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The Lived Space of Mathematics Learning: An Attempt for Change[1]

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Background Phenomenography suggests that more variation is associated with wider ways of experiencing phenomena. In the discipline of mathematics, broadening the "lived space" of mathematics learning might enhance students' ability to solve mathematics problems

Aims The aim of the present study is to:

1. enhance secondary school students' capabilities for dealing with mathematical problems; and
2. examine if students' conception of mathematics can thereby be broadened.

Sample 410 Secondary 1 students from ten schools participated in the study and the reference group consisted of 275 Secondary 1 students.

Methods The students were provided with non-routine problems in their normal mathematics classes for one academic year. Their attitudes toward mathematics, their conceptions of mathematics, and their problem-solving performance were measured both

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at the beginning and at the end of the year.
Results and conclusions Hierarchical regression analyses revealed that the problem-solving performance of students receiving non-routine problems improved more than that of other students, but the effect depended on the level of use of the non-routine problems and the academic standards of the students. Thus, use of non-routine mathematical problems that appropriately fits students' ability levels can induce changes in their lived space of mathematics learning and broaden their conceptions of mathematics and of mathematics learning.

Keywords: mathematics learning, conception of mathematics, mathematical beliefs, problem solving, open problems, approaches to mathematical problems, problem solving strategies, approaches to learning, Asian learners, mathematics education
ZDM Classification: D53
MSC2000 Classification: 97D50

BACKGROUND

Different efforts have been made to improve mathematics teaching and learning. At the turn of the Millennium, mathematics curriculum reforms have been taking place in almost all parts of the world, in which the enhancement of higher-order thinking and process abilities has been repeatedly addressed (Wong, Han & Lee 2004). The need for such curriculum change is also imminent in Hong Kong. Despite the outstanding performance in mathematics among Asian students (Wong 2004), their performance on non-routine problems remains dubious (see *e.g.*, Cai 1995). The Third International Mathematics and Science Study have also found that Hong Kong students performed less well in open problems and performance assessment tasks (Leung 2004; Wong 1995).

CONCEPTION OF MATHEMATICS AND MATHEMATICS LEARNING

Previous researches have supported that these may be related to the conceptions of mathematics among the students and the teachers alike. Their behaviors in learning and teaching mathematics are influenced by their (explicit or implicit) answers to the question "What is mathematics?," which is our operational definition of "conceptions of mathematics" in this paper (Grigutsch 1998; Pehkonen 1998; Zheng 1994).

Numerous studies have revealed that beliefs about mathematics as a discipline, beliefs about mathematics learning, beliefs about mathematics teaching, and beliefs about the self situated in a social context in which mathematics is taught and learned, all of which are components of conceptions of mathematics, are closely related to the students' motivation to learn mathematics and their performance in the subject (Cobb 1985; Crawford, Gordon, Nicholas & Prosser 1994, 1998a, 1998b; McLeod 1992; Pehkonen & Törner 1998; Underhill 1988). Indeed, students' conceptions are the key to understanding their actions (Wittrock 1986).

Research has repeatedly revealed that mathematics is regarded as a body of absolute truth (Fleener 1996), and also for some others, as a set of rules for playing around with symbols (Clay & Kolb 1983; Kloosterman 1991; McLeod 1992). Some 83% and 50% of the seventh-grade students participating in the Fourth U.S. National Assessment of Educational Progress respectively agreed or strongly agreed with the two statements "There is always a rule to follow in mathematics" and "Learning mathematics is mostly [memorizing]" (Dossey, Mullis, Lindquist & Chambers 1988). With such a conception, knowing mathematics, to these students, is done through memorization and algorithms, and learning is a transmission process (Underhill 1988).

CONCEPTION OF MATHEMATICS AND MATHEMATICS PROBLEM SOLVING

With the investigation of junior high school students, Frank (1988) came to the conclusion that, in students' eyes, (a) mathematics is computation; (b) mathematics problems should be solved in less than five steps (or else something is wrong with either the problem or the student); (c) the goal of doing a mathematical problem is to obtain the correct answer; (d) the role of the student is to receive mathematical knowledge and to demonstrate that it has been received; and (e) in the teaching-learning process, the student is passive while the teacher is active. Since there is always one correct way and/or rule to be followed to solve any mathematical problem, the task of the problem-solver is virtually the search of such a way/rule to solve the problem (Carpenter, Lindquist, Silver & Matthews 1983). Lester & Garofalo (1982) also found that third and fourth graders believed that mathematical problems could always be solved by using basic operations and in only a few minutes.

Schoenfeld (1989) investigated the relationship between students' conceptions of mathematics and their problem-solving behavior by using a set of 70 closed and 11 open questions administered to 230 Grade 10–12 mathematics students. Many students believed that "you must know certain rules, which are a part of all mathematics. Without knowing these rules, you cannot successfully solve a problem." Thus practices and memorization are important to learning. Students expect, or are expected, to master the subject "in bite-size bits and pieces" (Schoenfeld 1989, p. 344). Schoenfeld has called it a "rhetoric of mathematical understanding," and such an experience, year after year, has shaped students' conception of the subject. "Students come to expect typical homework and test problems to yield to their efforts in a minute or two, and most of them come to believe that any problem that fails to yield to their efforts in 1 to 2 minutes of work will turn out to be impossible" (Schoenfeld 1989, p. 348). In such a learning environment, students isolate school mathematics in its own compartment.

As mentioned above, such conceptions of mathematics can affect students' conceptions of mathematics learning and their approaches to solving mathematical problems. For instance, the conception of mathematics as a set of algorithms implies rote-memorization of facts, rules, and procedures of stereotypical problems as an appropriate learning strategy, resulting in a lower level of understanding (Confrey 1983; Peck 1984; Underhill 1988). Mathematics assessments have shown that failure to recognize a given problem-solving situation as mathematical also leads to unsuccessful problem solving (Brown et al. 1988a, 1988b; Kouba et al. 1988a, 1988b; Kouba & McDonald 1991).

CONCEPTIONS OF MATHEMATICS AND THE "LIVED SPACE" OF MATHEMATICS LEARNING

Naturally, such a classroom mathematics culture has been generated by a number of factors, among which how the teacher shapes the mathematics classroom is a prominent one. We call this the "*lived space*" of mathematics learning (N. Y. Wong, Marton, K. M. Wong, & Lam 2002). In fact, teachers' conceptions of mathematics are frequently reflected in their teaching acts (Pehkonen & Törner 1998; Shirk 1972; Thompson 1992). Those who see doing mathematics as following sets of procedures invented by others will provide students with little opportunity for making sense out of mathematics (Battista 1994). Some teachers think that students understand mathematics when they can successfully follow procedural instructions. For similar observations, Cobb et al. (1992) contrasted school mathematics with inquiry mathematics. They pointed out that the school, the society, and the textbooks enculturation students into the folk belief that it is impermissible to use any methods other than the standard procedures taught in school to solve school-like mathematical tasks, and the use of these procedures is the rational and objective way to solve mathematical tasks in any situation whatsoever (Lave 1988). The day to day experience of this "lived space", which is essentially shaped by the teacher (and the curriculum), influences students' conceptions of mathematics and how they approach mathematics problems. If most of the mathematical example and problems students meet in the daily mathematics classroom ("lived space") are routine, stereotyped and closed ended, they might tend to have a narrow conceptions of mathematics. As such, students might be less successful on non-routine and open problems.

BROADENING CONCEPTIONS OF MATHEMATICS AND ENHANCING PROBLEM SOLVING THROUGH THE SYSTEMATIC INTRODUCTION OF VARIATIONS

Despite the voluminous literature on students' conceptions of mathematics and their relations with mathematics performance, few studies have examined the relationship between students' conceptions of mathematics and how they actually tackle mathematical problems. Early in 1996, the authors' research team started the investigation of conceptions of mathematics under the general framework shown in Figure 1.

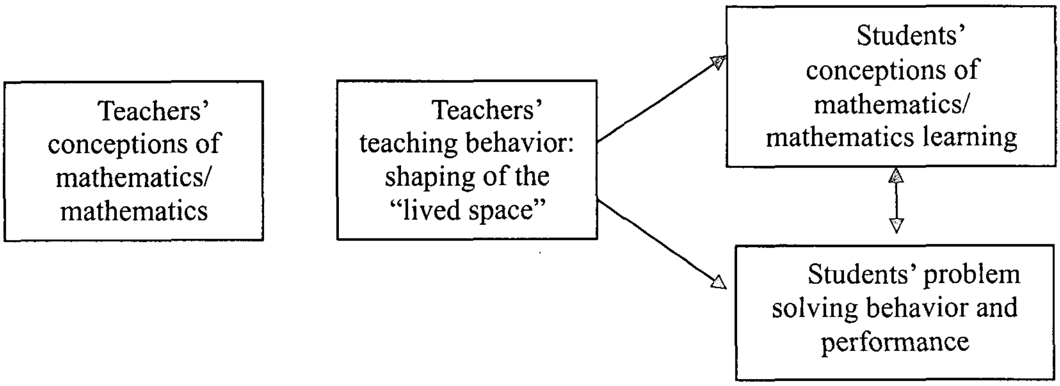

Figure 1. Our research model

The general picture we have obtained so far from this series of studies is that students' performance is closely related to their conceptions of mathematics, and Hong Kong students generally hold a relatively narrower conceptions of mathematics and mathematics learning. Teachers' conceptions of mathematics basically resemble those of the students although some teachers in Hong Kong hold a wider perspective. Inevitably, the conceptions of mathematics among students are both the antecedent and outcome of mathematics learning. If we see the "lived space" of mathematics learning as shaped by teachers, teachers' conceptions of mathematics may, in this sense, directly influence students' conceptions of mathematics. This in turn will affect students' problem-solving abilities and other learning outcomes of mathematics. In fact, it was revealed that an overwhelming portion of the mathematical problems given to Hong Kong students by their teachers is close-ended and stereotyped, demanding only low-level cognitive skills (see Lam, N. Y. Wong & K. M. P. Wong 1999; Wong 2001, 2002, 2004; Wong, Lam & Chan 2002; Wong, Marton et al. 2002). Thus, it is logical to hypothesize that students would become more capable mathematics problem-solvers if their "lived space" of mathematics learning were widened by systematically introducing variations. The aim of the present research is, therefore, to explore the effect of systematic introduction of more non-routine questions to students.

Revisiting the earlier findings of Marton & Booth (1997) and Bowden & Marton (1998) leads to the conclusion that a way of experiencing a phenomenon can be characterized in terms of those aspects of the phenomenon that are discerned and kept in focal awareness by the learner (see also Runesson 1999). Thus discernment is an essential element to learning, and variation is crucial to bringing about discernment, the lack of variation in the "lived space" of learning mathematics experienced by students would inevitably lead to a relatively narrow conception of mathematics (Marton, Watkins & Tang 1997). Furthermore, they would tend to hold a narrow conception of mathematics learning and would possess limited strategies when they are confronted with mathematical problems. So, we hypothesize that students' conceptions of mathematics would be broadened and their problem solving abilities enhanced if variation is systematically introduced into their "lived space."

To elaborate, the aim of the present study is to:

1. enhance secondary school students' capabilities for dealing with mathematical problems; and
2. examine if students' conception of mathematics can thereby be broadened,

by letting students, in the mathematics class, (a) engage with more varied and more open tasks; (b) explicate to the teacher and to their fellow students their ways of solving mathematical problems; and (c) compare and reflect on others' and their own ways of going about mathematical learning.

METHODOLOGY

Participants and procedure

The Secondary One[2] mathematics curriculum was the focus of our project on promotion to the secondary level; students could more easily accept new ways of doing

[2] Secondary One is equivalent to Grade 7 in American schools. In Hong Kong, after going through six years of primary education, students will continue their schooling at the secondary level, most probably in a different school.

mathematics. Furthermore, the concept of incorporating higher-order thinking abilities was in line with the new 2001 mathematics curriculum implemented in Hong Kong. A Secondary One class from each of ten schools took part in this study, among which 3 had lower-achievement students, 4 had medium-achievement students, and 3 had high-achievement students.

After having established shared views about the pedagogy of variation in the group of participating teachers through joint study activities and discussions, we designed mathematical tasks for the ten classrooms by considering the different ways in which students dealt with mathematics, discussed their views and strategies with one another, and their ways of reflecting on and learning from such discussions.

First, exemplars of non-routine mathematical problems were given to teachers, and they designed more of such problems afterwards. Non-routine mathematical problems are not frequently encountered by the students or found in standard textbooks. Such non-routine tasks may include mathematical problems that: (a) appear in unfamiliar formats; (b) have more than one answer; (c) allow openness in the solving process; (d) contain missing or irrelevant data; (e) can be solved by a variety of means (for instance, by algebraic means, geometric means, graphical methods, concrete objects, calculators, and micro-computers); or (f) involve problem-posing.

The problems provided include open problems (see Figures 2 and 3), problems found in overseas textbooks, problems found on the Internet, and even problems found in local textbooks that appear very different from those used in individual schools. Problem-posing here refers to both the formulation of new problems and the conversion of existing problems into new ones.

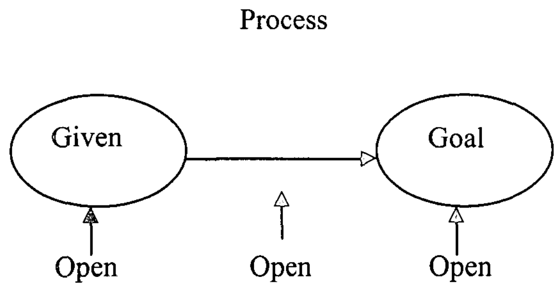

Figure 2. Definition of open problems (Becker 1998)

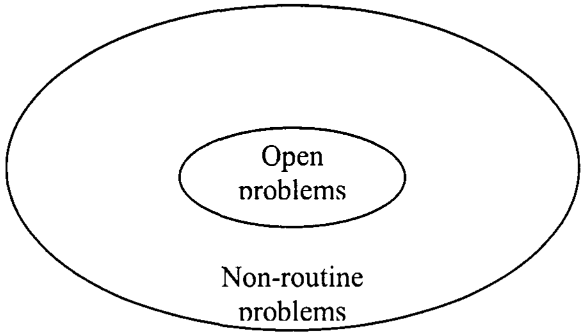

Figure 3. Relationship between Non-routine and Open Problems

Around half of all the tasks used in each classroom were taken from a common pool of tasks which were constructed by more teachers. These tasks were more varied and included a greater number of open problems than usual.

In sum, variation in the students' learning experience was produced through the openness of each task and the variation between tasks. This was achieved by: (a) the design of more open tasks by individual teachers; (b) shared tasks developed by different teachers; (c) reflection in sharing sessions within the teacher groups; or (d) sharing and discussion among the students of different solutions offered by others in the classroom. In this way, the students encountered not only different tasks authored by other teachers, but also different solutions offered by other students to the same problem. As a result, the "lived space" of variation of students' mathematical experience was broadened, as was that of the teachers.

At the end of the project, the teachers reported the extent of use of such non-routine problems, with low use in three classes (the number of non-routine problems is fewer than 10% of all the problems given to the class), medium use in four classes, and high use (the amount of non-routine problems is more than 30% of all the problems given to the class) in three classes.

Apart from the "experimental group," there was a "reference group" of eight other classes for the purpose of comparison. By and large, the same amount of time was used in all the classrooms for teaching mathematics. Furthermore, the textbooks used and the topics for consideration were similar. Background information regarding academic standards in general and those of mathematics in particular, attitude toward mathematics, and conceptions of mathematics has been collected for future analysis. The teachers in the reference group conducted their lessons as usual.

Scores obtained from different measures (see below) among all the students in the experimental and reference groups (comprising 410 and 275 students respectively) who agreed to participate were analyzed. The regression methods employed in this study accounted for the different numbers of students and classes through adjusted standard errors.

In classes of both groups, open problems were used to assess the cognitive learning outcomes of students at the start and toward the end of the academic year, with several conventional computation problems as warm ups.

Instruments

Conception of Mathematics Scale

The *Conception of Mathematics Scale*, a 5-point Likert scale, consists of 9 questions on "mathematics is a subject of calculable," 9 questions on "mathematics involves thinking," and 9 questions on "mathematics is useful." The scale was developed locally by grounded research, and prior application of the scale has yielded excellent reliability indices (Wong, Lam & Wong 1998).

Attitude Toward Mathematics

Aiken's (1974) *Mathematics Attitude Scale* comprises the four subscales of "enjoyment," "motivation," "importance of mathematics," and "free from fear." Each of these four subscales consists of 6 items. An additional subscale of "self-concept" consisting of 8 items was adapted for this purpose of measuring attitude. All the items were on a 5-point scale (from 1 to 5: strongly disagree, disagree, fairly agree, agree, and strongly agree).

Sample items of the above four subscales of *Mathematics Attitude Scale* include, respectively: "I have usually enjoyed studying mathematics in school," "I want to develop my mathematical skills and study this subject more," "Mathematics is a very worthwhile and necessary subject," and "I don't get upset when trying to work mathematics problems."

Minato's (1983) *Mathematics Semantic Differential* was used in the questionnaire to tap students' attitude toward mathematics. It consists of 14 bipolar statements on a 6-point response scale. Sample items include "School mathematics is (simple → complicated)," and "School mathematics is (beautiful → ugly)."

Problem-solving Items

Problem-solving items with openness in (a) the given information, (b) the goals to be achieved, or (c) the solving process (Becker 1998; Silver & Mamona 1989) were used in assessing students' performance in problem-solving. In our previous research, the following types of open problems were used: (a) problems with irrelevant information (Low & Over 1989); (b) "problematic word problems" (Verschaffel, Greer & DeCorte 2000); (c) problems allowing more than one solution; (d) problems allowing multiple methods; (e) problems with different interpretation of question possible; (f) problems asking for communication; (g) problems involving judgment; and (h) problems involving decision-making.

Those problems used in Cai (1995) and California State Department of Education (1989) were also taken into consideration. The scoring of open problems was developed according to the rubrics established in Cai, Lane and Jakabcsin (1996).

ANALYSIS

Hierarchical set, ordinary least squares regressions were used to model predictors of open problem solving scores at the end of the year (Cohen & Cohen 1983). Entering predictors in hierarchical sets allows the estimation of the portion of variance among student scores explained by each set of predictors. The sets of predictor variables were entered mostly in chronological order into a hierarchical regression. Predicting the outcome-variable post-intervention open problem score with the pre-intervention open problem score can give an effective estimation of the effects of later predictors on students' learning of open problem solving. Using the difference between the two scores assumes inevitably that the pre-intervention score has a unitary effect on post-intervention score (regression coefficient = 1).

Student gender is entered next as it is not affected by any of the later variables. Students' average academic standards of schools exist prior to the type of intervention, so it is entered before the intervention type. Next, types of intervention are entered. Lastly, interaction terms between the school standard and intervention type are entered. It should be noted that some combinations of school standard and intervention type did not occur, and in this case, only the occurring combinations are entered as interaction terms.

A nested hypothesis test (χ^2 log likelihood) was used to check whether each added set of variables was significant (Judge, Griffiths, Hill, Lutkepohl & Lee 1985, pp. 184–187). Non-significant variables were removed from each set to reduce the risk of Type I error. The results include not standardized coefficients to help readers understand the direct effects of predictors on post-intervention scores. A Wald χ^2 test was used to test for significant differences between coefficients (Davidson & MacKinnon 1993, p. 278).

RESULTS

Descriptive Statistics

Reliability Indices

The reliability indices of the instruments were calculated and shown in Table 1.

Table 1. The Cronbach alpha reliability indices of the instruments used in the pre-test and post-test

Subscales	Experimental group		Reference group	
	Pre-test	Post-test	Pre-test	Post-test
Enjoyment	0.88	0.88	0.80	0.82
Motivation	0.83	0.86	0.77	0.80
Importance	0.78	0.78	0.76	0.78
Free from fear	0.89	0.89	0.81	0.83
Self-concept	0.92	0.91	0.89	0.88
Calculable	0.66	0.70	0.70	0.65
Thinking	0.69	0.82	0.78	0.79
Useful	0.89	0.87	0.85	0.88
Attitude	0.94	0.94	0.92	0.94

The results show that the Cronbach alpha fell between 0.66 and 0.94, indicating that the instruments had satisfactory internal consistencies. Also, the instruments were generally stable, except for those on conceptions of mathematics, particularly when they were administered to the reference group.

Pre-test and Post-test Scores of the Experimental Group

The pre-test and post-test scores on conceptions of mathematics, attitude toward mathematics, and problem-solving performances were calculated and listed in Tables 2 to 4 respectively.

Table 2. Comparisons of pre-test and post-test scores on the conceptions of mathematics scores of the experimental group by paired *t*-test

Subscales	Pre-test		Post-test		*t* value
	M	*SD*	*M*	*SD*	
Calculable	29.00	5.07	27.33	5.22	5.81*
Thinking	35.25	5.30	34.23	5.82	3.29*
Useful	23.56	6.31	22.23	6.28	4.09*

$N \geq 380$; * $p < .05$

Table 3. Comparisons of pre-test and post-test scores on the attitude toward mathematics scores of the experimental group by paired *t*-test

Subscale	Pre-test		Post-test		*t* value
	M	*SD*	*M*	*SD*	
Enjoyment	1.54	5.26	1.05	5.26	2.18*
Motivation	3.32	4.57	2.29	4.86	4.85*
Importance	4.39	4.08	3.84	4.18	2.52*
Free from fear	1.84	5.79	0.95	5.68	3.80*
Self-concept	22.87	7.27	21.39	6.96	4.68*
Attitude	29.72	19.10	26.12	19.62	4.64*

$N \geq 382$; * $p < .05$

Table 4. Comparisons of pre-test scores on computational and open problems of the experimental group by paired *t*-test

Subscales	Pre-test		Post-test		*t* value
	M	*SD*	*M*	*SD*	
Computation Problems	1.14	0.64	0.73	0.72	9.45*
Open Problems	5.83	3.59	5.91	2.79	–0.41

$N \geq 380$; * $p < 0.05$

Their scores were also compared by paired *t*-test. The results revealed that there were significant differences between the pre-test and post-test scores in all the measures of conceptions of mathematics.

Hierarchical Regression Models

Here, we concentrate on students' performance on open problems. As expected, students with higher scores on the open problems before the intervention scored on average 0.31 points higher on the open problems after the intervention (see Table 5, Model 1). The results showed that girls tended to learn to do more open problems than boys, improving 0.62 points more on average (see Model 2). A small part of this effect is due to their greater enrollment in schools with higher academic standards (coefficient drops from 0.620 to 0.585, see Models 2 and 3). Students in schools with higher academic standards improved on average 0.68 points more than students in schools with low or medium academic standards (see Model 3). The amount of teacher's self-reported use of open problems showed a non-linear effect (see Model 4). Students tended to solve more open problems if the teacher reported low or medium use of open problems in their classes.

However, there are interaction effects among school standard and use of open problems that Model 4 does not consider. Model 5 shows that students who receive no instruction in open problems do not differ, regardless of the academic standards of their schools. Students in schools with high academic standards benefit from high and medium uses of open problems, scoring 2.9 and 1.6 points higher on average than students who receive no instruction (Wald tests show no significant differences between high and medium use, $\chi^2(1) = 2.56$, $p = 0.11$). In contrast, students in other schools benefit from low and medium uses of open problems in class (Wald tests show no significant differences between low and medium use, $\chi^2(1) = 2.23$, $p = 0.14$). The benefits of medium use of open problems showed no significant differences except that students in schools with high academic standards and with high exposure to open problems scored higher than students in the same type of schools who received low exposure to open problems (Wald test $\chi^2(1) = 4.23$, $p = 0.04$). Together, these variables explain over 26% of the variance among students' scores, with the open problem interventions explaining 9%.

Table 5. Summary results of five regression models predicting students' scores on open problems after the intervention.

Predictor	5 OLS regression models predicting post-open score									
	Model 1		Model 2		Model 3		Model 4		Model 5	
C	3.537 (0.180)	***	3.236 (0.206)	***	3.231 (0.219)	***	2.720 (0.230)	***	2.847 (0.240)	***
Pre-test score scores on open problems	0.311 (0.029)	***	0.305 (0.028)	***	0.288 (0.029)	***	0.231 (0.029)	***	0.217 (0.030)	***
Girl			0.620 (0.204)	**	0.585 (0.208)	**	0.345 (0.205)		0.336 (0.206)	
Hi_standard					0.681 (0.286)	*	0.834 (0.308)	**	–0.324 (0.470)	
Med_standard					–0.081 (0.237)		0.414 (0.293)		0.649 (0.337)	
Hi_use							0.264 (0.308)		–0.446 (0.370)	
Med_use							1.935 (0.278)	***	1.790 (0.320)	***
Low_use							1.246 (0.307)	***	1.064 (0.367)	**
Hi_standard × Hi_use									2.856 (0.709)	***
Hi_standard × Med_use									1.620 (0.679)	*
Hi_standard × Low_use									1.160 (0.696)	
R-squared	0.154		0.165		0.176		0.243		0.263	

Note. Standard errors are in parentheses; *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

DISCUSSION

The above results revealed that the systematic introduction of variation, *i. e.*, of non-routine problems, into students' "lived space" generally improves their open problem solving abilities, but the appropriate amount depends on their school standard. Students in high standard schools benefit from high and medium doses of non-routine problems, and low dose may not be sufficient. In contrast, students in the low standard schools benefit from medium and low doses rather than high dose. Undoubtedly, it is not sensible to think that the greater extent one implements a certain initiative (no matter what teaching/learning initiative it is), the greater will be the effect. Most likely, there could be an optimal level of dosage. Not only that, there could be individual differences too. The above results precisely reveal that the most effective extent of use varies with different academic standards of the students. For students in schools with low or medium standards, introducing too many non-routine problems could hinder their learning.

This is reflected in interviews with students participating in this study too (Lam, N. Y. Wong & K. M. P. Wong 2003). The teachers from five participating schools of diverse academic standards were asked to identify three students who had good, average and poor performance in mathematics respectively from their own class for the interview. It was found that most students who were not so good at mathematics disliked non-routine problems, and their interest in learning mathematics even declined after the experimental phase. The interview has shown that the non-routine problems were very challenging to the less able students, and even created frustration among them. This negative feeling, if prolonged, could become a demotivating force. Martino (2003) also remarks that "The student must understand that the new information is in contradiction with the belief, believe that it's necessary to overcome the contradiction and want to resolve the contradiction" (p. 6) before their attitude or belief can actually be changed. The same is also true for one who wants to change students' receptiveness of non-routine problems. In other words, students need to overcome a "cultural shock" of non-routine problems before really benefiting from them.

Compared to boys, girls tend to learn more about solving non-routine problems. With many studies on gender and mathematics in the last decades, the perceptions that boys learn better than girls still lack grounds. Rather, girls tend to run away from mathematics due to the fact that mathematics is socially shaped as a "male domain" (Leder 1992). However, it has been found that males hold more functional conceptions about themselves as learners (Leder, Forgasz & Solar 1996), and girls tend to use concrete solution strategies while boys, abstract solution (Fennema, Carpenter, Jacobs, Franke & Levi 1998). In the present study, students were not just asked to solve non-routine problems all at once. They were given prior exposure which might be more tangible among the girls.

In general, the present study has demonstrated that by familiarizing students with a more diverse variety of mathematical problems in their day-to-day classroom learning, students' mathematical problem-solving abilities were generally enhanced. They were found more capable of solving non-routine problems, a capability closely related to higher-order thinking. To optimize the learning outcome, the extent of such practice with non-routine problems can be adjusted according to the academic standards of the students. Students of higher mathematics achievement can benefit more from encountering more non-routine problems, and the reverse is true for students of lower achievement. In sum, the problem solving abilities of the students who took part in the project were enhanced as a result of a systematic introduction of variation, though how far such a variation should be introduced needs careful adjustment according to students' characteristics.

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