.15182	0.87968	0.98283	0.99636	0.99896	0.99964
	3	3.972	0.02447	0.7125	0.94746	0.98775	0.99635	0.99869
	3	4.712	0.0022	0.49935	0.87963	0.96865	0.99015	0.99636

Table 3. Minimum ratio of true mean life to specified mean life for the acceptability of a lot with producer's risk of 0.05

		t/σ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	5.55	5.89	5.56	6.94	7.36	9.81	12.27	14.72
	1	2.84	2.85	2.78	2.92	3.32	4.42	5.52	6.63
	2	2.2	2.21	2.4	2.43	2.46	3.28	4.1	4.92
	3	1.96	2.01	2.07	1.98	2.09	2.78	3.48	4.17
	4	1.79	1.84	1.89	1.91	2.27	2.49	3.12	3.74
	5	1.71	1.72	1.77	1.85	2.07	2.3	2.88	3.45
	6	1.63	1.64	1.69	1.7	1.92	2.17	2.71	3.25
	7	1.57	1.58	1.62	1.68	1.82	2.06	2.58	3.09
	8	1.52	1.54	1.58	1.58	1.73	1.98	2.48	2.97
	9	1.5	1.5	1.54	1.58	1.66	1.92	2.39	2.87
	10	1.47	1.47	1.51	1.51	1.61	1.86	2.32	2.79
0.90	0	6.8	7.21	6.8	6.94	7.36	9.81	12.27	14.72
	1	3.38	3.45	3.49	3.47	4.37	4.42	5.52	6.63
	2	2.6	2.66	2.59	2.73	3.12	3.28	4.1	4.92
	3	2.24	2.24	2.34	2.4	2.58	2.78	3.48	4.17
	4	2.03	2.07	2.09	2.07	2.27	2.49	3.12	3.74
	5	1.9	1.91	1.94	1.98	2.07	2.3	2.88	3.45
	6	1.8	1.8	1.83	1.91	1.92	2.17	2.71	3.25
	7	1.74	1.75	1.74	1.78	2.03	2.06	2.58	3.09
	8	1.68	1.68	1.68	1.75	1.92	1.98	2.48	2.97
	9	1.63	1.63	1.68	1.66	1.84	1.92	2.39	2.87
	10	1.59	1.61	1.63	1.64	1.77	1.86	2.32	2.79
0.95	0	7.85	7.79	7.85	8.5	10.41	9.81	12.27	14.72
	1	3.69	3.79	3.8	3.94	4.37	4.42	5.52	6.63
	2	2.78	2.86	2.95	3	3.12	3.28	4.1	4.92
	3	2.39	2.45	2.46	2.59	2.58	2.78	3.48	4.17
	4	2.17	2.18	2.19	2.22	2.27	2.49	3.12	3.74
	5	2.01	2.04	2.09	2.1	2.33	2.3	2.88	3.45
	6	1.92	1.91	1.96	2.01	2.16	2.17	2.71	3.25
	7	1.83	1.84	1.86	1.87	2.03	2.42	2.58	3.09
	8	1.77	1.77	1.78	1.83	1.92	2.31	2.48	2.97
	9	1.71	1.73	1.72	1.73	1.84	2.22	2.39	2.87
	10	1.67	1.68	1.71	1.71	1.77	2.14	2.32	2.79
0.99	0	9.61	9.76	9.62	9.81	10.41	9.81	12.27	14.72
	1	4.38	4.4	4.35	4.37	5.2	5.83	5.52	6.63
	2	3.22	3.3	3.26	3.24	3.64	4.16	4.1	4.92
	3	2.73	2.77	2.79	2.76	2.97	3.43	3.48	4.17
	4	2.43	2.47	2.45	2.49	2.58	3.02	3.12	3.74
	5	2.26	2.28	2.3	2.32	2.57	2.75	2.88	3.45
	6	2.11	2.15	2.13	2.2	2.36	2.56	2.71	3.25
	7	2.01	2.02	2.06	2.11	2.21	2.42	2.58	3.09
	8	1.94	1.95	1.96	1.97	2.09	2.31	2.48	2.97
	9	1.87	1.89	1.88	1.92	1.99	2.22	2.39	2.87
	10	1.82	1.82	1.85	1.88	1.91	2.14	2.32	2.79

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