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Recent Advances in Burn-in[1)]

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Abstract

Burn-in is an engineering method which is used to eliminate early failures of products or systems after they have been produced. Recently, various models for determining optimal burn-in times have been developed, where some preventive maintenance policies were considered together with burn-in problem. In this paper, a survey of recent research in burn-in is undertaken.

Keywords : burn-in, maintenance policy, repair, costs

1. Introduction

Burn-in is a method used to eliminate early failures of components before they are put into field operation. The burn-in procedure is stopped when a preassigned reliability goal is achieved. Since burn-in is usually costly, one of the major problem is to decide how long the procedure should continue. The best time to stop the burn-in process for a given criterion is called the optimal burn-in time. An introduction to this context can be found in Jensen and Petersen (1982). In the literature, certain cost structures have been proposed and the corresponding problem of finding the optimal burn-in time has been considered. See, for example, Clarotti and Spizzichino (1991), Mi (1994), Cha (2000), Cha (2001), Cha (2003), Na and Son (2002) and Na and Lee (2002).

Let $F(t)$ be a distribution function of a lifetime X. If X has density $f(t)$ on $[0,\infty)$, then its failure rate function $h(t)$ is defined as $h(t)=f(t)/\overline{F}(t)$, where $\overline{F}(t)=1-F(t)$ is the survival function of X. Based on the behavior of failure rate, various nonparametric classes of life distributions have been defined in the literature. The following is one definition

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of a bathtub-shaped failure rate function which we shall use in this article.

Definition 1.

A real-valued failure rate function $h(t)$ is said to be bathtub-shaped failure rate with change points t_1 and t_2, if there exist change points $0 \le t_1 \le t_2 < \infty$, such that $h(t)$ is strictly decreasing on $[0, t_1)$, constant on $[t_1, t_2)$ and then strictly increasing on $[t_2, \infty)$.

The failure rate is often high in the initial phase. This can be explained by the fact that there may be undiscovered defects(known as "infant mortality") in the units; these may soon show up when the units are activated. When the unit has survived the infant mortality period, the failure rate often stabilizes at a level where it remains for a certain amount of time until it starts to increase as the unit begins to wear out. From the shape of the bathtub failure rate, the lifetime of a unit may be divided into three typical intervals: the burn-in(infant mortality) period $[0, t_1]$, the usual life period $[t_1, t_2]$, and the wear-out period $[t_2, \infty)$. It is widely believed that many products, particularly electronic products or devices such as silicon integrated circuits, exhibit bathtub shaped failure rate functions. This belief is supported by much experience and extensive data collection by practitioners and researchers in many industries.

In this article, we give a survey of recent burn-in research with emphasis on cost models. Throughout this paper, we assume that the failure rate function $h(t)$ is differentiable and bathtub-shaped with change points t_1 and t_2.

2. Age Burn-in Strategy

Mi (1994) considers the following burn-in procedure. Consider a fixed burn-in time b and begin to burn-in a new component. If the item fails before a fixed burn-in time b, replace it with shop replacement cost c_s, and continue the burn-in procedure for the replaced item. If the item survives the burned-in item b then the item is to be put into field operation. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0.

Now the average cost incurred until the first item surviving burn-in is obtained. Let $\{X_i, i \ge 1\}$ be an i.i.d. sequence of random variables distributed according to F and let $\eta - 1$ be the random variable which is the number of shop repairs until the first item surviving burn-in is obtained. Then the cost incurred by this burn-in procedure is given by

$$g(b) = c_0\left(\sum_{i=1}^{\eta-1} X_i + b\right) + c_s(\eta - 1),$$

where by convention $\sum_{i=1}^{\eta-1} X_i = 0$ when $\eta = 1$, and the mean cost $c_1(b) = E[g(b)]$ is given by(see also Mi (1994))

$$c_1(b) = E[g(b)] = c_0 \frac{\int_0^b \overline{F}(t)dt}{\overline{F}(b)} + c_s \frac{F(b)}{\overline{F}(b)} . \tag{1}$$

2.1 Age Replacement and Complete Repair

For the burned-in item we adopt age replacement policy described in Barlow and Proschan (1965). Under this policy, the item is replaced by a burned-in item at planned time T, where T is a fixed number, or at failure, whichever occurs first. Let c_f denote the cost incurred for each failure in field operation. We assume that $c_s < c_f$; this means that the shop replacement cost during a burn-in procedure is lower than the cost incurred for each failure in field operation. Let c_r satisfying $c_r < c_s$ be the cost incurred for each non-failed item which is replaced at age T in field operation.

If we use only items which have survived the burn-in time, then total expected replacement cost is the sum of the expected cost incurred by replacement at age T and the expected cost incurred by failure replacement before T :

$$c_2(T) = c_f F_b(T) + c_r \overline{F}_b(T), \tag{2}$$

where $\overline{F}_b(T)$ is the conditional survival function, i.e. $\overline{F}_b(T) = \overline{F}(b+x)/\overline{F}(b)$. The total expected cycle length is the expected length of a replacement;

$$T\overline{F}_b(T) + \int_0^T t f_b(t)dt = \int_0^T \overline{F}_b(t)dt \tag{3}$$

From (1), (2) and (3), the expected cost per unit time $C_{11}(b, T)$ is given by

$$C_{11}(b, T) = \frac{c_1(b) + c_f F_b(T) + c_r \overline{F}_b(T)}{\int_0^T \overline{F}_{b(T)}dt}$$

$$= \frac{c_1(b)\overline{F}(b) + c_f(\overline{F}(b) - \overline{F}(b+T)) + c_r \overline{F}(b+T)}{\int_b^{b+T} \overline{F}(T)dt} .$$

The problem of finding optimal (b^*, T^*) minimizing $C_{11}(b, T)$ was considered in Mi(1994).

2.2 Block Replacement and Minimal Repair

For a burned-in component, block replacement policy with minimal repair at failure will be adopted. Under this replacement policy, the component is replaced by a burned-in component at planned time T, where T is a fixed positive number, and is minimally repaired at failure before a planned replacement. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0. Let c_2 denote the costs of a minimal repair during field operation. Let c_r be the cost of a planned replacement.

Let $N_r(t)$ be the random variable denoting the number of replacements in $[b, b+t]$. Then the number of replacements in $[b, b+t]$ during field operation is $N_r(t)=[t/T]$ where $[x]$ is the largest integer which is not greater than x. Since the cost of each burned-in component is $g(b)$, the associated cost is $(N_r(t)+1)g(b)$. Hence the corresponding long-run average cost is

$$\lim_{t\to\infty}\frac{F(N_r(t)+1)g(b)}{t} = \lim_{t\to\infty}\frac{(N_r(t)+1)Eg(b)}{t} = \frac{1}{T}\left(c_0\frac{\int_0^b \overline{F}(t)dt}{\overline{F}(b)} + c_s\frac{F(b)}{\overline{F}(b)}\right) \quad (4)$$

since $N_r(t)=[t/T]$ is not random. The total expected cost incurred by replacement is the sum of the cost of a planned replacement, c_r, and the expected cost of minimal repairs, $c_2\int_b^{b+T} h(t)dt$,

$$c_3(T)=c_r+c_2\int_b^{b+T} h(t)dt. \quad (5)$$

where $\int_b^{b+T} h(t)dt$ is the expected number of minimal repairs during the interval $[b, b+t]$. Combining (4) and (5) the long-run average cost $C_{12}(b, T)$ is given by

$$C_{12}(b,T)=\frac{1}{T}\left(c_0\frac{\int_0^b \overline{F}(t)dt}{\overline{F}(b)} + c_s\frac{F(b)}{\overline{F}(b)} + c_r + c_2\int_b^{b+T} h(t)dt \right)$$

The problem of finding optimal (b^*, T^*)minimizing $C_{12}(b, T)$ was also considered in Mi(1994).

3. Block Burn-in with Minimal Repair

Cha (2000) and Na and Son (2002) consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, we repair it minimally, and continue the burn-in procedure for the repaired component. Note that the total burn-in time of this procedure is a constant b. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0. Let c_1 denote the cost of a minimal repair during burn-in.

Then the total expected cost incurred by burn-in is the sum of the cost for burn-in, $c_0 b$, and the expected cost of minimal repairs, $c_1 \int_0^b h(t)dt$,

$$c_4(b) = c_0 b + c_1 \int_0^b h(t)dt \,. \tag{6}$$

where $\int_0^b h(t)dt$ is the expected number of minimal repairs during the interval $[0, b]$.

3.1 Age Replacement and Complete Repair

After the fixed burn-in time b, the component is put into field operation. For a burned-in component we adopt age replacement policy with complete repair described in Section 2.1. Let c_f denote the cost incurred for each failure in field operation and c_r satisfying $0 < c_r < c_f$ the cost incurred for each non-failed item which is replaced at age $T > 0$ in field operation. If we use only items which have survived the burn-in time b, then total expected replacement cost and the total expected cycle length are given in (2) and (3), respectively.

From (2), (3) and (6) the long-run average cost $C_{21}(b, T)$ is given by

$$\begin{aligned} C_{21}(b, T) &= \frac{c_0 b + c_1 \int_0^b r(t)dt + c_f F_{b(T)} + c_r \overline{F}_b(T)}{\int_0^T \overline{F}_b(T)dt} \\ &= \frac{(c_0 b + c_1 \int_0^b r(t)dt\,)\overline{F}(b) + c_f(\overline{F}(b) - \overline{F}(b+T)) + c_r \overline{F}(b+T)}{\int_0^T \overline{F}(b+T)dt} \,. \end{aligned}$$

3.2 Block Replacement and Minimal Repair

For a burned-in component, block replacement policy with minimal repair at failure described in Section 2.1 will be adopted. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0. Let c_2 denote the cost of a minimal repair in field operation. We assume that $c_1 < c_2$; this means that the cost of a minimal repair during a burn-in procedure is lower than that of a minimal repair in field operation. Let c_r be the cost of a planned replacement.

The total expected cost in the interval $[0, b+T]$ is the sum of the expected burn-in cost in (6) and the expected replacement cost in (5). Thus, the total expected cost per unit time $C_{22}(b, T)$ is given by

$$C_{22}(b, T) = \frac{c_0 b + c_1 \int_0^b h(t)dt + c_r + c_2 \int_b^{b+T} h(t)dt}{T}.$$

The problem of finding optimal (b^*, T^*) minimizing $C_{22}(b, T)$ was considered in Cha(2000).

4. Burn-in When Minimal Repair Costs Vary With Time

Na and Lee (2002) consider that the cost of a minimal repair to the component which fails at age t is is a continuous nondecreasing function of t. Hence, as the component ages it becomes more expensive to perform minimal repair.

Suppose that the burn-in time is fixed as b and begin to burn-in a new component. If the component fails before burn-in time b, we repair it minimally, and continue the burn-in procedure with the repaired component. After the fixed burn-in time b, the component is put into field operation. For a burned-in component, block replacement policy with minimal repair at failure is adopted.

Let $N_x(y)$ be the random variable denoting the number of minimal repairs performed on the interval $[x, x+y]$. Then $N_x(y)$ has a Poisson distribution with parameter $H_x(y) = H(x+y) - H(x)$ where $H(t) = \int_0^t h(s)ds$. Let $C(t)$ be the cost of minimal repair to the component which fails at time t, where $C(t)$ is a continuous nondecreasing function of t. Then the expected minimal repair cost in the interval $[x, x+y]$ is

$$E\left(\sum_{i=1}^{N_x(y)} C(t_i) \right) = \int_x^{x+y} C(t)h(t)dt. \quad (7)$$

Let $C_1(t)$ and $C_2(t)$ denote the costs of a minimal repair which fails at time t during burn-in period and in field operation, respectively. We assume that $C_1(t) \le C_2(t)$ for all $t \ge 0$, then this means that the cost of a minimal repair during a burn-in procedure is lower than that of a minimal repair in field operation. Then from (7) the expected minimal repair costs during burn-in period $[0, b]$ and field operating time interval $[b, b+T]$ are given by

$$\int_0^b C_1(t)h(t)\,dt \quad \text{and} \quad \int_b^{b+T} C_2(t)h(t)\,dt,$$

respectively. Let c_r be the cost of a replacement and $c_0(b)$ be the cost for burn-in, where $c_0(b)$ is a continuous nondecreasing function of b. Hence the long-run average cost per unit time $C_3(b, T)$ is given by

$$C_3(b, T) = \frac{1}{T}\left(c_0(b) + \int_0^b C_1(t)h(t)dt + c_r + \int_b^{b+T} C_2(t)h(t)dt \right).$$

5. Burn-in Procedures for a Generalized Model

Cha (2002) consider two types of burn-in procedures for a general failure model. In this model, when the unit fails, Type I failure occurs with probability $1-p$ and Type II failure occurs with probability p, $0 \le p \le 1$. It is assumed that Type I failure is a minor one thus can be removed by a minimal repair or a complete repair(or a replacement), whereas Type II failure is a catastrophic one thus can be removed only by a complete repair. Two types of burn-in procedure are as follows.

- Burn-In Procedure I : Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then repair it completely regardless of the type of failure with shop complete repair cost c_s and then burn-in the repaired component again, and so on.
- Burn-In Procedure II : Consider a fixed burn-in time b and begin to burn-in a new component. On each component failure, only minimal repair is done for the Type I failure with shop minimal repair cost $0 < c_{sm} \le c_s$, and a complete repair is performed for the Type II failure with shop complete repair cost c_s. And then continue the burn-in procedure for the repaired component.

Note that the Procedure I stops when there is no failure during a fixed burn-in time $(0, b]$ at the first time, whereas the Procedure II stops when there is no Type II failure during a fixed burn-in time $(0, b]$ at the first time. In the field use the component is replaced by a new burned-in component at the 'field use age' T or at the time of the first Type II failure, whichever occurs first. For each Type I failure occurring during field use, only minimal repair is done.

Denote random variable X the lifetime of a component with the distribution function $F(t)$, density $f(t)$ and failure rate $r(t)$. Also denote random variable Y_b the time to first Type II failure of a burned-in component with fixed burn-in time b. If we define $G_b(t)$ as the distribution function of Y_b and $\overline{G}_b(t)$ as $1-G_b(t)$, then $\overline{G}_b(t)$ is given by

$$\overline{G}_b(t) = \exp\{-\int_0^t pr(b+u)du\},\ \forall\ t \ge 0. \tag{8}$$

Also define random variable Z_b as $\min\{Y_b, T\}$ then the expectation of Z_b is given by

$$E(Z_b) = \int_0^T \overline{G}_b(t)dt\ . \tag{9}$$

Let random variable $N(b;T)$ be the total number of minimal repairs of a burned-in component which occur during field operation under burn-in time b and replacement policy T. Then the unconditional expectation of $N(b;T)$ is given by

$$E[N(b;T)] = \left(\frac{1}{p}-1\right)\left(1-\exp\{-\int_0^t pr(b+u)du]\}\right). \tag{10}$$

First consider the Burn-In Procedure I and replacement model. The component is burned-in by the Burn-In Procedure I and is replaced by a new burned-in component at the 'field use age' T or at the time of the first Type II failure, whichever occurs first. Let c_f denote the cost incurred for each Type II failure in field operation and c_a satisfying $0 < c_a < c_f$ the cost incurred for each non-failed item which is replaced at field use age $T > 0$. And also denote c_m the cost of a minimal repair which is performed in field operation. Note that if $p=1$ then this burn-in and replacement model reduces to the case of Section 3.1 of Mi(1994) and if $p=0$ with $c_a = c_r$ it reduces to the case of Section 3.2 of Mi(1994). Then, by the results of (1), (9) and (10), the long-run average cost rate $C_{31}(b, T)$ is given by

$$C_{31}(b,T) = \frac{1}{\int_0^T \overline{G}_b(t)\,dt}\left[c_0\frac{\int_0^b \overline{F}(t)dt}{\overline{F}(b)} + c_s\frac{F(b)}{\overline{F}(b)}\right]$$
$$+ c_m\left[(\frac{1}{p}-1)(1-\exp -p[\Lambda(b+T)-\Lambda(b)])\right]$$
$$+ c_f G_b(T) + c_a \overline{G}_b(T).$$

Now consider the Burn-In Procedure II and replacement model. The component is burned-in by the Burn-In Procedure II and is replaced by a new burned-in component at the 'field use age' T or at the time of the first Type II failure, whichever occurs first.

And we observe that the long run average cost rate $C_{32}(b, T)$is given by

$$C_{32}(b,T) = \frac{1}{\int_0^T \overline{G}_b(t)\,dt}(\,[c_0\frac{\int_0^b \overline{G}(t)dt}{\overline{G}(b)} + c_s\frac{G(b)}{\overline{G}(b)}$$
$$+ c_{sm}(\frac{1}{p}-1)[\exp p\Lambda(b)-1\,]$$
$$+ c_m[(\frac{1}{p}-1)(1-\exp p[\Lambda(b+T)-\Lambda(b)])]$$
$$+ c_f G_b(T) + c_a \overline{G}_b(T)).$$

Cha (2003) extended this generalized burn-in and replacement model to the case in which the probability of Type II failure is time dependent.

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